

Answers Homework 3

1) Optimise the following Boolean functions by finding all the prime implicants and essential prime implicants and applying the selection rule. After you have optimized the functions, implement it using the corresponding logic gates.

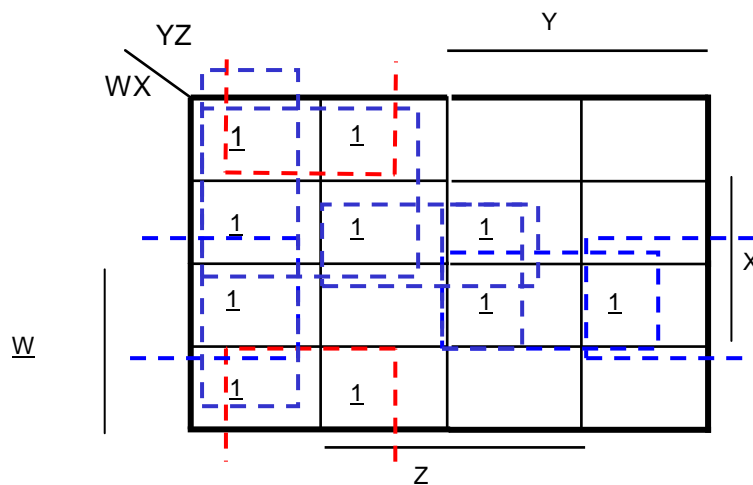
a) $F(W,X,Y,Z) = \sum m(0,1,4,5,7,8,9,12,14,15)$
 (draw it with AND,OR, NOT gates)

b) $F(W,X,Y,Z) = \sum m(1,5,6,7,11,12,13,15)$
 (draw it with NAND gates)

c) $F(W,X,Y,Z) = \sum m(0,2,3,4,5,7,8,10,11,12,13,15)$
 (draw it with NOR gates)

For convenience I will show the prime and essential implicants. In the tables the prime implicants are drawn in blue and the essential ones in red. (of course all essential prime implicants are also prime implicants). The simplified functions do not have to be unique! (but have to be minimal)

a)



Our prime implicants are:

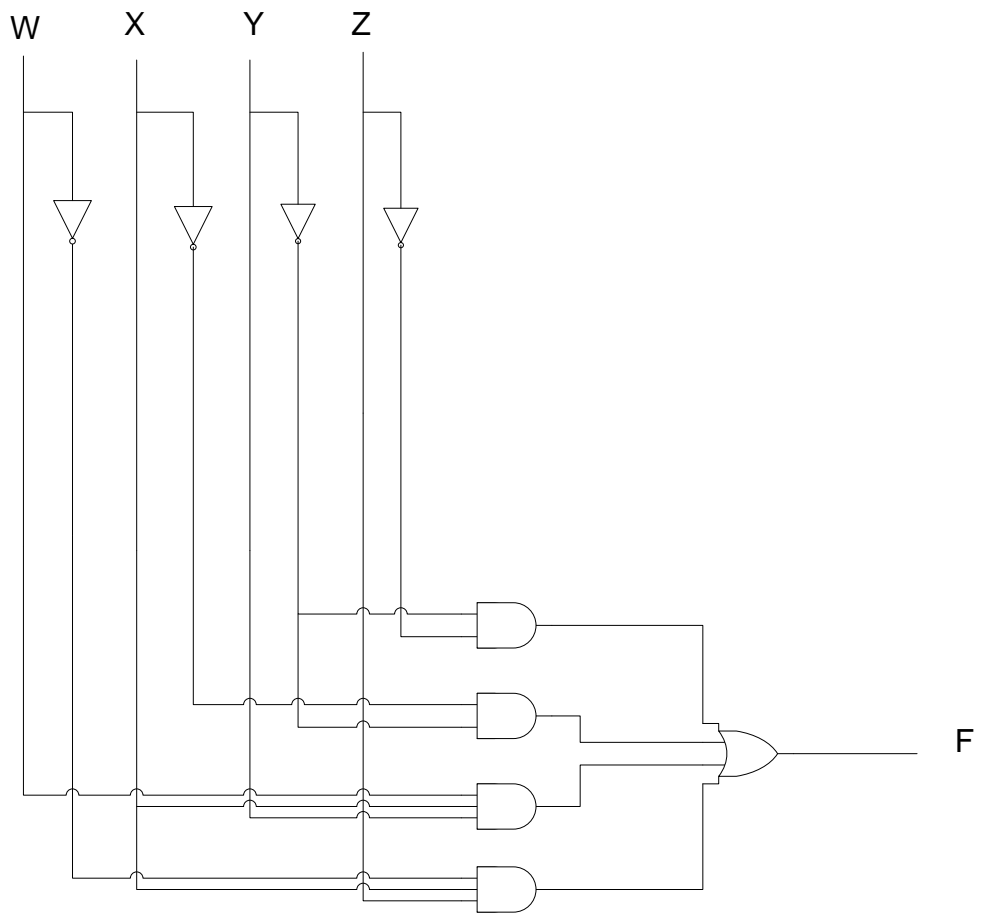
$$W'Y', W'XZ, XYZ, Y'Z', X'Y', WXY, WXZ'$$

