1. Consider the LTL formula $\phi = GFp$ and let $A_{\neg\phi}$ be the Büchi automata below, representing the negation of the LTL formula $\phi$.

\[
\begin{array}{c}
\text{Let } B \text{ be the transition system} \\
\end{array}
\]

Does the system $B$ satisfy the specification $\phi$? [Hint: construct the automata synchronizing $A_{\neg\phi}$ and $B$] [20 pts]

2. Using the labelling algorithm, give the set of all states of the following transition system satisfying the CTL formula $A[p U AX q]$ [20 pts]
3. Show that any CTL formula can be transformed into a semantically equivalent CTL formula which uses only the logical operators $\bot, \land, \neg, \text{AF}, \text{EU}, \text{AX}$.  [10 pts]

4. Calculate the weakest precondition $P$ of the following commands:
   a) $\{ P \} \text{ if } x < 0 \text{ then } x := x+2 \text{ else skip fi } \{ x > 0 \} \quad [5 \text{ pts}]$
   b) $\{ P \} \text{ if } x = y \text{ then } x := 0 \text{ else } y := 0 \text{ fi } \{ x = y \} \quad [10 \text{ pts}]$
   c) $\{ P \} z := 0 \{ z = 0 \land \exists z. x^2 + y^2 = z \} \quad [15 \text{ pts}]$

6. The following is an algorithm for the calculation of the square root of a non-negative integer $N$: when the algorithm terminates, $x$ stores the greatest integer approximating the square root of $N$.

$$\{ N \geq 0 \}$$
$$x := N;$$
$$y := 1;$$
$$\text{while } x > y \text{ do}$$
$$\quad x := (x+y) \div 2;$$
$$\quad y := N \div x$$
$$\text{od}$$
$$\{ x^2 \leq N < (x+1)^2 \}$$

   a) Give an invariant for the while command implying the above postcondition. [10 pts]
   b) Give a proof outline for the partial correctness of the above algorithm. [10 pts]
   c) Give a proof that the above algorithm terminates if $N$ is non-negative. [10 pts]

The final score is given by the sum of the points obtained divided by 10 (with a maximum of 10).

---

**Proof system for partial correctness:**

1. $\{ \phi \} \text{skip } \{ \phi \}$
2. $\{ \phi[e/x] \} \ x := e \{ \phi \}$
3. $\{ \phi \} \ c1 \{ \phi \} \ { \phi } \ c2 \{ \psi \}$
4. $\{ \phi \} \ if \ b \ then \ c1 \ else \ c2 \ fi \{ \psi \}$
5. $\{ \phi \} \ while \ b \ do \ c \ od \{ \phi \land \neg b \}$
6. $\phi \Rightarrow \phi \land \{ \phi \land \psi \} \ \psi \Rightarrow \psi$

---

$\{ \phi \} \ c \{ \psi \}$