

n -Queens

This paper contains references to the well-known n -queens problem. It is available through:

<http://www.liacs.nl/home/kosters/nqueens.html>

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