

GP&C—Rush Hour

Games, Puzzles, & Computation

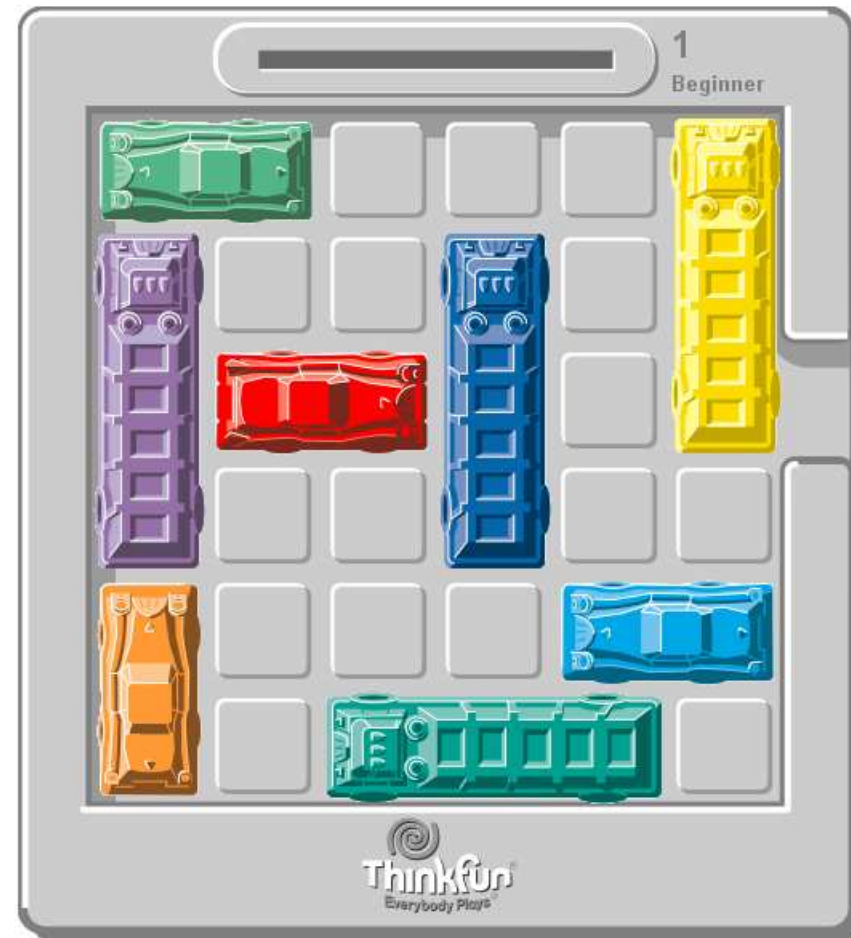
Rush Hour

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IPA, Eindhoven, Friday July 8, 2011

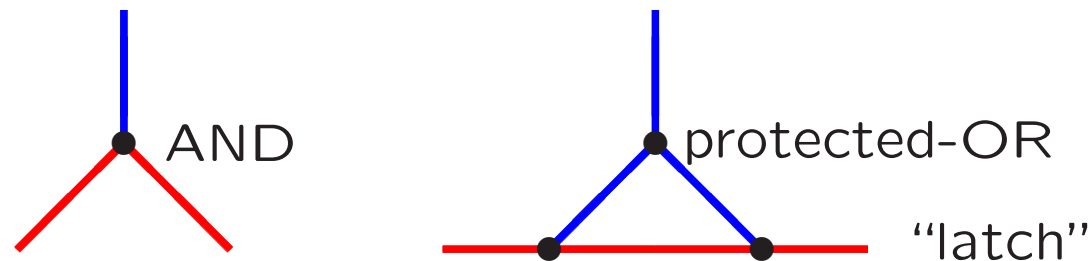
Having seen the general picture and some gadgetry, we now examine particular games and puzzles, like **Rush Hour®**:



