1. **[1 point]** For each of the following four pairs of CTL formulas say if they are equivalent or find a model of one of the pair which is not a model of the other:
   a) $AF(\phi \lor \psi)$ and $AF\phi \lor AF\psi$
   b) $\neg AF(\phi \lor \psi)$ and $EF(\neg \phi \land \neg \psi)$
   c) $true$ and $AG\phi \rightarrow EF\phi$
   d) $true$ and $EG\phi \rightarrow AF\phi$.

2. **[1,5 points]** Use the labelling algorithm to give the set of all states of the following transition system satisfying the CTL formula $AGAF(p \lor q)$:

   ![Diagram of a transition system](image)

3. **[1,5 points]** Prove the equivalence between the following three pairs of LTL formulas by explicitly referring to their formal semantics:
   a) $\neg X\phi$ and $X\neg\phi$
   b) $\neg F\phi$ and $G\neg\phi$
   c) $G\neg\phi$ and $\neg(true U \phi)$.

4. **[1,5 points]** Let $C$ be the command $while \ x \neq 0 \ do \ x := x+1 \ od$.
   a) Is the partial correctness assertion $\{false\} C \{true\}$ valid? Justify your answer.
   b) For which preconditions $\phi$ is the partial correctness assertion $\{\phi\} C \{true\}$ valid?
   c) For which preconditions $\phi$ is the total correctness assertion $\{\phi\} C \{true\}$ valid?

5. **[2 points]** Exhibit a proof tree for the partial correctness of the following Hoare triple:

   $\{y \geq 0\} \ if \ x>y \ then \ x:=y \ else \ if \ x<0 \ then \ x:=0 \ fi \ fi \ \{0 \leq x \leq y\}$

6. **[2,5 points]** Consider the following Hoare triple for a command computing the greatest common divisor $gcd(m,n)$ of two positive integers $m$ and $n$:

   $\{ x \geq 1 \land y \geq 1 \land x = x_0 \land y = y_0 \}$
   while $x \neq y$ do
     if $(x < y)$ then
       $y := y - x$;
     else
       $x := x - y$;
     fi
   od
   $\{ x = gcd(x_0,y_0) \}$

   Give a proof outline for total correctness. Clearly identify the invariant and the variant.
   Remember that $gcd(m,n) = gcd(m,m-n)$ for positive integers $m$ and $n$ with $n < m$.

The final score is given by the sum of the points obtained.