Theorie van Concurrency

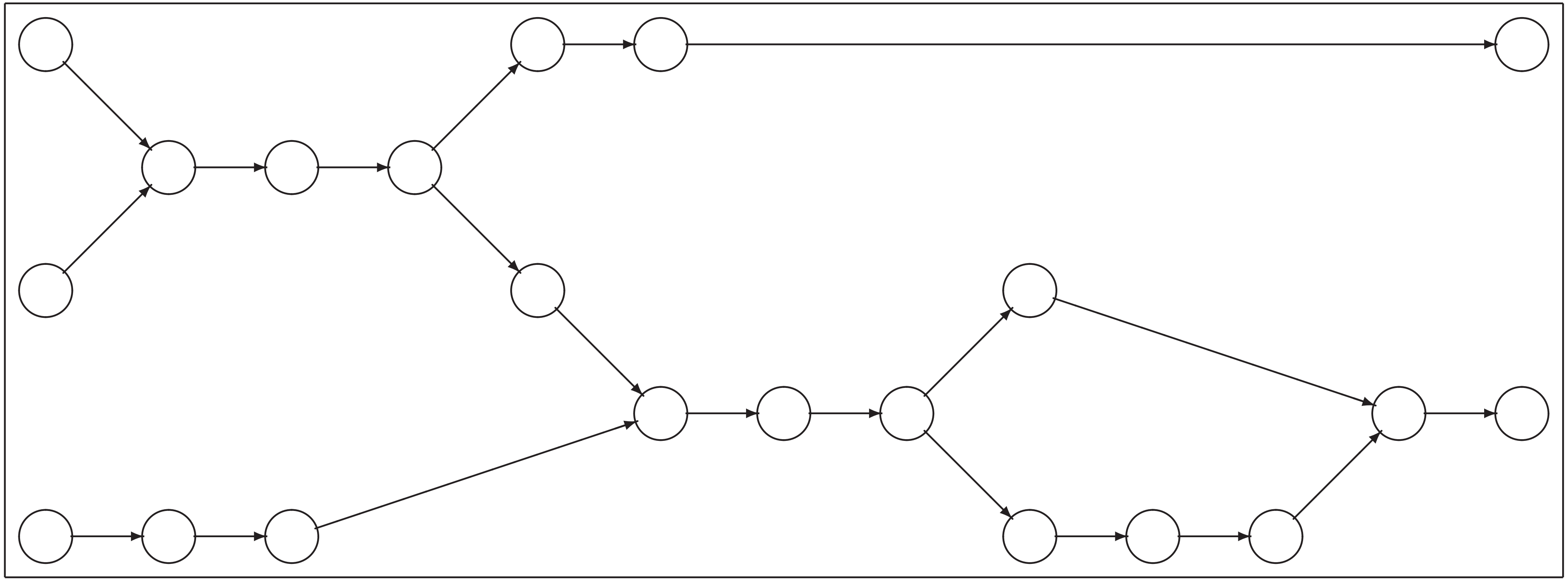
najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

negende college: 13 oktober 2011

6. Processes

6.2 Process Nets



Graphical representation of a partially ordered set (A, ρ) (except for . . .).

$\mathbf{li}_\rho$, $\mathbf{co}_\rho$, cliques, lines, cuts, ${}^\circ A$, A°

Definition 73. A net $N = (P, T, F)$ is a *process net* if:

(1) N is acyclic, and

(2) $\#({}^\bullet p) \leq 1$ and $\#(p^\bullet) \leq 1$ for all $p \in P$.

