VAK Theorie van Concurrency
DATUM 7-1-11
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1 (a) In the firing sequence $t_2 t_1 t_5 t_4 t_3$,

$$p_2p_3p_6 \xrightarrow{t_2} p_1p_4p_6 \xrightarrow{t_1} p_2p_4p_6 \xrightarrow{t_5} p_2p_4p_5 \xrightarrow{t_4} p_3p_5p_6 \xrightarrow{t_3} p_2p_3p_6$$

all transitions occur once.
We see that

$C_{in}[t_2t_1t_5\rangle \{p_2, p_4, p_5\}$

$C_{in}[t_2t_5t_3\rangle \{p_1, p_2, p_4\}$

$C_{in}[t_2t_5t_3t_4t_5t_3\rangle \{p_1, p_2, p_3\}$

so $\{p_2, p_4, p_5\}$, $\{p_1, p_2, p_4\}$ and $\{p_1, p_2, p_3\}$ are reachable.
(for verification also see the SCG in (b))

(b) see next page

(c) ~~In~~ Let $t \in T$ and $C \in \mathbb{C}_M$.
We say that t has contact in C if t has input-concession but no output-concession:

$${}^\bullet t \subseteq C \text{ and } t^\bullet \cap C \neq \emptyset.$$

M is contact-free if no such t and C exist; but with

$$C = \{p_1, p_2, p_3\} \in \mathbb{C}_M$$

we have

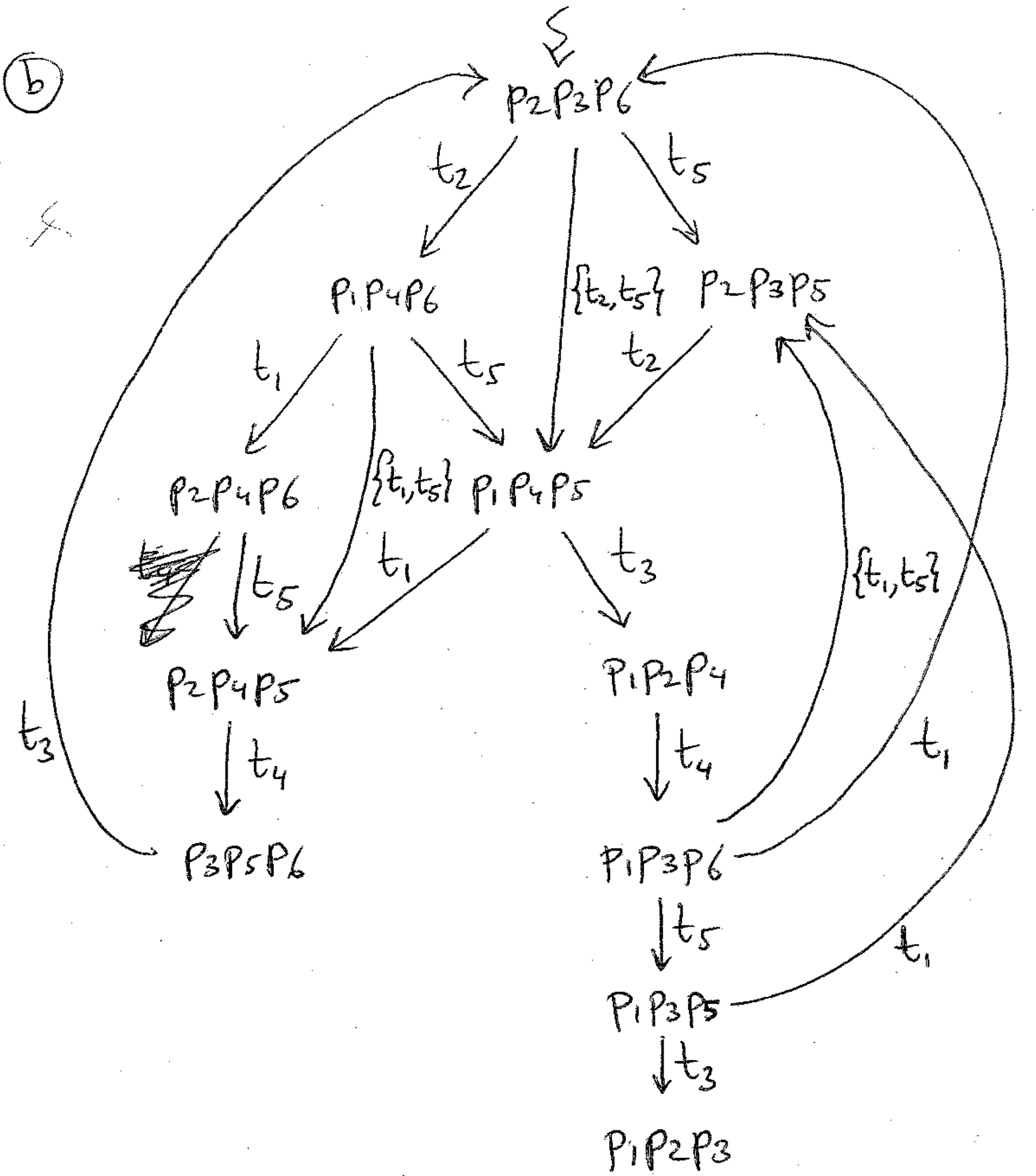
$${}^\bullet t_2 = \{p_2, p_3\} \subseteq C$$

