1. **[1 point]** Give a formula in conjunctive normal form equivalent to the formula $\phi$ specified by the following truth table:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<td>F</td>
<td>F</td>
<td>F</td>
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</tr>
</tbody>
</table>

We consider the lines where $\phi$ is false and obtain the following clauses by negating the atoms that are true in those lines:

$(\neg p \lor \neg q \lor r)$ from line 2
$(\neg p \lor q \lor r)$ from line 4
$(p \lor \neg q \lor \neg r)$ from line 5
$(p \lor q \lor r)$ from line 8

The conjunction of these clauses is the formula

$$(\neg p \lor \neg q \lor r) \land (\neg p \lor q \lor r) \land (p \lor \neg q \lor \neg r) \land (p \lor q \lor r)$$

which is in conjunctive normal form and equivalent to $\phi$.

2. **[2 points]** Transform the formula $p \lor \neg(q \lor p)$ in conjunctive normal form and check its validity.

$$p \lor \neg(q \lor p) \equiv p \lor (\neg q \land \neg p)$$

$$\equiv (p \lor \neg q) \land (p \lor \neg p)$$

It is not valid because in the leftmost clause neither q nor the negation of p occurs.

3. **[3 points]** Transform the following sentences into a Horn formula and apply the marking algorithm to decide its satisfiability:

a. If Jan has a dog, then Betty has a cat.
b. If Kees has a dog, then he has a cat, too.
c. If Jan has a dog and Kees has a cat, then Sarah has a dog.
d. If Betty and Kees share a pet of the same species, then Jan has a cat.
e. Sarah does not have a cat.
f. All the men have dogs.

We use the atomic propositions Kd, Kc, Jd, Jc, Sd, Sc, Bd, and Bc to indicate that Kees, Jan, Sarah or Betty have a dog or cat, respectively. The above statements can thus be translated into the Horn formula

$$(Jd \rightarrow Bc) \land$$
$$(Kd \rightarrow Kc) \land$$
$$(Jd \land Kc) \rightarrow Sd) \land$$
$$((Bc \land Kc) \rightarrow Jc) \land ((Bd \land Kd) \rightarrow Jc) \land$$
$$Sc \rightarrow \bot$$
We now apply the marking algorithm to the above Horn formula and have the following marking steps:

1. T is marked,
2. Jd and Kd are marked
3. Bc, and Kc are marked
4. Sd and Jc are marked
5. Nothing else can be marked

The algorithm thus stops and the formula is satisfiable via the valuation mapping all marked atomic propositions to T and all other atoms to F.

4. **[4 points]** Transform the formula \((p \rightarrow \neg q) \land (q \land (q \rightarrow \neg r))\) in one without implication and disjunctions (do not simplify it!), and use a SAT solver to compute a valuation witness for the satisfiability of the formula.

We first translate the formula into one without implication and disjunction:

\[
T((p \rightarrow \neg q) \land (q \land (q \rightarrow \neg r))) = T(p \rightarrow \neg q) \land T(q \land (q \rightarrow \neg r))
\]

\[
= \neg(T(p) \land \neg T(q)) \land (T(q) \land T(q \rightarrow \neg r))
\]

\[
= \neg(p \land \neg q) \land (q \land \neg(T(q) \land \neg T(\neg r))
\]

\[
= \neg(p \land \neg q) \land (q \land \neg(q \land \neg\neg r))
\]

We now construct a DAG for the above formula and label it starting from T at its root:

Note that we do not have contradicting labels. Thus the formula is satisfiable and a witness for it is given by any valuation mapping p to either F, q to T and r to F.

The final score is given by the sum of the points obtained.

Return your homework solution as a single pdf file to Sander van Rijn (svr003@gmail.com). The deadline is on Friday April 10, 2015. This deadline is strict, thus homework solutions sent after the deadline will not be considered.

Remember to write in your solution your name, surname and student number.