Our essential prime implicants are:

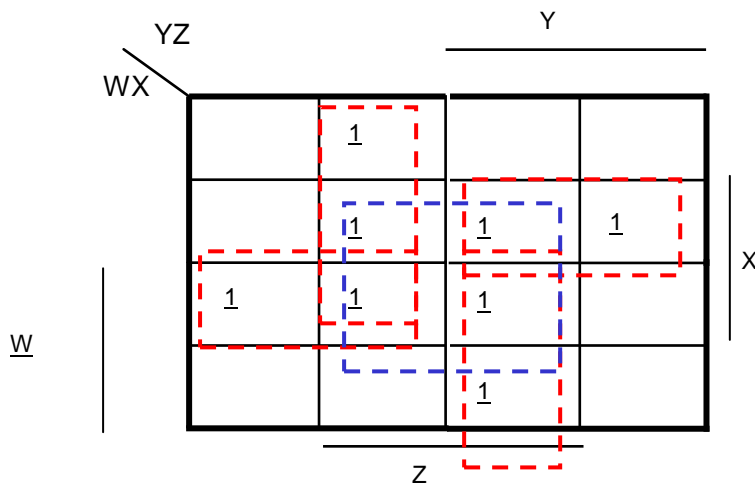
$$X'Y'$$

Our simplified function is:

$$F(W,X,Y,Z) = Y'Z' + X'Y' + WXY + W'XZ$$



b)