<http://www.puzzles.com/products/rushhour.htm>

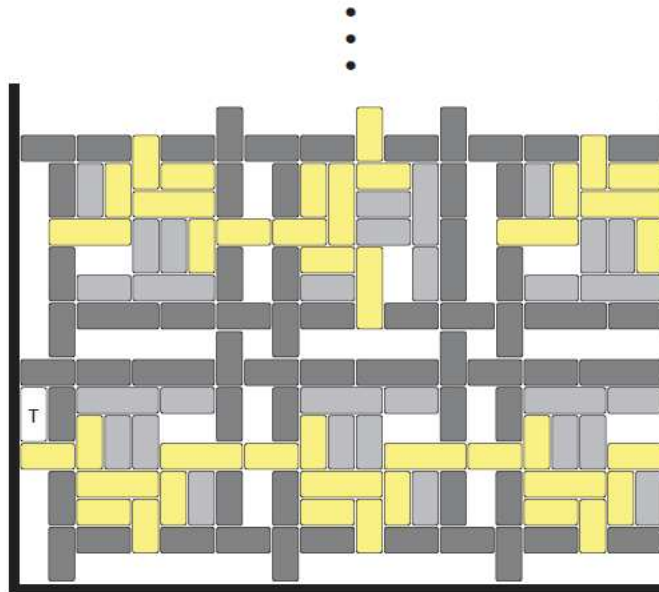
Theorem Rush Hour is PSPACE-complete.
 (Remember that PSPACE = NPSPACE (Savitch).)

The proof proceeds by reduction from Nondeterministic Constraint Logic (NCL): NCL is PSPACE-complete for planar graphs using only ANDs and protected-ORs.

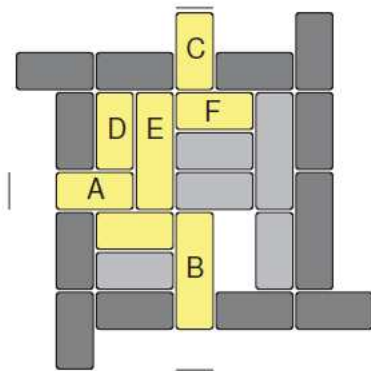


The decision problem is: Given a constraint graph G and a distinguished edge e in G ; is there a sequence of edge reversals that eventually reverses e ?

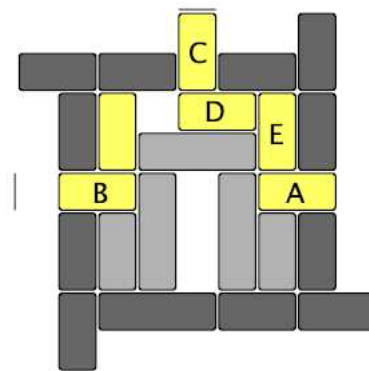
Moves may be repeated: it is an *unbounded game*.



(a) Layout



(b) AND

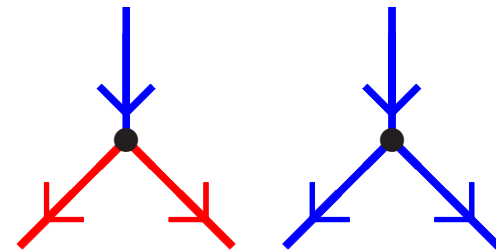


(c) Protected OR

“car” is in

\Leftrightarrow

edge points out

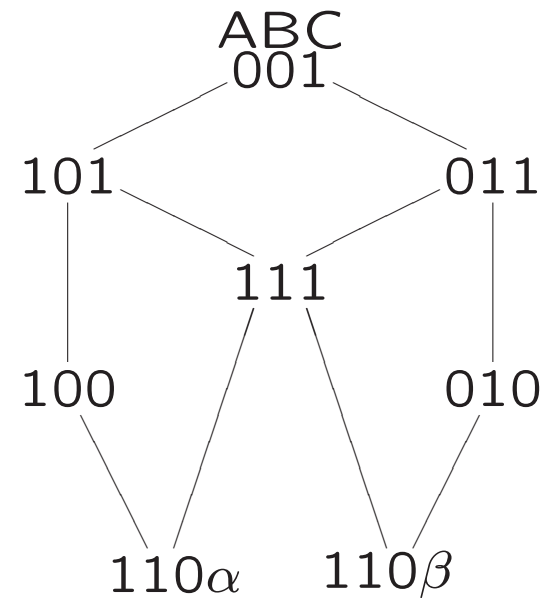
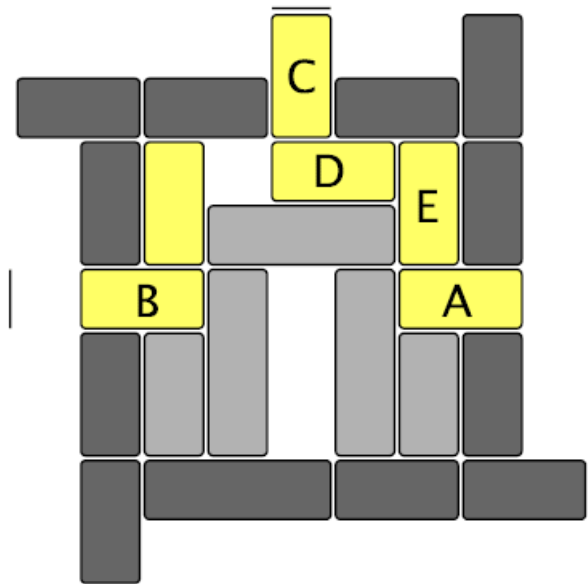


Exercise: Fill in the proof details.