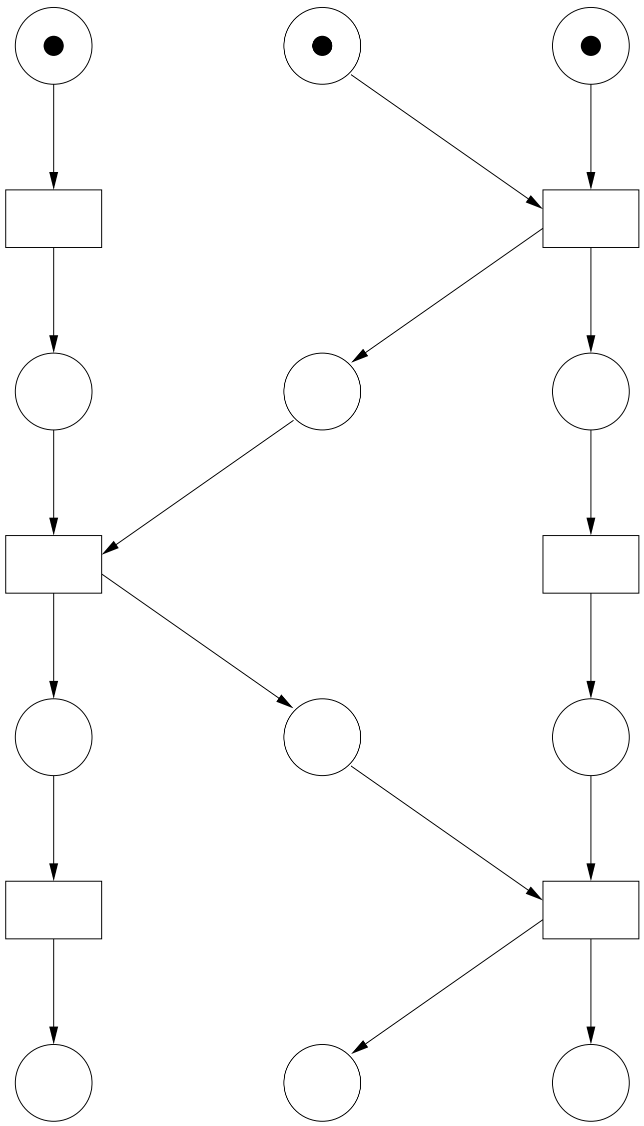


Fig. 52. A process net.

Definition 73. A net $N = (P, T, F)$ is a *process net* if:

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Definition 62. Let A be a finite set.

A binary relation $\rho \subseteq A \times A$ is a *partial order on A* if
ρ is irreflexive and transitive;
(A, ρ) is also called a *partially ordered set*.

Partial order 'is' transitive, directed acyclic graph.

Lemma 74. For every process net N, F_N^+ is a partial order on X_N.

$$\mathbf{li}_N = \mathbf{li}_{F+}$$

$$\mathbf{co}_N = \mathbf{co}_{F+}$$

Definition 75. A *slice* of a process net N is a cut C of N such that $C \subseteq P_N$.

Lemma 76. Let $N = (P, T, F)$ be a process net and let $C \subseteq P$.

C is a slice of N iff

(1) for all $p, q \in C$, $p \ \mathbf{co}_N \ q$, and

(2) for every $p \in P - C$ there exists $q \in C$ such that $\neg \ p \ \mathbf{co}_N \ q$.

Lemma 76. Let $N = (P, T, F)$ be a process net and let $C \subseteq P$.
C is a slice of N iff
(1) for all $p, q \in C$, $p \ \mathbf{co}_N \ q$, and
(2) for every $p \in P - C$ there exists $q \in C$ such that $\neg \ p \ \mathbf{co}_N \ q$.

A cut C is a maximal $\mathbf{co}$-clique

Definition 65'.
C is a $\mathbf{co}$-*clique* if
$a \ \mathbf{co} \ b$ for all $a, b \in C$, and

C is a *maximal* $\mathbf{co}$-*clique* if
C is a $\mathbf{co}$-clique and
for every $a \in X_N - C$ there exists $b \in C$ such that $\neg \ a \ \mathbf{co} \ b$.

