Program correctness

LTL equivalences

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LTL equivalences

We say that two LTL formulas $\phi$ and $\psi$ are semantically equivalent, writing $\phi \equiv \psi$ if for all models $M$ and for all paths $\pi$ of $M$ we have

$$\pi \models \phi \text{ iff } \pi \models \psi$$
De Morgan-based equivalences

\[ \neg F \phi \equiv G \neg \phi \]
\[ \neg G \phi \equiv F \neg \phi \]
\[ \neg X \phi \equiv X \neg \phi \]

X-self duality: on a path each state has a unique successor

\[ \neg (\phi \cup \psi) \equiv \neg \phi \cup \neg \psi \]
\[ \neg (\phi \circ \psi) \equiv \neg \phi \cup \neg \psi \]
Distributivities

\[ F(\phi \lor \psi) \equiv F\phi \lor F\psi \]
\[ G(\phi \land \psi) \equiv G\phi \land G\psi \]
Reductions

\[ F\phi \equiv T \cup \phi \]
\[ G\phi \equiv \bot \cap R \phi \]

\[ \phi \cup \psi \equiv \phi \wedge \psi \wedge F\psi \]
\[ \phi \wedge \psi \equiv \phi \vee \psi \vee F\psi \]

\[ \phi \wedge \psi \equiv \psi \vee (\phi \vee \psi) \]
\[ \phi \vee \psi \equiv \psi \wedge (\phi \wedge \psi) \]
LTL: Adequate sets of connectives

- A set of operators $S$ is **adequate for LTL** if every formula in LTL can be expressed as an equivalent one using only the operators in $S$.

- **Theorem**: The set of operators $T, \neg, \land, X, U$
  
  is adequate for LTL.

- **Without negation**, the set of operators $T, \perp, \lor, \land, X, U, R$
  
  is adequate but $T, \perp, \lor, \land, X, R, G$ is not (because one cannot define $F$).
Other LTL equivalences

- $G\phi \equiv \phi \land XG\phi$
- $F\phi \equiv \phi \lor XF\phi$
- $\phi U\psi \equiv \psi \lor (\phi \land X(\phi U\psi))$

**Theorem:** $\phi U\psi \equiv \neg(\neg\neg\neg\psi U(\neg\phi \land \neg\psi)) \land F\psi$