This includes

- proper inner working of the gadgets,
- proper communication between gadgets,
- proper glueing together (in polynomial space),
- . . .

The statespace for the Rush-Hour protected-OR gadget is somewhat strange (where 1: car out; 0: car in):



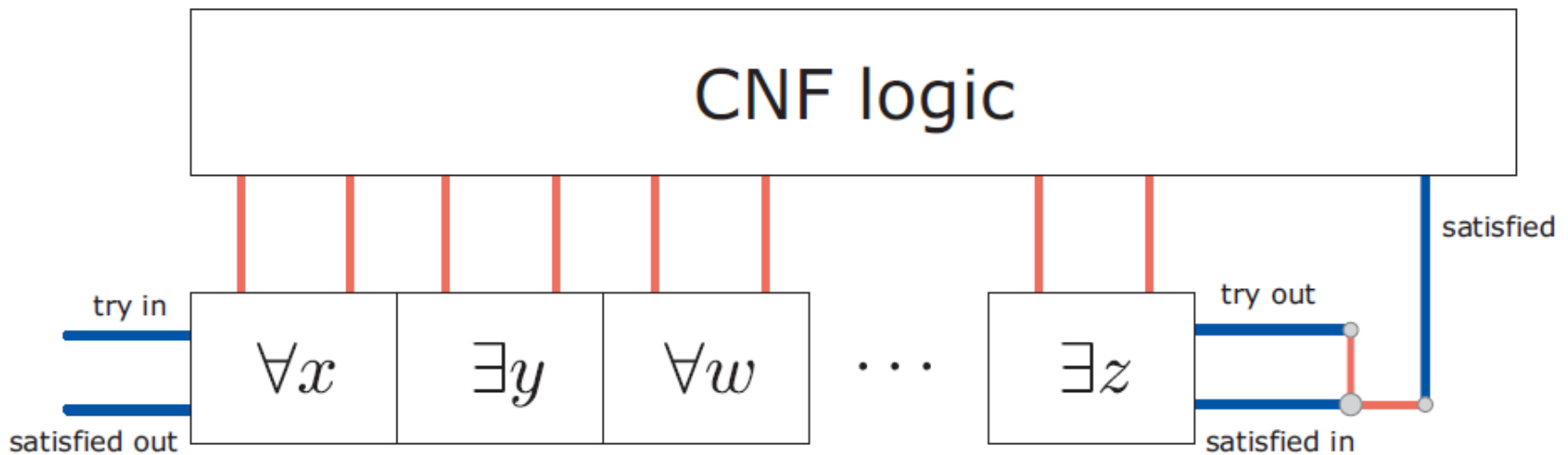
Clearly, NCL is in NPSPACE (= PSPACE).