$$t_2^\bullet \cap C = \{p_1, p_4\} \cap C = \{p_1\} \neq \emptyset.$$

so M is not contact-free.

(continued)

(b)



(c) (continued)

All transitions of M are useful, because all occur in the firing sequence $t_2t_1t_5t_4t_3$.

No transition is live, because in the reachable configuration $C = \{p_1, p_2, p_3\}$ no transition can be fired; in particular, for all transitions $t \in T$, there is no $x \in T^*$ such that

xt con C.

(d) See previous page; we have added the concurrent steps in red.
(they can be found from SCG by the sequentialization property)

(e) if t con C, then ~~cfl~~ $cfl(t,C) = \{s \in T : s \text{ con } C,\ \neg\{s,t\} \text{ con } C\}$.

Using the CG we find:

$cfl(t_1, \{p_1, p_4, p_5\}) = \{t_3\}$

$cfl(t_3, \{p_1, p_4, p_5\}) = \{t_1\}$

$cfl(t_1, \{p_1, p_3, p_5\}) = \{t_3\}$

$cfl(t_3, \{p_1, p_3, p_5\}) = \{t_1\}$

(f) A triple (C, t, u) with $C \in \mathcal{C}_M$, $t, u \in T$, is called a confusion if

- $\{t, u\}$ con C, and
- if $D \in \mathcal{C}_M$ is such that $C[u\rangle D$, then $cfl(t,C) \neq cfl(t,D)$.

$t \neq u$

The first condition gives only ~~three~~ six candidates (using the CG):

- $(\{p_2, p_3, p_6\}, t_2, t_5)$. but
  $cfl(t_2, \{p_2, p_3, p_6\}) = \emptyset = cfl(t_2, \{p_2, p_3, p_5\})$
- $(\{p_2, p_3, p_6\}, t_5, t_2)$ but
  $cfl(t_5, \{p_2, p_3, p_6\}) = \emptyset = cfl(t_5, \{p_1, p_4, p_6\})$
- $(\{p_1, p_4, p_6\}, t_1, t_5)$
  $cfl(t_1, \{p_1, p_4, p_6\}) = \emptyset \neq \{\underset{t_3}{p_3}\} = cfl(t_1, \{p_1, p_4, p_5\})$!

(continued)

- $(\{p_1, p_4, p_6\}, t_5, t_1)$ but
  $cfl(t_5, \{p_1, p_4, p_6\}) = \emptyset = cfl(t_5, \{p_2, p_4, p_6\})$
- $(\{p_1, p_3, p_6\}, t_1, t_5)$
  $cfl(t_1, \{p_1, p_3, p_6\}) = \emptyset \neq \{t_3\} = cfl(t_1, \{p_1, p_3, p_5\})$!
- $(\{p_1, p_3, p_6\}, t_5, t_1)$ but
  $cfl(t_5, \{p_1, p_3, p_6\}) = \emptyset = cfl(t_5, \{p_2, p_3, p_6\})$.

