LIACS
Leiden University

Examination Fundamentele Informatica 2
24 August 2009, 14:00 - 17:00

Question 1: **[1,5 points]**

a) Find at least four different pairs of languages A and B over the alphabet $\Sigma = \{0, 1\}$ for which their concatenation AB is the set of strings { 10, 111, 1010, 1000, 10111, 101000 }.
b) Give regular expressions for the intersection, union, and concatenation of the two languages $A = \{ w \in \Sigma^* \mid w \text{ begins with } 11\}$ and $B = \{ w \in \Sigma^* \mid w \text{ ends with } 00\}$.
c) Let R be a regular expression. Do $R + \varnothing$ and $R\Lambda$ denote the same language? And what about $R\varnothing$ and $\varnothing R$? And $\Lambda+R(R^*)$ and R^*? Justify your answers.

Question 2: **[2,5 points]**

a) Give a non-deterministic finite automaton with at most three states accepting the language of the regular expression $\Lambda + ((0 + 00^*1)1^*)$.
b) Using the powerset construction, find a deterministic finite automaton that recognizes the same language as the non-deterministic finite automaton in the above item a).
c) Suppose $M = (Q, \Sigma, q_0, A, \delta)$ is a non-deterministic finite automaton with Λ-transitions recognizing the language L. Let M_1 be the automaton obtained from M by adding a Λ-transition from each state in A to q_0. Describe the language $L(M_1)$ in terms of L.

Question 3: **[2,5 points]**

For each statement below decide if it is true or false. If it is true, prove it, otherwise give a counterexample.

a) If $L_1 \subseteq L_2$ and L_1 is not regular, then also L_2 is not regular.
b) If L is not regular then also its complement L' is not regular.
c) If L_1 is regular, L_2 is not regular and $L_1 \cap L_2$ is regular, then $L_1 \cup L_2$ is not regular.

Question 4: **[2 points]**

a) Is the language of all odd-length strings over the alphabet {a,b,c} regular? Justify your answer.
b) Is the language of all odd-length strings over the alphabet {a,b,c} with middle symbol b regular? Justify informally your answer.
c) Use the pumping lemma for regular languages to show that the language $L = \{a^n b^m c^k \mid n+m < k\}$ is not regular.

Question 5: **[1,5 points]**

a) Give the definitions of context free grammar and of regular grammar.
b) Find a *context-free* grammar for language $L_1 = \{a^n b^m c^k \mid n+2m \geq k\}$.
c) Find a *regular* grammar for the language $L_2 = \{a^n b^m \mid n \geq 0, m \geq 2\}$.

The final score is given by the sum of the points obtained.