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Continuous Lattices and Domains

(Encyclopedia of Mathematics and Its Applications, 93)

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In 1980, the same groups of authors published *A Compendium of Continuous Lattices*. In few years, the *Compendium* was out of print. Since then, many researchers in the fields of order theory, algebra, topology, topological algebras, analysis and theoretical computer science have cited the *Compendium*, often owned only as a photocopy, as one of the most comprehensive reference on continuous lattices.

A continuous lattices is a partially ordered set such that every subset has least upper bound (completeness) and every element can be approximated by other elements that are, in a suitable sense, smaller (continuity). For many applications in the area of computing, computability and semantics of programming languages, the first of the above conditions was too strict, and has been generalized to directed completeness. Continuous directed completed partial orders are referred to as domains.

In this book, the authors successfully succeed in presenting a new long-awaited edition of the *Compendium* containing the original information on continuous lattices reformulated and supplemented in the more general context of domains. Since the literature in this area in the last twenty years is really impressive, the authors release their initial encyclopedic aspiration of the *Compendium*, and drop this word from the title of the new edition.

The book comprises seven chapters and an introduction to order sets and lattices. The core chapters are Chapter II on the Scott topology, where the Hofmann-Mislove theorem is discussed as well as the Isbell topology for function spaces, and Chapter III on the Lawson topology, which is crucial in linking domains and continuous lattices to topological algebra (extensively treated in Chapter VI and VII). Domain equations for recursive data types, and powerdomains (including the probabilistic powerdomain construction) are discussed in Chapter IV. Almost unchanged with respect to the previous edition, but still excellent, is Chapter V, dealing with the spectral theory of lattices.

The book gives a rather complete picture of the mathematical foundations of the theory of continuous lattices from the concrete point of view of order theory, topology and algebra, deliberately avoiding the abstraction of category theory. Applications of domains and continuous lattices in the areas of, e.g., logic, lambda-calculus, computational analysis, and semantics of programming languages are not treated in this book.

This excellent book is written with authority, is self-comprehensive, and comprises an extensive bibliography, including reference to books, proceedings, articles, dissertations and memos of the Seminar on Continuity in Semilattices. Similar to the *Compendium*, it will have considerable influence to all researchers in this area for many years to come. It may be useful for educational purpose, too, especially for mathematicians, but it would be necessary to complement it with a book with more exercises and more applications oriented to the course needs.

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