Question 1: [3 points]
Find a regular expression for each of the following languages over the alphabet $\Sigma = \{0,1\}$:

a) $L = \{ x \in \Sigma^* \mid x \text{ begins and ends with } 0 \}$
$0(0+1)^*0 + 0$

b) $L = \{ x \in \Sigma^* \mid \text{no consecutive } 1\text{'s appear in } x \}$
$(0 + 10)^*(1 + \Lambda)$

c) $L = \{ x \in \Sigma^* \mid 1 \text{ appears twice in } x \text{ and only after a } 0 \}$
$00^*100^*10^*$

Question 2: [3 points]
Find a deterministic finite automaton equivalent to the following non-deterministic one by first eliminating $\Lambda$-transitions and then by using the powerset construction. Label the state of the deterministic automaton so as to make it clear how they are obtained from the powerset construction.

We first eliminate the $\Lambda$-transitions. The automaton obtained is the following

Next we determinize the above NFA obtaining the automaton:
The language accepted by the automaton is $\Lambda + bb^* (a+b)b^*$.

**Question 3:**  
Find a non-deterministic finite automaton with $\Lambda$-transitions accepting the same language as the one denoted by the regular expression $(a + aab)^*b$ using the construction described in class.

The automata for $a$ and $b$ are

From which we obtain the automata for $aab$

and for $(a + aab)$

The automaton for $(a + aab)^*$ is

And finally we obtain the required automaton for the expression $(a+aab)^*b$
Question 4: Calculate the a-derivative and the a-partial derivative of \((a + aab)^b\).

First we calculate the a-derivative of \((a + aab)^b\):

\[
\delta_a((a + aab)^b) = \delta_a((a + aab)^b) + \delta_b(b) \quad \text{because } \Lambda((a + aab)^b) = 1
\]

\[
= \delta_a(a + aab) (a + aab)^b + \emptyset
\]

\[
= (\delta_a(a) + \delta_a(aab)) (a + aab)^b + \emptyset
\]

\[
= (\Lambda + \delta_a(a)ab) (a + aab)^b + \emptyset \quad \text{because } \Lambda(aab) = \min\{\Lambda(a), \Lambda(ab)\} = \min\{0, \Lambda(ab)\} = 0
\]

\[
= (\Lambda + \Lambda ab) (a + aab)^b + \emptyset.
\]

Finally, the a-partial derivative of \((a + aab)^b\) is calculated as follows:

\[
\partial_a((a + aab)^b) = \partial_a((a + aab)^b) \cup \partial_a(b) \quad \text{because } \Lambda((a + aab)^b) = 1
\]

\[
= \partial_a(a + aab) \cdot (a + aab)^b \cup \emptyset
\]

\[
= (\partial_a(a) \cup \partial_a(aab)) \cdot (a + aab)^b
\]

\[
= (\{\Lambda\} \cup \partial_a(a) \cdot ab) \cdot (a + aab)^b \quad \text{because } \Lambda(aab) = 0
\]

\[
= (\{\Lambda\} \cup \{\Lambda\} \cdot ab) \cdot (a + aab)^b
\]

\[
= \{\Lambda, ab\} \cdot (a + aab)^b
\]

\[
= \{(a + aab)^b, ab(a + aab)^b\}.
\]

The final score is given by the sum of the points obtained.

Return your homework solution to Sander van Rijn (svr003@gmail.com). The deadline is on Tuesday October 21, 2014. This deadline is strict, thus homework solutions sent after the deadline will not be considered.

Remember to write in your solution your name, surname and student number.