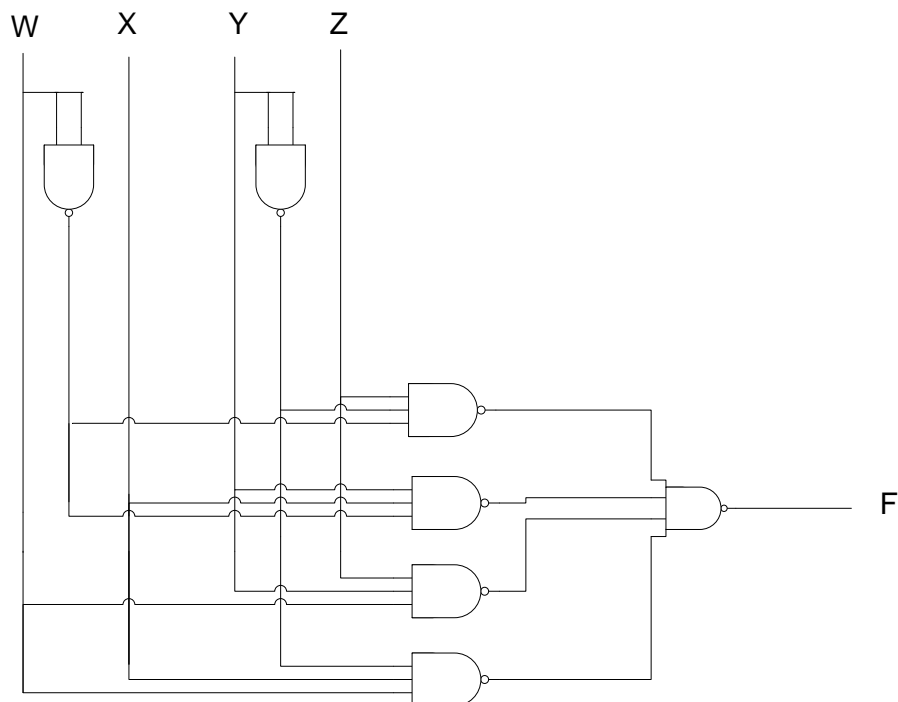


Our prime implicants are:
 $ZX, W'Y'Z, W'XY, WYZ, WXY'$

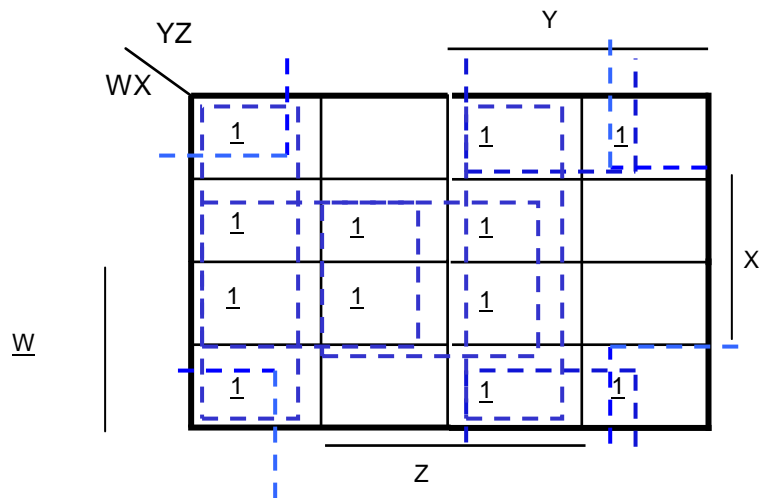
Our essential prime implicants are:
 $W'Y'Z, W'XY, WYZ, WXY'$

Our simplified function is:
 $F(W,X,Y,Z) = W'Y'Z + W'XY + WYZ + WXY'$

Now we convert to NAND:
 $F = ((W'Y'Z + W'XY + WYZ + WXY'))'$
 $= ((W'Y'Z)'(W'XY)'(WYZ)'(WXY'))'$



c)



Our prime implicants are:

$X'Z'$, $X'Y$, YZ , $Y'Z'$, XZ , XY'

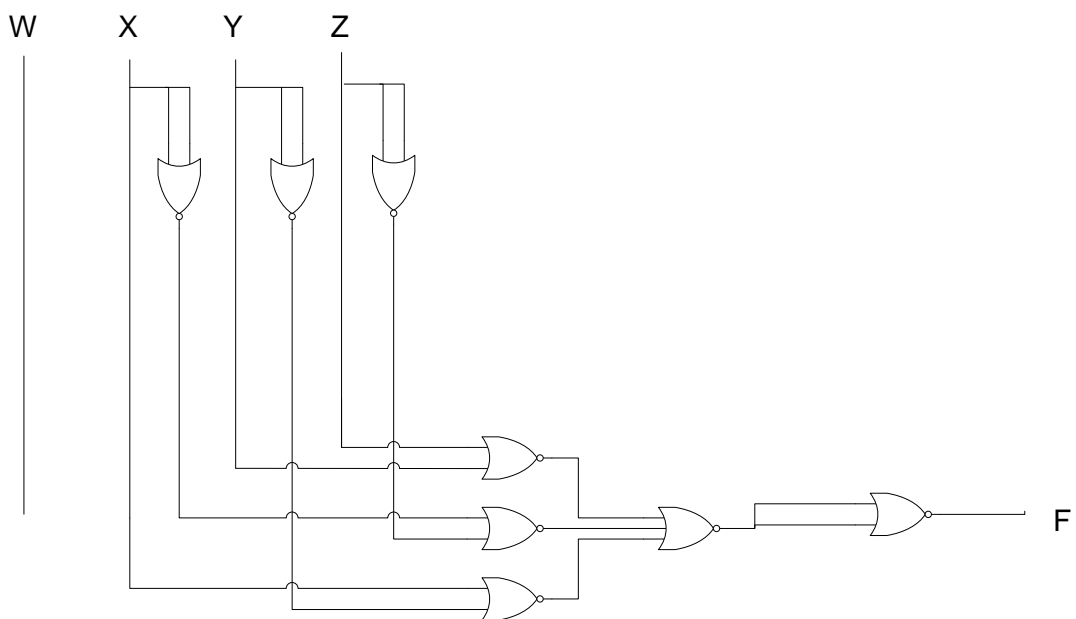
None of the prime implicants is essential!!!