So the conflicts are:

$(\{p_1, p_4, p_6\}, t_1, t_5)$

conflict-increasing since $cfl(t_1, \{p_1, p_4, p_6\}) \subsetneq cfl(t_1, \{p_1, p_4, p_5\})$
asymmetric since $(\{p_1, p_4, p_6\}, t_5, t_1)$ is no conflict.
> CONFUSION

$(\{p_1, p_3, p_6\}, t_1, t_5)$

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asymmetric since $(\{p_1, p_3, p_6\}, t_5, t_1)$ is no conflict.
> CONFUSION

2 (a) if ${}^\bullet S = S^\bullet$ holds.

(b) $S \subseteq P$ determines a sequential component if $\#(C \cap S) = 1$ for all $C \in \mathcal{C}_M$; this can be checked by the SCG in 1(b).

preset conditions:		postset conditions:	
${}^\bullet p_1 = \{t_2\}$	$(p_2$ or $p_3)$	$p_1^\bullet = \{t_1\}$	(p_2)
${}^\bullet p_2 = \{t_1, t_3\}$	$(p_1$ and $p_5)$	$p_2^\bullet = \{t_2, t_4\}$	$((p_1$ or $p_4)$ and $(p_3$ or $p_6))$
${}^\bullet p_3 = \{t_4\}$	$(p_2$ or $p_4)$	$p_3^\bullet = \{t_2\}$	$(p_1$ or $p_4)$
${}^\bullet p_4 = \{t_2\}$	$(p_2$ or $p_3)$	$p_4^\bullet = \{t_4\}$	$(p_3$ or $p_6)$
${}^\bullet p_5 = \{t_5\}$	(p_6)	$p_5^\bullet = \{t_3\}$	(p_2)
${}^\bullet p_6 = \{t_4\}$	$(p_2$ or $p_4)$	$p_6^\bullet = \{t_5\}$	(p_5)

(continued)

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2 (b) (continued)

Hence ~~the~~ subsystems are determined by the following $S \subseteq P$:

$S = \emptyset$ not sequential: $\#(C_{in} \cap \emptyset) = 0$

$S = P$ not sequential: $\#(C_{in} \cap P) = 3$

$S = \{p_1, p_2, p_5, p_6\}$ not sequential: $\#(C_{in} \cap S) = 2$

$S = \{p_1, p_2, p_3, p_5, p_6\}$ not sequential: $\#(C_{in} \cap S) = 3$

$S = \{p_1, p_2, p_4, p_5, p_6\}$ not sequential: $\#(C_{in} \cap S) = 2$

$S = \{p_3, p_4\}$ sequential (obvious)

(c) Yes: take $C = \{p_2\}$. Clearly, no transition can be fired from C, so $\mathcal{C}_{M'} = \{C\}$.

The ~~subsystem~~ subset $P \subseteq P$ determines a subsystem of M' and

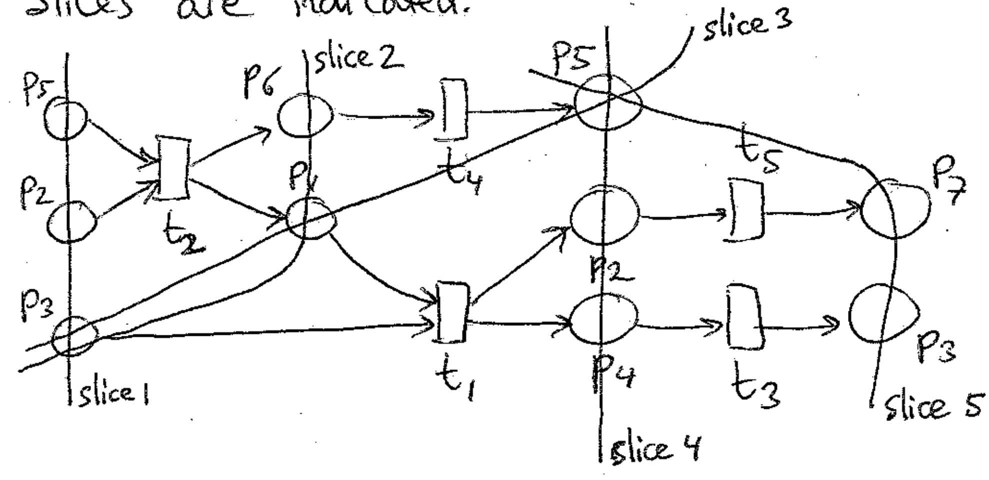
$\forall D \in \mathcal{C}_{M'} : \#(D \cap P) = 1$