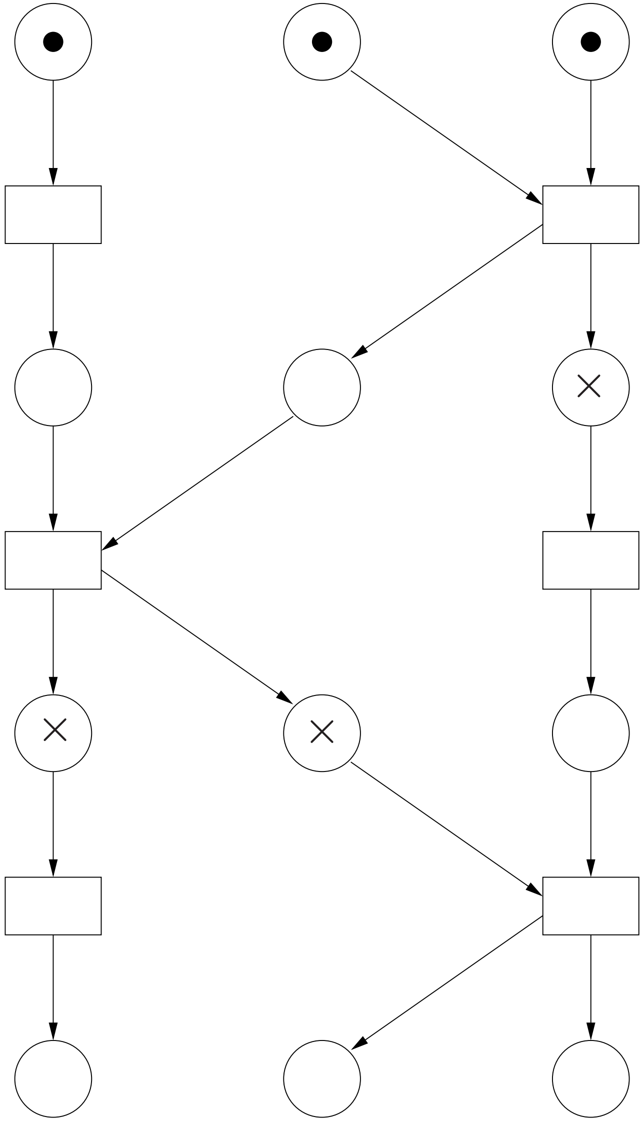

Fig. 52. A process net.

Lemma 66. Let A be a finite set and
let $\sigma \subseteq A \times A$ be a reflexive symmetric relation.

For every σ-clique B
there exists a maximal σ-clique C with $B \subseteq C$.

Lemma 77. Let $N = (P, T, F)$ be a process net.
For every $\mathbf{co}_N$-clique $B \subseteq P$
there exists a slice C of N with $B \subseteq C$.

${}^\circ N = {}^\circ X_N$

$N^\circ = X_N{}^\circ$

Process net $N = (P, T, F)$ corresponds to EN system $(P, T, F, {}^\circ N)$.

$(P, T, F, {}^\circ N)$ is conflict-free

If $N = (P, T, F)$ is isomorphic to $N' = (P', T', F')$,
with bijection α on places,
then $\alpha({}^\circ N) = {}^\circ N'$.

Aim:

Theorem 79. Let $N = (P, T, F, {}^{\circ}N)$ be a process net and let $C \subseteq P$.

$C \in \mathbb{C}_N$ iff C is a slice of N.

Lemma 78. Let $N = (P, T, F)$ be a process net.

(1) For every $U \subseteq T$, if U is a $\mathbf{co}$-clique,
then ${}^\bullet U$ and $U^\bullet$ are $\mathbf{co}$-cliques.
In particular, ${}^\bullet t$ and $t^\bullet$ are $\mathbf{co}$-cliques for every $t \in T$.

(2) For every $\mathbf{co}$-clique $U \subseteq T$ there exists a slice C such that ${}^\bullet U \subseteq C$.

(3a) For every slice C and every $t \in T$,
if ${}^\bullet t \subseteq C$, then
$t^\bullet \cap C = \varnothing$ and
$D = (C - {}^\bullet t) \cup t^\bullet$ is a slice such that ${}^\rightarrow D = {}^\rightarrow C \cup \{t\} \cup t^\bullet$.

