1. Alice and Bob are two persons seated at a table with one single fork. Each of them can be hungry and each of them can hold a fork. Write a CTL specification for the following properties:

   a. Initially neither one of them holds the fork. [2 points]
   b. They cannot hold the fork at the same time. [2 points]
   c. If one of them gets hungry then that person will eventually pick up the fork. [2 points]
   d. Each person cannot hold the fork continuously. [2 points]
   e. Each person will be hungry infinitely often. [2 points]

   f. Draw a transition system satisfying the above properties. [10 points]

2. Let p and q be two atomic propositions. Using the fixed-point method give the set of all states of the following transition system satisfying the CTL formula EF (p \land AX q). [15 points]

3. Give an informal definition of operational, denotational and axiomatic semantics explaining their differences. [10 points]

4. Consider the command iterate N times C end, where N is a natural number and C is a command intended to be executed N times.

   a. Formulate rules for the derivation of the execution

      \[ \langle \text{iterate N times C end}, \sigma \rangle \rightarrow \sigma' \]

      in an initial state \( \sigma \) and terminating in the final state \( \sigma' \). [10 points]

   b. Formulate a deduction rule for the partial correctness of the Hoare triple

      \[ \{ \varphi \} \text{iterate N times C end} \{ \psi \} \].

      Give an informal justification of soundness. [10 points]
5. Calculate the weakest precondition of the following assignments with respect to the postcondition \( a[i]=i \), where \( a \) is an array of positive integers, and \( i \) and \( j \) are two positive integers:

a. \( a[i]:= a[i] + 1 \) \([5\text{ points}]

b. \( a[a[j]]:= a[i] +1 \) \([10\text{ points}]

c. \( a[a[i]]:= a[j] + 1 \) \([10\text{ points}]

6. The following is a division algorithm. It assumes that \( x \) is non-negative and \( y \) is a positive integer. When the algorithm terminates, \( \text{quot} \) is the integer quotient and \( \text{rest} \) the remainder of \( x \) divided by \( y \). Give a proof outline for the total correctness of the following Hoare triple:

\[
\{ x \geq 0 \land y > 0 \} \quad \text{quot} := 0; \quad \text{rest} := x; \quad \text{while} \ \text{rest} \geq y \ \text{do} \quad \text{rest} := \text{rest} - y; \quad \text{quot} := \text{quot} + 1 \quad \text{od} \quad \{ \exists n. 0 \leq n < y \land \text{quot} \times y + n = x \}
\]

[20 points]

The final score is given by the sum of the points obtained divided by 10 (with a maximum of 10).

Proof system for partial correctness:

1. \( \{ \phi \} \text{skip} \{ \phi \} \)

2. \( \{ \phi[e/x]\} x:= e \{ \phi \} \)

3. \( \{ \phi \} \text{c}1 \{ \phi \} \quad \{ \phi \} \text{c}2 \{ \psi \} \)

4. \( \{ \phi \land b \} \text{c}1 \{ \psi \} \quad \{ \phi \land \neg b \} \text{c}2 \{ \psi \} \)

5. \( \{ \phi \} \text{while} \ b \ \text{do} \quad \text{od} \{ \phi \land \neg b \} \)

6. \( \phi \Rightarrow \phi \text{l} \quad \{ \phi \text{l} \} \text{c} \{ \psi \text{l} \} \quad \psi \text{l} \Rightarrow \psi \)

\( \{ \phi \} \text{c} \{ \psi \} \)