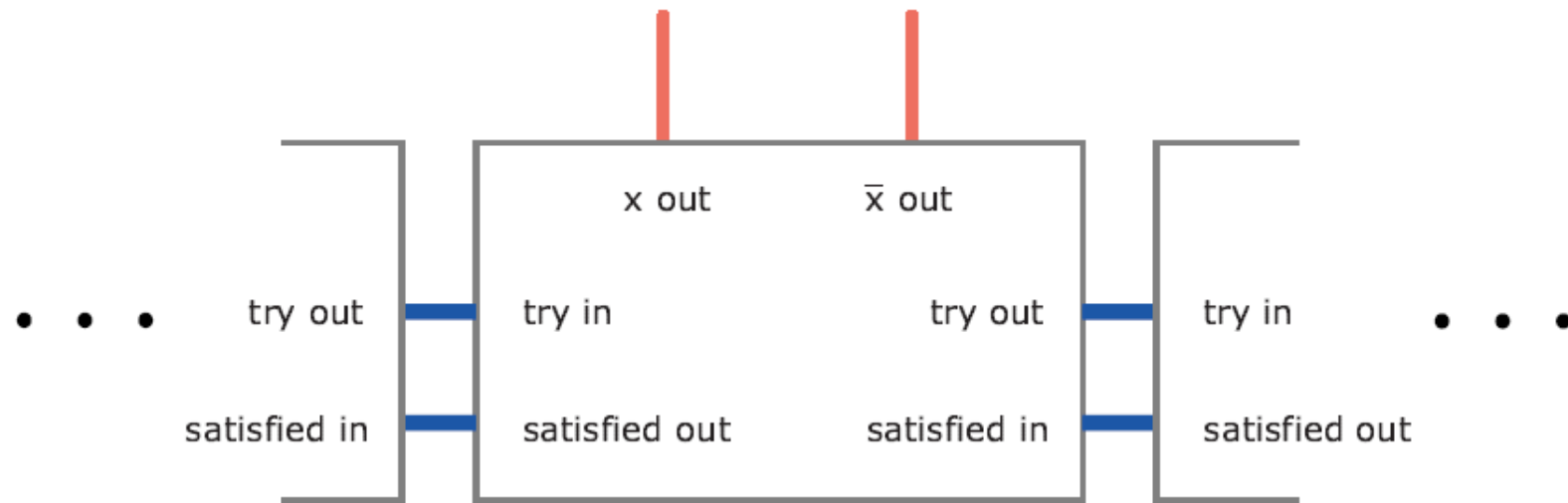
To prove that NCL is PSPACE-hard, we reduce from **Quantified Boolean Logic (QBF)**:

Given a quantified Boolean formula ϕ (with \exists and \forall , and of course variables, \vee , \wedge , \neg and brackets), is this ϕ true?

And QBF is known to be PSPACE-complete: it is an element of PSPACE and every problem in PSPACE reduces to QBF — and so to NCL.

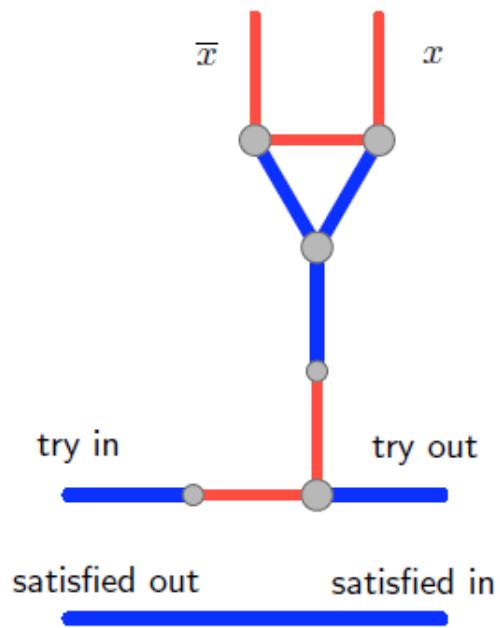
$$\forall x \exists y \forall w \dots \exists z [(x \vee y) \wedge \dots \wedge (\bar{z} \vee x \vee \bar{w})]$$



