1. Consider a following generalization of CTL that permits reasoning about past state. It contains two new operators $AP\phi$ and $EP\phi$, with intended meaning given by

- $AP\phi$ is true in state $s$ if $\phi$ holds in all states immediately preceding $s$,
- $EP\phi$ is true in state $s$ if $\phi$ holds in some state immediately preceding $s$.

For each of the following propositions, indicates if they are true or not, providing either a justification or a model with a state $s_0$ in which the proposition is false:

a) $\phi \implies AX(AP\phi)$

b) $\phi \implies AX(EP\phi)$

c) $EX(AP\phi) \implies \phi$

d) $EX(EP\phi) \implies \phi$

2. Using the fixed point method, give the set of states of the following transition system satisfying the CTL formula $AG(AFq)$:

![Transition System Diagram]

3. For each of the following pairs of CTL formulae exhibit a model with a state $s_0$ in which one formula is true but not the other:

a) $EFp$ and $EGp$

b) $AF(p \lor q)$ and $AFp \lor AFq$

c) $EF\neg p$ and $\neg AFp$

d) $True$ and $EGp \implies AGp$


4. Consider the command \( c = \text{while } x \neq 4 \text{ do } x := x + 2 \text{ od:} \)
   a) Give a precondition \( P \) such that \( c \) does not terminates if \( P \) is satisfied. [5 pts]
   b) Prove \( \{ P \} \ c \ { \text{false} } \) using the proof system for partial correctness. [10 pts]
   c) Prove \( \{ x = 0 \} \ c \ { \text{true} } \) using the proof system for total correctness. [10 pts]

6. The following is an algorithm for the calculation of the factorial of a non-negative integer \( N \).

   \[
   \begin{align*}
   \{ x = N \land N > 0 \} \\
   y := 1; \\
   \text{while } x \neq 1 \text{ do} \\
     y := y \times x; \\
     x := x - 1 \\
   \text{od} \\
   \{ y = N! \land N > 0 \}
   \end{align*}
   \]
   a) Give an invariant for the while command implying the above postcondition. [5 pts]
   b) Give a proof outline for the partial correctness of the above algorithm. [10 pts]
   c) Give a proof that the algorithm terminates. [10 pts]

The final score is given by the sum of the points obtained divided by 10 (with a maximum of 10).

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**Proof system for partial correctness:**

1. \( \{ \phi \} \ skip \{ \phi \} \)
2. \( \{ \phi[e/x] \} \ x := e \{ \phi \} \)
3. \( \{ \phi \} \ c_1 \{ \phi \} \{ \psi \} \ c_2 \{ \psi \} \)
4. \( \{ \phi \land b \} \ c_1 \{ \psi \} \{ \phi \land \neg b \} \ c_2 \{ \psi \} \)
5. \( \{ \phi \} \ while \ b \ do \ c \ od \{ \phi \land \neg b \} \)
6. \( \phi \Rightarrow \phi_1 \{ \phi_1 \} \ c \{ \psi_1 \} \ \psi_1 \Rightarrow \psi \)