The resulting simplified function is

$$F(WXYZ) = Y'Z' + XZ + YX'$$

Now we convert to NOR:

$$\begin{aligned} F &= ((Y'Z')')' + ((XZ)')' + ((YX')')' \\ &= (Y + Z)' + (X' + Z')' + (Y' + X)' \\ &= (((Y + Z)' + (X' + Z')' + (Y' + X)')')' \end{aligned}$$



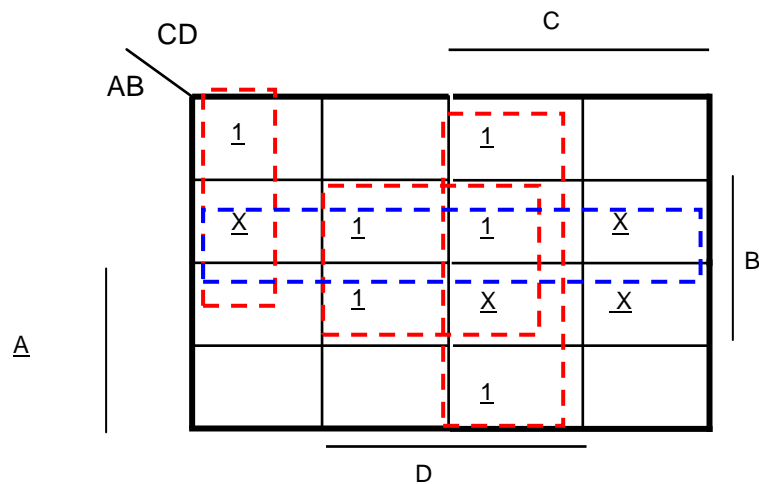
2) The same as task 1, only now we also have don't care conditions d.

a) $F(A,B,C,D) = \sum m(0,3,5,7,11,13)$, $d(A,B,C,D) = \sum m(4,6,14,15)$
(AND,OR,NOT)

b) $F(A,B,C,D) = \sum m(0,6,8,13,14)$, $d(A,B,C,D) = \sum m(2,4,7,10,12)$
(NAND)

c) $F(A,B,C,D) = \sum m(0,1,2,4,5)$, $d(A,B,C,D) = \sum m(3,6,7)$
(NOR)

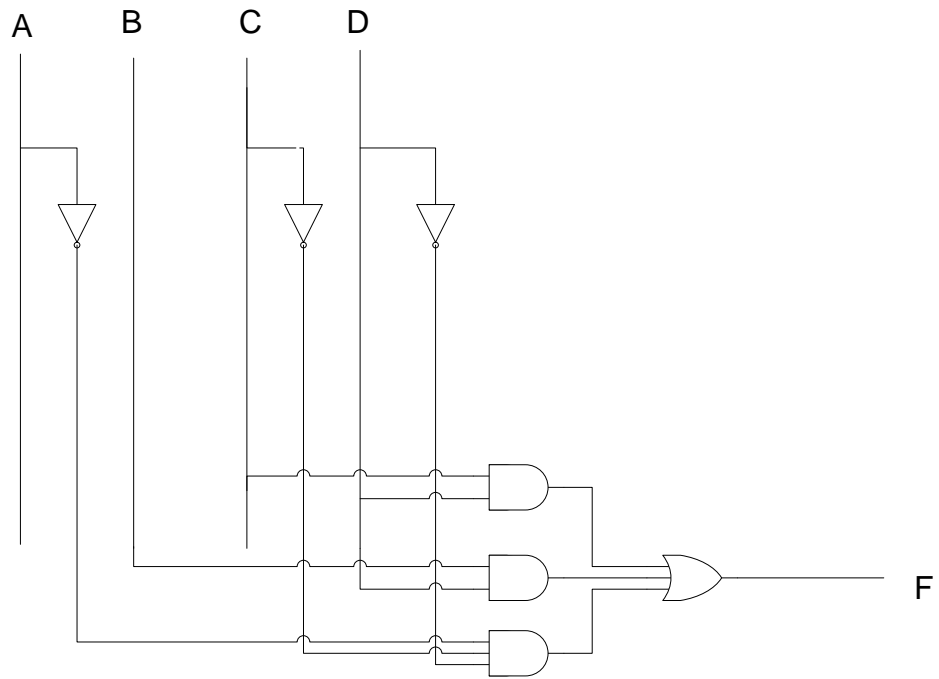
a)