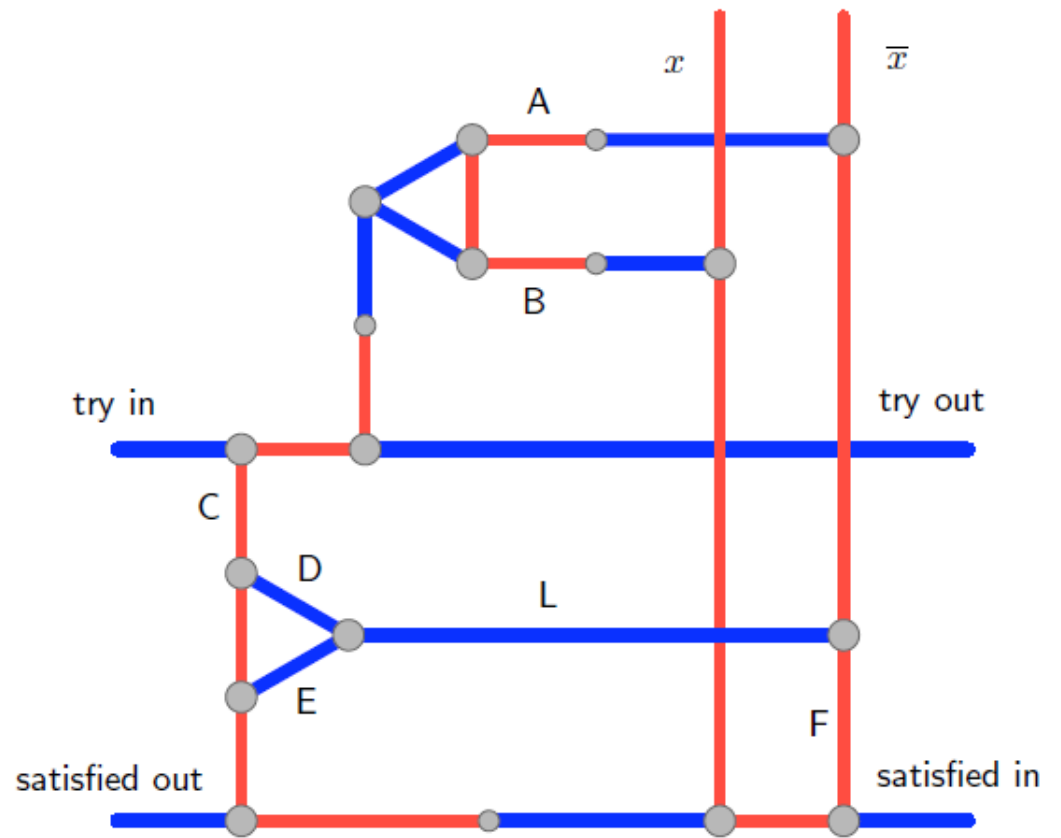


In a \exists gadget, you try (fix) both x and \bar{x} to be true, and then try the next quantifier.

In a (more complicated) \forall gadget, you do both, meanwhile “recursively” doing the rest twice.

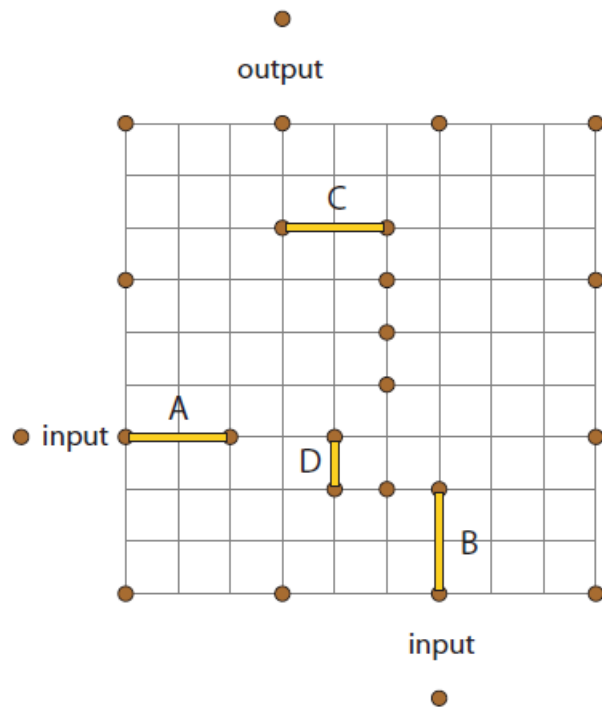


(a) Existential quantifier

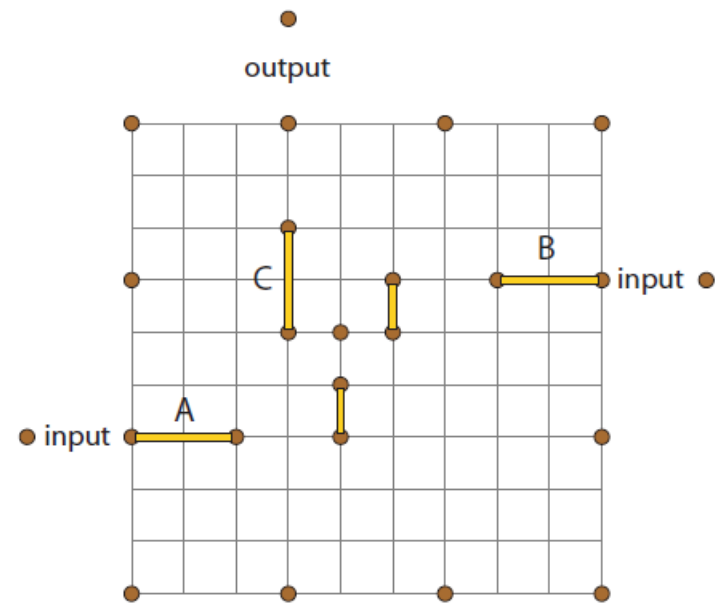


(b) Universal quantifier

And the **Plank puzzle** is also PSPACE-complete:



(a) AND



(b) OR