(for only $D = C$ occurs)

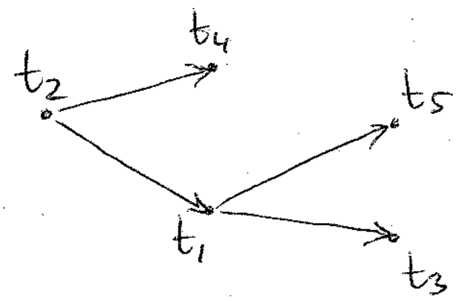
So P ~~is~~ determines a sequential component; this component covers M'.

Inderdaad (reduced word niet geëist!)

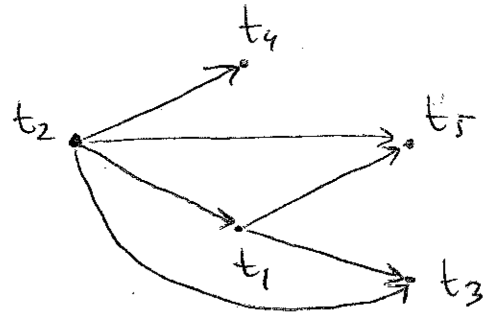
(a) Slices are indicated:



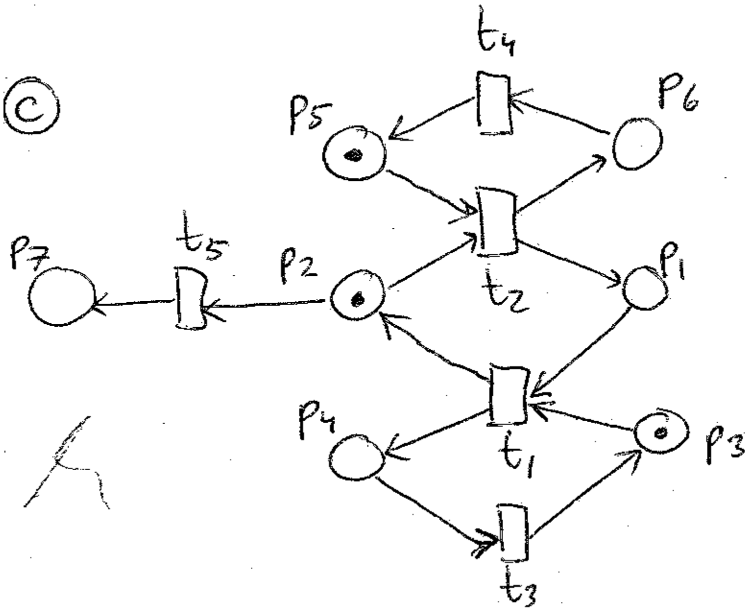
(b) ctr(N):



tra(ctr(N)):



(c)



