Question 1: [2 points]

Use one of the methods studied in class to find a regular expression for the language accepted by the following automaton.

According the algebraic method, the above automaton corresponds to the following system of equations:

\[
\begin{align*}
  x &= ay + bx + \Lambda \\
  y &= bz \\
  z &= ax + by
\end{align*}
\]

Eliminating the variable \( z \) we obtain

\[
\begin{align*}
  x &= ay + bx + \Lambda \\
  y &= b(ax + by) = bax + bby = bby + bax
\end{align*}
\]

From the last equation we obtain that the unique solution of \( y \) is \( y = (bb)^*bax \). We can now substitute \( y \) in the first equation obtaining

\[
\begin{align*}
  x &= a(bb)^*bax + bx + \Lambda = (a(bb)^*ba + b)x + \Lambda
\end{align*}
\]

We thus have that \( x = ((bb)^*ba + b)^*\Lambda = (a(bb)^*ba + b)^* \).

Alternatively, we could find a regular expression for the above automaton using the state elimination method. We first add an initial and a final state obtaining the following automaton:

We first remove the state \( z \), obtaining the automaton
Question 2: [3 points]

Find context-free grammars generating the following languages over $\Sigma = \{a,b\}$:

a) $L_1 = \{ uav | u,v \in \Sigma^*, |u| \geq |v| \}$

$$S \rightarrow aSa | aSb | bSa | bSb | Ua$$

$$U \rightarrow aU | bU$$

b) $L_2 = \{ uv | u,v \in \Sigma^*, |u| \neq |v| \}$

Since $|u| \neq |v|$ if and only if $|u| < |v|$ or $|u| > |v|$ we have that

$$L_2 = \{ uav | u,v \in \Sigma^*, |u| < |v| \} \cup \{ ubbv | u,v \in \Sigma^*, |u| > |v| \}.$$  

A grammar generating the leftmost language is, for example,

$$X \rightarrow aXa | aXb | bXa | bXb | aV$$

$$V \rightarrow aV | bV | a | b$$

while a grammar for the rightmost language is

$$Y \rightarrow aYa | aYb | bYa | bYb | Ua$$

$$U \rightarrow aU | bU | a | b.$$  

The grammar for $L_2$ is thus obtained by adding the extra production $S \rightarrow X | Y$.

c) $L_3 = \{ auava | u,v \in \Sigma^*, |u| = |v| \}.$

$$S \rightarrow aXa$$

$$X \rightarrow aXa | aXb | bXa | bXb | a.$$  

Question 3: [2 points]

Construct a regular grammar generating the same language as the one accepted by the following nondeterministic finite automaton.
The regular grammar is given by \(<\{S,V,X,Y,Z\}, \{a,b\}, \{S\}, P>\) where the productions P are

\[
S \rightarrow aX | bV \\
X \rightarrow bY | bZ | \Lambda \\
Y \rightarrow bZ \\
Z \rightarrow bZ | aV | \Lambda \\
V \rightarrow aV | aZ .
\]

**Question 4:** [3 points]
Construct for the grammar \( G \) with productions \( S \rightarrow SS | SaSb | \Lambda \) a grammar \( G' \) in Chomsky Normal Form (CNF) with \( L(G') = L(G) \setminus \{ \Lambda \} \).

We first eliminate the nullable productions from \( G \) which yields \( G_1 \) with productions

\[
S \rightarrow SS | S | SaSb | aSb | Sab | ab .
\]

The resulting grammar has no nullable productions and \( L(G_1) = L(G) \setminus \{ \Lambda \} \). Next we remove the newly introduced unit production \( S \rightarrow S \), yielding the grammar \( G_2 \) with productions

\[
S \rightarrow SS | SaSb | aSb | Sab | ab .
\]

We have \( L(G_2) = L(G_1) \). Finally, we adapt \( G_2 \) to CNF in two steps. First we get

\[
S \rightarrow SS | SXaSXb | XaSXb | SXaXb | XaXb \\
X_a \rightarrow a \\
X_b \rightarrow b .
\]

Finally we get the the productions of \( G' \):

\[
S \rightarrow SS | SX | X_aY | SZ | X_bX_b \\
X \rightarrow X_aY \\
Y \rightarrow SX_b \\
Z \rightarrow XaX_b \\
X_a \rightarrow a \\
X_b \rightarrow b .
\]

that are in Chomsky normal form. We have \( L(G') = L(G_2) = L(G_1) = L(G) \setminus \{ \Lambda \} \).

The final score is given by the sum of the points obtained.

Return your homework solution to Sander van Rijn (svr003@gmail.com). The deadline is on Tuesday November 11, 2014. This deadline is strict, thus homework solutions sent after the deadline will not be considered.

Remember to write in your solution your name, surname and student number.