LIACS
Leiden University

Examination Program Correctness
5 June 2007, 14:00 - 17:00

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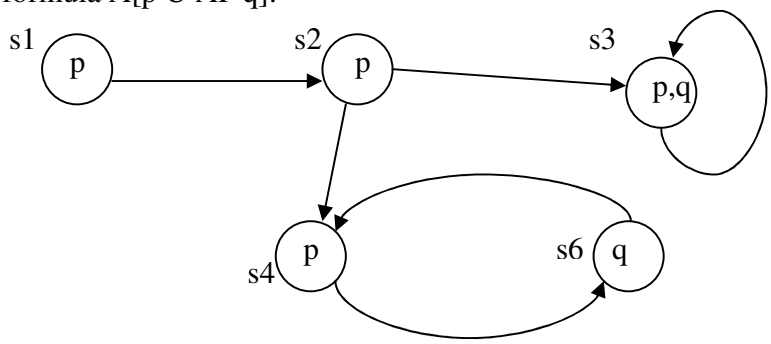
1. **[1 point]** Give a simple CTL model with a state satisfying the CTL formula ¬EG p but not satisfying AG¬p

2. **[2 points]** Use the *fixpoint method* to find the states of the following transition system that satisfy the CTL formula A[p U AF q]:



3. **[1 point]** Show that any LTL formula using the operators F,G, R, and W can be transformed into a semantically equivalent one which uses only the Boolean operators, X, and U.

4. **[1 point]** Describe a finite method for checking if the language of a Büchi automaton is empty.

5. **[2 points]** Let `a[1..k]` be an array of integer, and consider the following two commands swapping two elements of the array, one with the help of a temporary variable and another without it:

```
SWAP1  =   tmp := a[j];          SWAP2  =   a[i]:= a[i]+a[j];
           a[j]:= a[i];                     a[j]:= a[i]-a[j];
           a[i]:= tmp                       a[i]:= a[i]-a[j]
```

   a) Prove `{a[i]=x ∧ a[j]=y} SWAP1 {a[i]=y ∧ a[j] =x}`.
   b) Try to prove `{a[i]=x ∧ a[j]=y} SWAP2 {a[i]=y ∧ a[j] =x}`. Where is the error? Modify the precondition so that you can obtain a correct proof.

6. **[3 points]** Consider the following following Hoare triple of a command computing the sum of the first m integers (recall that 1+2+…+m = m*(m+1) div 2):

```
{m ≥ 0}
 x := 0;
 y := 1;
 while (y ≤ m) do
       x := x + y;
       y := y + 1
 od
{x = m*(m+1) div 2}
```

   a) Find an invariant and give a proof outline for *partial* correctness.
   b) Find a variant and give a proof outline for *total* correctness.

The final score is given by the sum of the points obtained.