1. **[2 points]** Give a proof by means of natural deduction of the following sequents:
   a) \( p \vdash \neg p \rightarrow q \)
   b) \((p \lor q), \neg p \vdash q\)
   c) \((p \lor q), p \rightarrow r, q \rightarrow s \vdash (r \lor s)\)
   d) \(\vdash (p \rightarrow q) \lor (q \rightarrow p)\)

2. **[1.5 points]**
   a) Give the complete truth table for the formula \( p \rightarrow (q \rightarrow \neg p) \).
   b) Construct a formula \( \phi \) in conjunctive normal form from the truth table
<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
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</tr>
</tbody>
</table>
   c) Convert the formula \((p \lor q) \rightarrow (r \rightarrow p)\) in conjunctive normal form. Is it valid?

3. **[1.5 points]** Prove that the following sequents are **not** valid:
   a) \( \vdash (p \land \neg q) \lor (q \rightarrow \neg p) \)
   b) \( r \rightarrow (q \rightarrow p) \vdash q \rightarrow (p \rightarrow r) \)

4. **[1 point]** Draw the parse trees of the following formulas, and identify all free and bound variable leaves in each of these trees:
   a) \( \exists y \ (P(x,y) \rightarrow \forall x \ P(x,y)) \)
   b) \( \forall x \ x=x \land \exists x \exists y \neg(x = z \lor y = z) \)

5. **[3 points]** For each of the following sequent either show the validity by natural deduction or give a model where it is not valid:
   a) \( \vdash \forall x \forall y \forall z \ (x = y \rightarrow f(x,z) = f(y,z)) \)
   b) \( \vdash \exists x \ (P(x) \rightarrow \forall y \ P(y)) \)
   c) \( \forall x \exists y \ (P(x,y) \rightarrow \forall z \ P(z,x)), \exists x \exists y \ P(x,y) \vdash \forall x \forall y \ P(x,y) \)
   d) \( \forall x \forall y \forall z \ ((P(x,y) \land P(y,z)) \rightarrow P(x,z)) \vdash \forall x \forall y \ ((P(x,y) \land P(y,x)) \rightarrow x = y) \)

6. **[1 point]** Let \( F = \{ 0, s, f, \} \) be a set of function symbols where \( 0 \) is a constant symbol and \( s \) and \( f \) are unary function symbols. Consider the model \( M \) consisting of the set \( N \) of the natural numbers, with interpretations \( 0^M = 0, s^M(n) = n+1, \) and \( f^M(n) = n/2 \) if \( n \) is even, and \( f^M(n) = 3n+1 \) if \( n \) is odd.
   a) Does \( M \vdash s(0) = f(f(s(0))) \) hold? Explain your answer.
   b) Does \( M \vdash \exists x \ f(x) = s(x) \) hold? Explain your answer.

The final score is given by the sum of the points obtained.