LIACS
Leiden University

Examination Program Correctness
8 June 2004, 9:00 - 12:00

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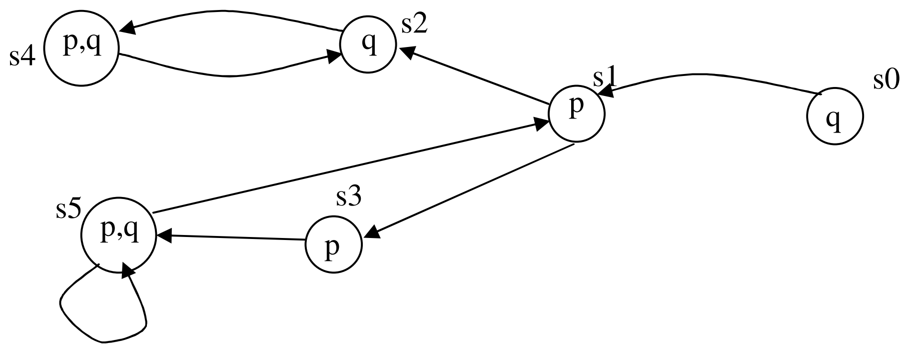
1. Alice and Bob are two persons seated at a table with one single fork. Each of them can be hungry and each of them can hold a fork. Write a CTL specification for the following properties:
   a. Initially neither one of them holds the fork . **[2 points]**
   b. They cannot hold the fork at the same time. **[2 points]**
   c. If one of them gets hungry then that person will eventually pick up the fork. **[2 points]**
   d. Each person cannot hold the fork continuously. **[2 points]**
   e. Each person will be hungry infinitely often. **[2 points]**

   f. Draw a transition system satisfying the above properties. **[10 points]**

2. Let p and q be two atomic propositions. Using the fixed-point method give the set of all states of the following transition system satisfying the CTL formula EF ($p \wedge$ AX q). **[15 points]**



3. Give an informal definition of operational, denotational and axiomatic semantics explaining their differences. **[10 points]**

4. Consider the command `iterate N times C end`, where `N` is a natural number and C is a command intended to be executed `N` times.
   a. Formulate rules for the derivation of the execution

   $$< \underline{\texttt{iterate}}\ \texttt{N}\ \underline{\texttt{times}}\ \texttt{C}\ \underline{\texttt{end}}, \sigma> \rightarrow \sigma'$$

   in an initial state σ and terminating in the final state σ'. **[10 points]**
   b. Formulate a deduction rule for the partial correctness of the Hoare triple

   $$\{ \phi \}\ \underline{\texttt{iterate}}\ \texttt{N}\ \underline{\texttt{times}}\ \texttt{C}\ \underline{\texttt{end}}\ \{ \psi \}.$$

   Give an informal justification of soundness. **[10 points]**

5. Calculate the weakest precondition of the following assignments with respect to the postcondition `a[i]=i`, where `a` is an array of positive integers, and `i and j` are two positive integers:
   a. `a[i]:= a[i] + 1` **[5 points]**
   b. `a[a[j]]:= a[i] +1` **[10 points]**
   c. `a[a[i]]:= a[j] + 1` **[10 points]**

6. The following is a division algorithm. It assumes that `x` is non-negative and `y` is a positive integer. When the algorithm terminates, `quot` is the integer quotient and `rest` the remainder of `x` divided by `y`. Give a proof outline for the total correctness of the following Hoare triple:

```
{ x≥0 ∧ y>0 }
quot := 0;
rest := x;
while rest ≥ y do
     rest := rest-y;
     quot := quot+1
od
{ ∃n.0≤n<y ∧ quot*y+n=x }
```

[20 points]

The final score is given by the sum of the points obtained divided by 10 (with a maximum of 10).

Proof system for partial correctness:

1. $\{ \phi \}\ \texttt{skip}\ \{ \phi \}$

2. $\{ \phi[e/x] \}\ x := e\ \{ \phi \}$

3. $$\frac{\{ \phi \}\ c1\ \{ \varphi \} \quad \{ \varphi \}\ c2\ \{ \psi \}}{\{ \phi \}\ c1\ ;\ c2\ \{ \psi \}}$$

4. $$\frac{\{ \phi \wedge b \}\ c1\ \{ \psi \} \quad \{ \phi \wedge \neg b \}\ c2\ \{ \psi \}}{\{ \phi \}\ \underline{\texttt{if}}\ b\ \underline{\texttt{then}}\ c1\ \underline{\texttt{else}}\ c2\ \underline{\texttt{fi}}\ \{ \psi \}}$$

5. $$\frac{\{ \phi \wedge b \}\ c\ \{ \phi \}}{\{ \phi \}\ \underline{\texttt{while}}\ b\ \underline{\texttt{do}}\ c\ \underline{\texttt{od}}\ \{ \phi \wedge \neg b \}}$$

6. $$\frac{\phi \Rightarrow \phi 1 \quad \{ \phi 1 \}\ c\ \{ \psi 1 \} \quad \psi 1 \Rightarrow \psi}{\{ \phi \}\ c\ \{ \psi \}}$$