A firing sequence of M defining N is $t_2 t_4 t_1 t_5 t_3$

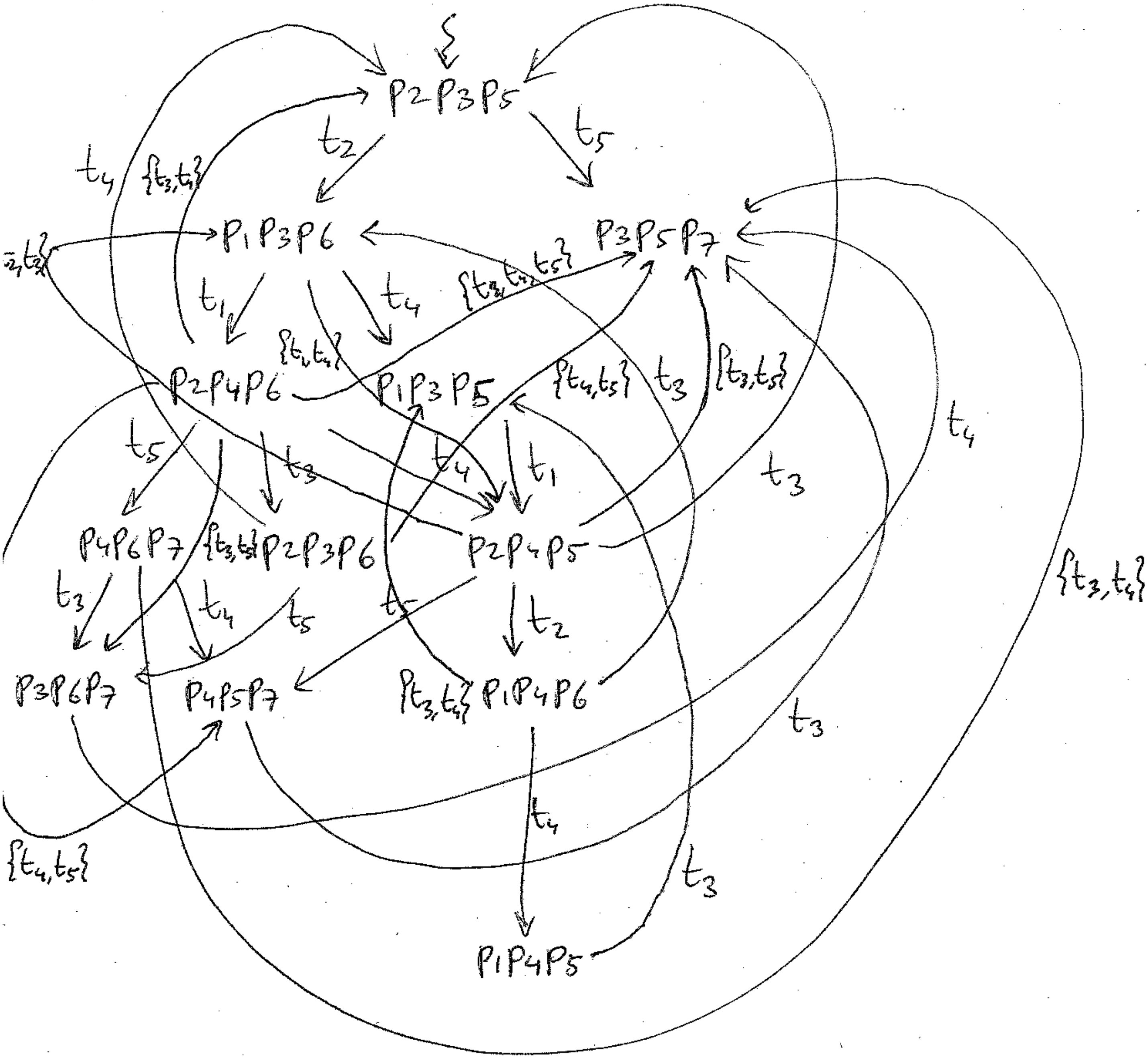
(this is also a word of ctr(N)).

(d) $ind(M) = \{(s,t) \in useT \times useT \; : \; s \neq t, \exists C \in \mathcal{C}_M : \{s,t\} \text{ con } C\}$.

We draw the CG of M (see next page).

We read from it:

$ind(M) = \{(t_1,t_4), (t_4,t_1), (t_3,t_4), (t_4,t_3), (t_3,t_5), (t_5,t_3),$
$(t_4,t_5), (t_5,t_4), (t_2,t_3), (t_3,t_2)\}$



ⓔ $pru(ctr(N)) = ctr(N)$ (see ⓑ).

$$coords(pru\cancel{(N)}(ctr(N))) = \{ t_2 t_4 t_1 t_5 t_3,\ t_2 t_4 t_1 t_3 t_5,\ t_2 t_1 t_4 t_5 t_3,\ t_2 t_1 t_4 t_3 t_5,\ t_2 t_1 t_5 t_4 t_3,\ t_2 t_1 t_3 t_4 t_5,\ t_2 t_1 t_5 t_3 t_4,\ t_2 t_1 t_3 t_5 t_4 \}.$$

