Question 1: [1.5 points]

a) Give a regular expression for the language $L$ of alternating 0’s and 1’s.

\[(0+\Lambda)(10)^*(1+\Lambda)\]

b) Give a deterministic finite automaton $M$ for the complement of language $L$ above.

\[\text{We remove state } q_3;\]

\[\text{Next we remove state } q_1;\]

\[\text{We remove state } q_2;\]
We finish by removing state $q_0$:

**Question 2:**  
Use the subset construction to convert the following nondeterministic automaton to a deterministic one.

**Question 3:**  
Consider the language $L = \{ w \in \{a,b\}^* \mid abw = wba \}$. Is the language empty? Is it regular? Motivate your answers.

It is not the empty set since, for instance, $w = a$ and $w = aba$ satisfies $abw = wba$. Further, the condition $abw = wba$ says that $w$ has to start with $ab$ since this is the prefix on the left hand side. In the same way, the string has to end in $ba$ since that is the postfix on the right hand side. The only exception is $w = a$ because it satisfies the condition $aba = aba$. In other words, the string $w$ begins with an $a$ followed by zero or more $ba$’s, that is $L = L(a(ba)*)$. Since there exists a regular expression describing $L$, it is regular.

**Question 4:**  
Given the following context free grammar

\[ S \rightarrow AB \mid aaB \]
\[ A \rightarrow a \mid Aa \]
\[ B \rightarrow b \]

a) Give a string generated by the grammar that has two leftmost derivations.

\[ S \Rightarrow aaB \Rightarrow aab \] and \[ S \Rightarrow AB \Rightarrow AaB \Rightarrow aaB \Rightarrow aab \]

b) Give an equivalent unambiguous regular grammar.

\[ S \rightarrow aS \mid b \]
It is not ambiguous because at any derivation step there is only one choice to make. This grammar is equivalent to the previous one, because both grammars generate the same language: all the strings that start with one or more \( a \) and end with a single \( b \).

c) Transform the grammar into an equivalent non-deterministic finite automaton.

\[
\begin{align*}
\text{S} & \rightarrow a\text{S}b & \text{if there are at least as many b as a}\n\text{S} & \rightarrow a\text{S}bb & \text{if there cannot be more than twice as many b as a}.
\end{align*}
\]

Question 5: [2.5 points]

a) Consider the language \( L = \{ a^n b^m \mid 0 \leq n \leq m \leq 2n \} \). Use the pumping lemma to show that it is not regular.

Let \( k \) be the number from the pumping lemma, and consider the string \( x = a^k b^{2k} \in L \). Note that \( |a^k b^{2k}| \geq k \). For every decomposition \( x = uvw \) with \( |uv| \leq k \) and \( v \neq \Lambda \), it holds that both \( u \) and \( v \) consist of only \( a \)'s. Thus \( uv^3w \) will add at least \( 3k \) \( a \)'s, and it is of the form \( a^{i+3k} b^{2k} \) for some \( 0 \leq i \leq k \). Thus \( 2k < 3k+i \), implying that \( uv^3w \) is not in \( L \). This contradicts the pumping lemma for regular languages and thus \( L \) cannot be regular.

b) Find a context-free grammar \( G \) generating the language \( L \).

\[
S \rightarrow aSb \mid aSbb \mid \Lambda
\]

Because of the first production there are at least as many \( b \) as \( a \), while the second production says that there cannot be more than twice as many \( b \) as \( a \).

c) Transform your grammar \( G \) into an equivalent pushdown automaton using the top-down construction.

Apply definition 5.17 from the textbook to obtain the desired automaton:

Question 6: [2 points]

a) Construct a pushdown automaton with two states that recognizes the set of binary strings containing an equal number of 1’s and 0’s.
b) Give a computation showing that 010 is not recognized by your pushdown automaton.

\[(p, 010, Z_0) \Rightarrow (q, 10, 0Z_0) \Rightarrow (q, 0, Z_0)\]

Since \(q\) is not an accepting state and there are no other moves the string is not accepted.

c) Give a computation showing that 0011 is recognized by your pushdown automaton.

\[(p, 0011, Z_0) \Rightarrow (q, 011, 0Z_0) \Rightarrow (q, 11, 00Z_0) \Rightarrow (q, 1, 0Z_0) \Rightarrow (p, \Lambda, Z_0)\]

Since \(p\) is an accepting state and there is no more input the string is accepted.

The final score is given by the sum of the points obtained.