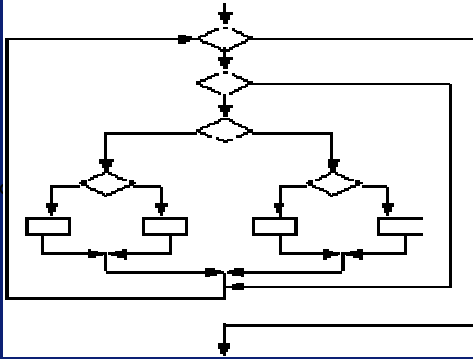


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Program correctness

LTL equivalences

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LTL equivalences

- We say that two LTL formulas ϕ and ψ are **semantically equivalent**, writing $\phi \equiv \psi$ if for all models M and for all paths π of M we have

$$\pi \models \phi \text{ iff } \pi \models \psi$$



De Morgan-based equivalences

$$\square \neg F\phi \equiv G\neg\phi$$

$$\square \neg G\phi \equiv F\neg\phi$$

$$\square \neg X\phi \equiv X\neg\phi$$

X-self duality: on a path each state has a unique successor

$$\square \neg(\phi U \psi) \equiv \neg\phi R \neg\psi$$

$$\square \neg(\phi R \psi) \equiv \neg\phi U \neg\psi$$



Distributivities

- $F(\phi \vee \psi) \equiv F\phi \vee F\psi$
- $G(\phi \wedge \psi) \equiv G\phi \wedge G\psi$



Reductions

$$\square F\phi \equiv T U \phi$$

$$\square G\phi \equiv \perp R \phi$$

$$\square \phi U \psi \equiv \phi W \psi \wedge F\psi$$

$$\square \phi W \psi \equiv \phi U \psi \vee F\psi$$

$$\square \phi W \psi \equiv \psi R (\phi \vee \psi)$$

$$\square \phi R \psi \equiv \psi W (\phi \wedge \psi)$$



LTL: Adequate sets of connectives

- A set of operators S is **adequate for LTL** if every formula in LTL can be expressed as an equivalent one using only the operators in S .

- Theorem: The set of operators

\top, \neg, \wedge, X, U

is adequate for LTL.

- Without negation, the set of operators

$\top, \perp, \vee, \wedge, X, U, R$

is adequate but $\top, \perp, \vee, \wedge, X, R, G$ is not (because one cannot define F).



Other LTL equivalences

- $G\phi \equiv \phi \wedge XG\phi$
- $F\phi \equiv \phi \vee XF\phi$
- $\phi U \psi \equiv \psi \vee (\phi \wedge X(\phi U \psi))$
- Theorem: $\phi U \psi \equiv \neg(\neg\psi U(\neg\phi \wedge \neg\psi)) \wedge F\psi$