(continued)

(used pairs of ind(M) are indicated)

They indeed belong to the same trace, since:

$$t_2 t_4 t_1 t_5 t_3 \;\underset{(t_3,t_5)}{\doteq}\; t_2 t_4 t_1 t_3 t_5$$

$(t_1,t_4)\ \|\cdot$

$$t_2 t_1 t_4 t_5 t_3 \;\underset{(t_3,t_5)}{\doteq}\; t_2 t_1 t_4 t_3 t_5$$

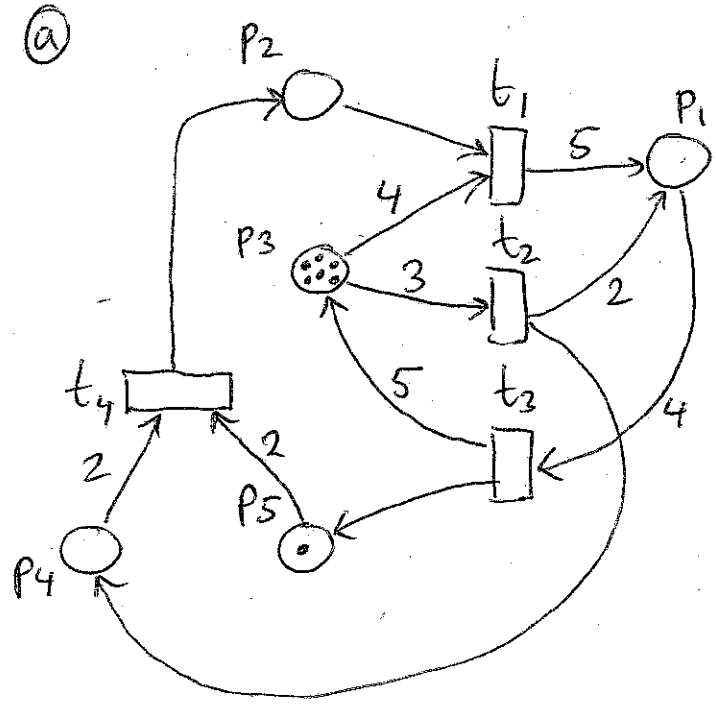
$(t_4,t_5)\ \|\cdot$ $\qquad$ $(t_4,t_3)\ \|\cdot$

$$t_2 t_1 t_5 t_4 t_3 \qquad\qquad t_2 t_1 t_3 t_4 t_5$$

$(t_4,t_3)\ \|\cdot$

$$t_2 t_1 t_5 t_3 t_4 \;\underset{(t_3,t_5)}{\doteq}\; t_2 t_1 t_3 t_5 t_4 .$$

4 (a)



~~(b)~~

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(b) We apply the finiteness algorithm:

$$\begin{tikzcd}
 & (0,0,6,0,1) \arrow[dl, "t_2"'] \\
(2,0,3,1,1) \arrow[d, "t_2"'] & \\
(4,0,0,2,1) \arrow[d, "t_3"] & \\
(0,0,5,2,2) \arrow[d, "t_2"] \arrow[dr, "t_4"] & \\
(2,0,2,3,2) \arrow[d, "t_4"] & (0,1,5,0,0) \arrow[dl, "t_2"'] \arrow[d, "t_1"] \\
(2,1,2,1,0) & (5,0,1,0,0) \arrow[d, "t_3"] \\
 & (1,0,6,0,1)
\end{tikzcd}$$

we find : $(0,0,6,0,1)\,[t_2t_2t_3t_4t_1t_3\rangle\,(1,0,6,0,1)$

and $(0,0,6,0,1) < (1,0,6,0,1)$

so C_M is infinite.

