Answers Homework 2

Task1
Prove the equality of the following Boolean expressions:

1) $AB + BC'D' + A'BC + C'D = B + C'D$

2) $WY + W'YZ' + WXZ + W'XY' = WY + W'XZ' + X'YZ' + XY'Z$

3) $AC' + A'B + B'C + D' = (A' + B' + C' + D')(A + B + C + D')$

Answer:

1) If you had trouble figuring out how to do this cleverly, don't worry because it's a hard problem to solve with reductions. Instead you can do it using the conversion of SOP from standard to canonical form. You transform both sides of the equation into canonical SOP and because every function has a unique canonical SOP you can prove the functions equal if both sides have the same canonical SOP.

Left side:

- $AB + BC'D' + A'BC + C'D$
- $AB(C'+C)(D'+D) + (A'+A)BC'D' + A'BC(D'+D) + (A'+A)(B'+B)C'D$
- $ABC'D' + ABC'D + ABCD' + ABCD + A'BC'D' + ABC'D' + A'BCD' + A'BCD + A'B'C'D + A'BC'D + AB'C'D + ABC'D$
- Now remove the duplicates
- $ABC'D' + ABC'D + ABCD' + ABCD + A'BC'D' + A'BCD' + A'BCD + A'B'C'D + A'BC'D + AB'C'D$

Right side:

- $B + C'D$
- $(A + A')B(C + C')(D + D') + (A + A')(B + B')C'D$
- $A'BC'D' + A'BC'D + A'BCD' + A'BCD + ABC'D' + ABC'D + ABCD' + ABCD + A'B'C'D + A'BC'D + AB'C'D + ABC'D$
- Now remove the duplicates
- $ABC'D' + ABC'D + ABCD' + ABCD + A'BC'D' + A'BCD' + A'BCD + A'B'C'D$ $A'BC'D + AB'C'D$

Now the proof is complete, note that both sides are the same so the equation is true.

2) see 1
3) see 1

Task2
Find the complement of the following expressions

1) $F1 = AB' + A'B$

2) $F2 = (V'W + X)Y + Z'$

3) $F3 = WX(Y'Z + YZ') + W'X'(Y'+Z)(Y + Z')$

4) $F4 = (A + B' + C)(A'B' + C)(A + B'C')$

Answer:

1) we apply the Demorgen's theorem or we can find the dual form and then complement the literals. For the examples below we apply the DeMorgans theorem, although the other approach is simpler.

- $(AB' + A'B)'$
- $(AB')'(A'B)'$
- $(A' + B)(A + B')$

2)

- $((V'W + X)Y + Z')'$
- $((V'W + X)Y)'Z$
- $((V'W + X)' + Y')Z$
- $((V'W)' X' + Y')Z$
- $((V + W') X' + Y')Z$

3)

- $(WX(Y'Z+YZ')+W'X'(Y'+Z)(Y+Z'))'$
- $(WX(Y'Z+YZ'))'(W'X'(Y'+Z)(Y+Z'))'$
- $(W'+X'+(Y'Z+YZ')')(W+X+(Y'+Z)'+(Y+Z')')$
- $(W'+X'+(Y'Z)'(YZ')')(W+X+YZ'+Y'Z)$
- $(W'+X'+(Y+Z')(Y'+Z))(W+X+YZ'+Y'Z)$

4)

- ((A+B'+C)(A'B'+C)(A+B'C'))'
- (A+B'+C)'+(A'B'+C)'+(A+B'C')'
- A'BC'+(A'B')'C'+A'(B'C')'
- A'BC'+(A+B)C'+A'(B+C)

Task3

Convert the following expressions into canonical SOP and Canonical POS

1) $F1 = (AB + C)(B + C^{'}D)$
2) $F2 = X^{'} + X(X + Y^{'})(Y + Z^{'})$
3) $F3 = (A + BC^{'} + CD)(B^{'} + EF)$

1) First we simplify the function so That we can get it in the standard SOP form.

- (AB+C)(B+C'D)
- ABB+ABC'D+CB+CC'D distribution
- AB+ABC'D+CB using idepotence and complement we get
- AB+CB absorption

Now we are ready to convert to canonical SOP

- AB(C'+C)(D'+D)+(A'+A)BC(D'+D)
- ABC'D'+ABC'D+ABCD'+ABCD+A'BCD'+A'BCD+ABCD'+ABCD
- ABC'D'+ABC'D+ABCD'+ABCD+A'BCD'+A'BCD

∑M(6,7,12,13,14,15)

from SOP we can convert to POS

∏M(0,1,2,3,4,5,8,9,10,11)

2) I will only show the needed reduction

- $X' + X(X + Y')(Y + Z')$
- $X' + XX(Y + Z') + XY'(Y + Z')$ distribution
- $X' + X(Y + Z') + XY'(Y + Z')$ idepotence
- $X' + XY + XZ' + XY'Y + XY'Z'$ distribution
- $X' + XY + XZ' + XY'Z'$ complement

This should be enough for you to convert to SOP and POS

3) I will only show the needed reduction

- $(A + BC' + CD)(B' + EF)$
- $AB'+AEF+BC'B'+BC'EF+CDB'+CDEF$ distribution
- $AB'+AEF+BC'EF+CDB'+CDEF$ complement

This should be enough for you to convert to SOP and POS