LIACS
Leiden University

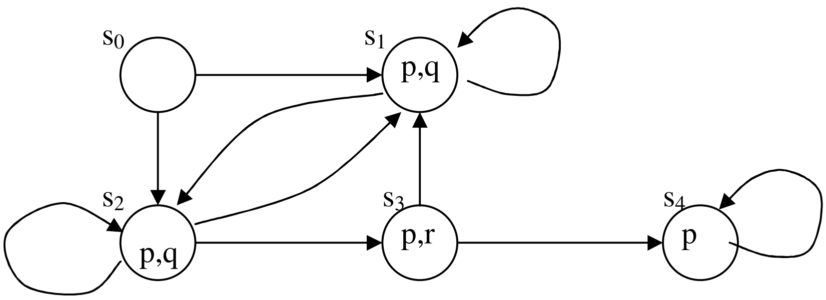
Examination Program Correctness
13 August 2003, 09:00 - 12:00

1. Consider a mutual exclusion problem for two processes: a process is either in the critical section (C), in the attempting section (T), or in the non-critical section (N). A process starts in a non-critical section, and indicates that it wants to enter its critical section by going into the attempting section. It remains in the attempting section until it obtains access to the critical section, and from its critical section it moves to the non-critical section. The states of the process P_i are labelled by the atomic propositions C_i, T_i, and N_i, for i=1,2, denoting a critical section, an attempting section or a non-critical section of the process i, respectively. Specify in CTL the following properties

   a) It is not possible for both processes to be in their critical section at the same time. **[10 points]**
   b) A process that wants to enter in its critical section is eventually able to do so. **[10 points]**

2. Give a translation of an arbitrary CTL formula into the basic operators AX, AG, and AU. **[10 points]**

3. Consider the following transition system



   where p, q and r are atomic propositions and, for each state s, the atomic propositions labelling it are given inside the circle representing s.

   a) Using the labelling algorithm explained in the course, give the set of all states satisfying the CTL formulas A[p U EG(p $\Rightarrow$ q)]. **[10 points]**
   b) Give the set of all states satisfying the CTL* formula EG(p U AFr). Justify informally your answer. **[10 points]**

4. Prove the partial correctness of following Hoare triple:

   (| `x=0` |) **`while`** `x<9` **`{`**`x:=x+1; x:=x+1`**`}`** (| `even(x)` |)

   with the understanding that even(x) means the content of x is an even number. **[10 points]**

5. a) Give the formal definition of validity with respect to partial correctness of an Hoare triple (that is, define the meaning of $\models_{par}$ (| ϕ |) C (| φ |)). **[10 points]**

   b) Use the above definition to prove that if $\models_{par}$ (| ϕ_1 |) C (| φ |)) and $\models_{par}$ (| ϕ_2 |) C (| φ |)) are both valid, then also $\models_{par}$ (| $\phi_1 \vee \phi_2$ |) C (| φ |)) is valid. **[10 points]**

6. For n ≥ 0, let in the n-th Fibonacci number *fib*(n) defined as follows,

   fib(0) = 0
   fib(1) = 1
   fib(n+2) = *fib*(n) + *fib*(n+1)

   The program FIB below is intended to calculate the k-th Fibonacci number (for k a given constant)

```
x:=1;
y:=1;
z:=0;
while x ≠ k {
x:=x+1;
y:=y+z;
z:=y
}
```

   Give a proof outline for partial correctness of (| k ≥ 1 |) FIB (| y=*fib*(k) |). **[30 points]**

The final score is given by the sum of the points obtained divided by 10 (with a maximum of 10).

Proof system for partial correctness:

1. (| ϕ |) skip (| ϕ |)

2. (| ϕ[e/x] |) x:= e (| ϕ |)

3.
$$\frac{(|\ \phi\ |)\ c1\ (|\ \varphi\ |) \quad (|\ \varphi\ |)\ c2\ (|\ \psi\ |)}{(|\ \phi\ |)\ c1\ ;\ c2\ (|\ \psi\ |)}$$

4.
$$\frac{(|\ \phi \wedge b\ |)\ c1\ (|\ \psi\ |) \quad (|\ \phi \wedge \neg b\ |)\ c2\ (|\ \psi\ |)}{(|\ \phi\ |)\ \textbf{if}\ b\ \textbf{then}\ \{\ c1\}\ \textbf{else}\ \{c2\}\ (|\ \psi\ |)}$$

5.
$$\frac{(|\ \phi \wedge b\ |)\ c\ (|\ \phi\ |)}{(|\ \phi\ |)\ \textbf{while}\ b\ \textbf{do}\ \{\ c\ \}\ (|\ \phi \wedge \neg b\ |)}$$

6.
$$\frac{\phi \Rightarrow \phi 1 \quad (|\ \phi 1\ |)\ c\ (|\ \psi 1\ |) \quad \psi 1 \Rightarrow \psi}{(|\ \phi\ |)\ c\ (|\ \psi\ |)}$$