LIACS
Leiden University

Examination Program Correctness
13 August 2004, 9:00 - 12:00

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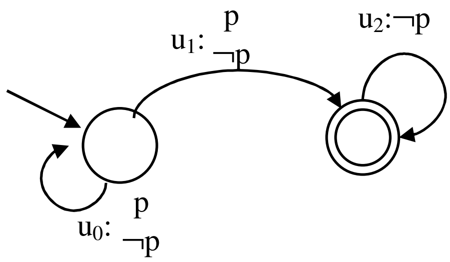
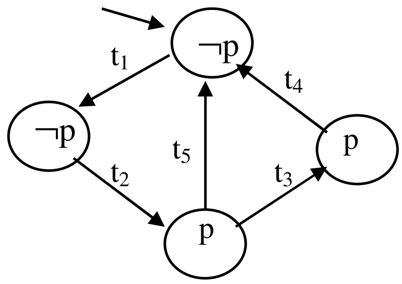
1. Consider the LTL formula ϕ = GFp and let $A_{\neg\phi}$ be the Büchi automata below, representing the negation of the LTL formula ϕ.

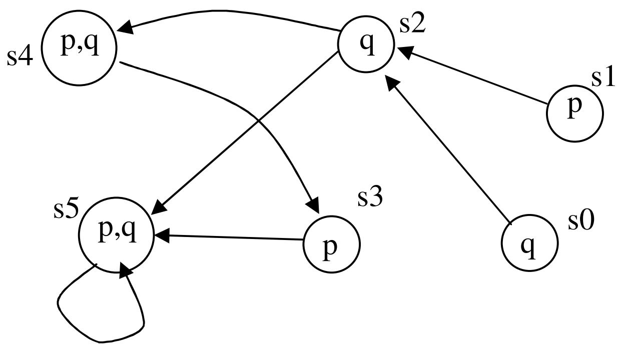


Let B be the transition system



Does the system B satisfy the specification ϕ? [Hint: construct the automata synchronizing $A_{\neg\phi}$ and B] **[20 pts]**

2. Using the labelling algorithm, give the set of all states of the following transition system satisfying the CTL formula A[p U AX q] **[20 pts]**



3. Show that any CTL formula can be transformed into a semantically equivalent CTL formula which uses only the logical operators $\bot$, $\wedge$, $\neg$, AF, EU, and AX. **[10 pts]**

4. Calculate the weakest precondition P of the following commands:
   a) `{ P } if x < 0 then x := x+2 else skip fi` $\{ x > 0 \}$ **[5 pts]**
   b) `{ P } if x=y then x := 0 else y:= 0 fi` $\{ x = y \}$ **[10 pts]**
   c) `{ P } z := 0` $\{ z = 0 \wedge \exists z . x^2+y^2=z \}$ **[15 pts]**

6. The following is an algorithm for the calculation of the square root of a non-negative integer N: when the algorithm terminates, x stores the greatest integer approximating the square root of N.

```
{ N ≥ 0 }
x := N;
y := 1;
while x > y do
     x := (x+y) div 2;
     y := N div x
od
{ x² ≤ N < (x +1)² }
```

   a) Give an invariant for the while command implying the above postcondition. **[10 pts]**
   b) Give a proof outline for the partial correctness of the above algorithm. **[10 pts]**
   c) Give a proof that the above algorithm terminates if N is non-negative. **[10 pts]**

The final score is given by the sum of the points obtained divided by 10 (with a maximum of 10).

Proof system for partial correctness:

1. $\{ \phi \}$ `skip` $\{ \phi \}$

2. $\{ \phi[e/x] \}$ `x:= e` $\{ \phi \}$

3. $$\frac{\{ \phi \}\ c1\ \{ \varphi \} \quad \{ \varphi \}\ c2\ \{ \psi \}}{\{ \phi \}\ c1\ ;\ c2\ \{ \psi \}}$$

4. $$\frac{\{ \phi \wedge b \}\ c1\ \{ \psi \} \quad \{ \phi \wedge \neg b \}\ c2\ \{ \psi \}}{\{ \phi \}\ \underline{if}\ b\ \underline{then}\ c1\ \underline{else}\ c2\ \underline{fi}\ \{ \psi \}}$$

5. $$\frac{\{ \phi \wedge b \}\ c\ \{ \phi \}}{\{ \phi \}\ \underline{while}\ b\ \underline{do}\ c\ \underline{od}\ \{ \phi \wedge \neg b \}}$$

6. $$\frac{\phi \Rightarrow \phi1 \quad \{ \phi1 \}\ c\ \{ \psi1 \} \quad \psi1 \Rightarrow \psi}{\{ \phi \}\ c\ \{ \psi \}}$$