Lemma 78 Ctd. Let $N = (P, T, F)$ be a process net.

(3b) For every slice C and every $t \in T$,
if $t^\bullet \subseteq C$, then
${}^\bullet t \cap C = \varnothing$ and
$D = (C - t^\bullet) \cup {}^\bullet t$ is a slice such that ${}^\rightarrow D = {}^\rightarrow C - t^\bullet - \{t\}$.

(4) For every slice C and every transition t,
if $t \in {}^\rightarrow C$, then
$\mathbf{nbh}(t) \subseteq {}^\rightarrow C$.

(5) For every slice $C \neq {}^\circ N$
there exists $t \in T$ such that $t^\bullet \subseteq C$.

Theorem 79. Let $N = (P, T, F, {}^{\circ}N)$ be a process net and let $C \subseteq P$.

$C \in \mathbb{C}_N$ iff C is a slice of N.

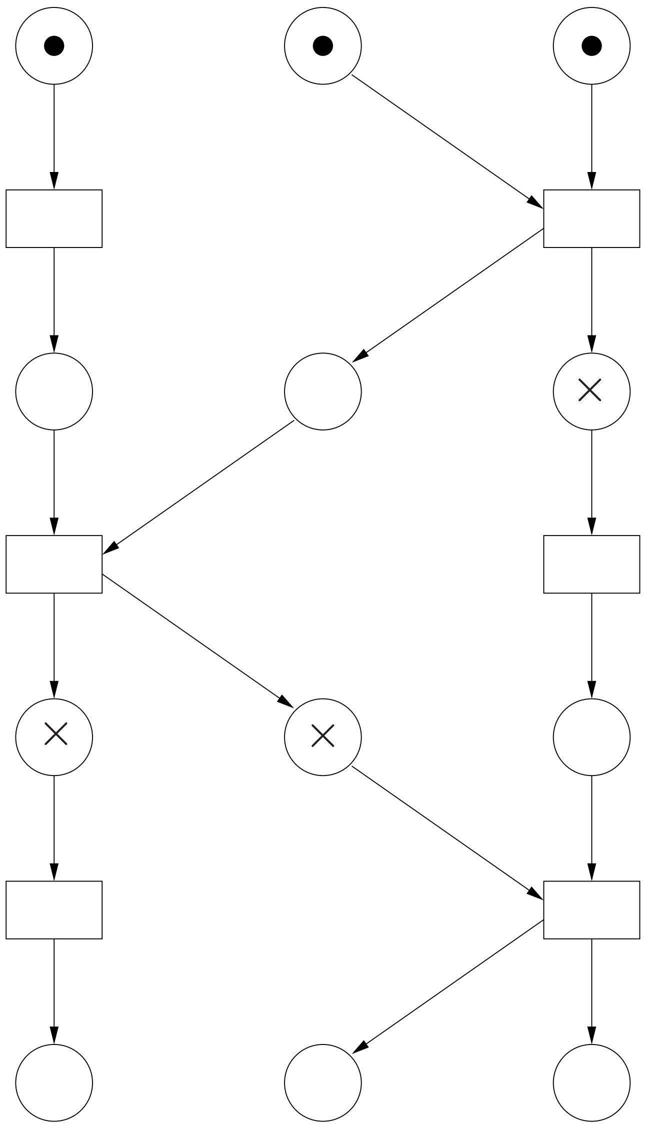

Fig. 52. A process net, reachable configurations.

Since process nets are conflict-free and contact-free:

Theorem 80. Let N be a process net, let $C \in \mathbb{C}_N$, and let $U \subseteq T_N$ with $U \neq \emptyset$.

If ${}^\bullet U \subseteq C$, then $U \;\mathbf{con}\; C$.

Theorem 81. Let $N = (P, T, F, {}^\circ N)$ be a process net.

(1) N is reduced.

(2) For every $U \subseteq T$,
$(\exists C \in \mathbb{C}_N : U \;\mathbf{con}\; C)$ iff U is a **co**-clique.