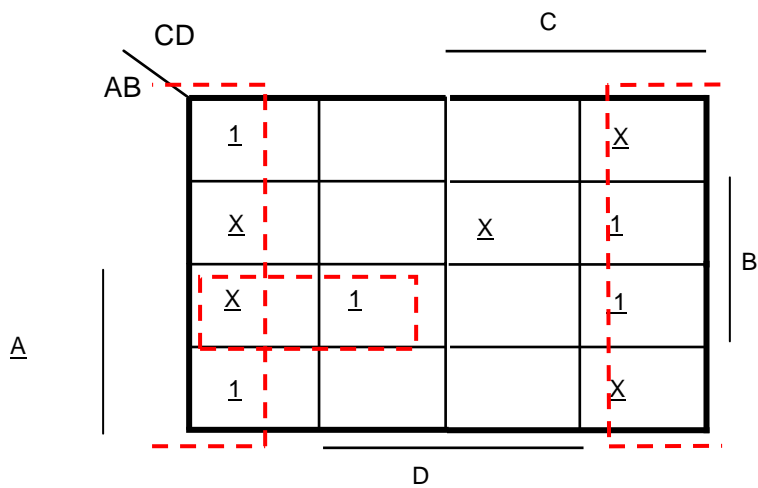
Our prime implicants are:
CD, BD, A'B, A'C'D'

Our essential prime implicants are:
CD, BD, A'C'D'

Our simplified function is:
 $F(A,B,C,D) = CD + BD + A'C'D'$



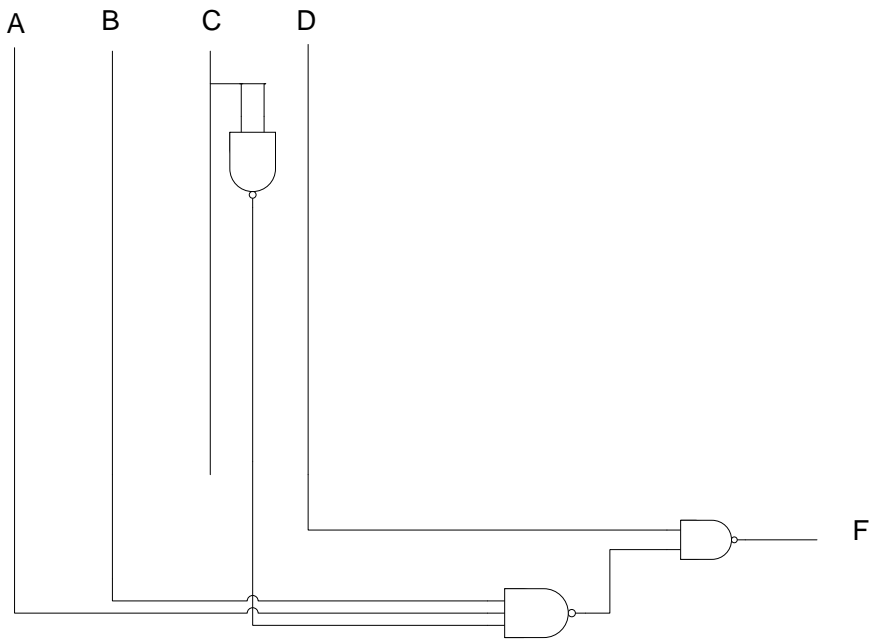
b)



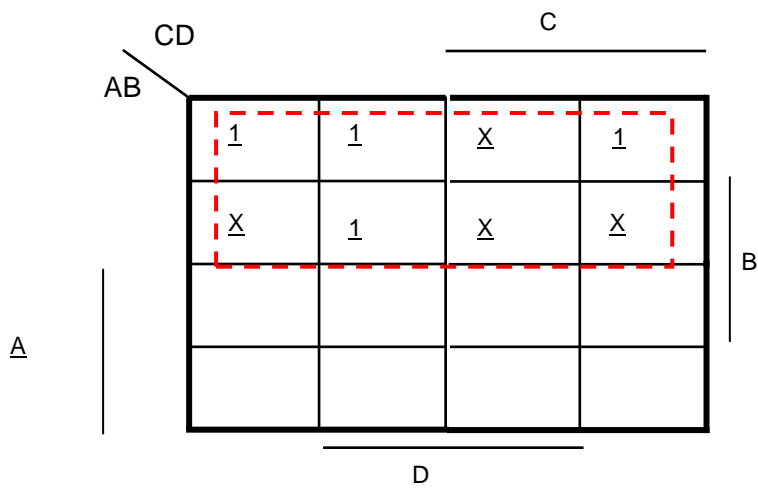
Our (essential) prime implicants are:
 D' , ABC'

Our simplified function is:
 $F(A,B,C,D) = D' + ABC'$

Now convert to NAND:
 $F = ((D' + ABC')')'$
 $= (D(ABC'))'$



c)



Our (essential) prime implicants is
 A'

Our simplified function is
 $F(A,B,C,D) = A'$

Now convert to NOR (you don't have to do anything actually)
 $F = (A + A)'$