ⓒ $$M' = \begin{pmatrix} 4 & -1 & -4 & 0 & 0 \\ 2 & 0 & -3 & 1 & 0 \\ -4 & 0 & 5 & 0 & 1 \\ 0 & 1 & 0 & -2 & -2 \end{pmatrix} \begin{array}{l} -2 \text{ times} \\ +2 \text{ times} \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{pmatrix} 0 & -1 & 2 & -2 & 0 \\ 2 & 0 & -3 & 1 & 0 \\ 0 & 0 & -1 & 2 & 1 \\ 0 & 1 & 0 & -2 & -2 \end{pmatrix} \quad +1 \text{ times} \quad -3 \text{ times} \quad \text{times } -1$$

$$\downarrow$$

$$\begin{pmatrix} 0 & 0 & 2 & -4 & -2 \\ 2 & 0 & 0 & -5 & -3 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 1 & 0 & -2 & -2 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 2 & 0 & 0 & -5 & -3 \\ 0 & 1 & 0 & -2 & -2 \\ 0 & 0 & 2 & -4 & -2 \\ 0 & 0 & 1 & -2 & -1 \end{pmatrix} \begin{array}{l} \\ \\ -2 \text{ times} \\ \\ \end{array}$$

$$\downarrow$$

$$\begin{pmatrix} 2 & 0 & 0 & -5 & -3 \\ 0 & 1 & 0 & -2 & -2 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Its kernel is generated by $(3, 4, 2, 0, 2)$ and $(5, 4, 4, 2, 0)$.
So the p-invariants are

$$\lambda(3,4,2,0,2) + \mu(5,4,4,2,0), \quad \lambda, \mu \in \mathbb{Z}.$$

(d) $i = (3,4,2,0,2) + (5,4,4,2,0) = (8,8,6,2,2)$

is a positive p-invariant of M' that covers M' and has value $8\cdot 0 + 8\cdot 0 + 6\cdot 6 + 2\cdot 0 + 2\cdot 1 = 38$.
So in all reachable configurations of M' we have

$$8\cdot C(p_1) + 8\cdot C(p_2) + 6\cdot C(p_3) + 2\cdot C(p_4) + 2\cdot C(p_5) = 38.$$

In particular, for every $p \in P$ we see that

$$C(p) \leq 38.$$

Hence, M' is bounded.

(e) $(5,4,4,2,0) - (3,4,2,0,2) = (2,0,2,2,-2)$

is a ~~positive~~ p-invariant of M' with value

$$2\cdot 0 + 0\cdot 0 + 2\cdot 6 + 2\cdot 0 - 2\cdot 1 = 10.$$

So all reachable configurations of M' satisfy

$$2\cdot C(p_1) + 2\cdot C(p_3) + 2\cdot C(p_4) - 2\cdot C(p_5) = 10$$

and so

$$\cancel{2}\, C(p_1) + C(p_3) + C(p_4) - C(p_5) = 5.$$

So we can take $p = 1$, $q = -1$, $n = 5$.

5 ⓐ A P/T-system $M = (P, T, F, W, C_{in})$ is a marked graph if:

- for all $(x,y) \in F$, $W(x,y) = 1$ and
- for all $p \in P$, $\#({}^\circ p) = \#(p^\circ) = 1$.

We see immediately from the diagram that these conditions are satisfied, so M is a marked graph.

ⓑ A cycle is a sequence $(q_0, q_1, \rightarrow q_m)$, $m \geq 0$, of places such that $q_i^\circ = {}^\circ q_{i+1}$ for $i = 0, \rightarrow m-1$, and $q_m^\circ = {}^\circ q_0$, and all q_i° $(i = 0, \rightarrow, m)$ are different.

cycles:

$p_1 p_6$ value 1

~~$p_1 p_3 p_4 p_5$~~

~~$p_1 p_7 p_2 p_5 p_4$~~

~~$p_2 p_5 p_4$~~

~~$p_3 p_4 p_5$~~

$p_4 p_5$ value 1

~~$p_5 p_4$~~ (same as above)

~~$p_6 p_1$~~ (same as above)

~~$p_7 p_2 p_5 p_4$~~

So the only cycles are (p_1, p_6) and (p_4, p_5), both with value 1.

ⓒ M is live iff all cycles have value > 0. This is the case (see ⓑ) so M is live.

Given that M is life: M is safe iff each place lies in a cycle with value 1. But for instance p_3 doesnot lie in such a cycle: so M is not safe.
(indeed, after transitions ~~$p_1 p_6 p_1$~~ $t_3 t_1 t_3$ we are in a configuration with $C(p_3) = 2$.)

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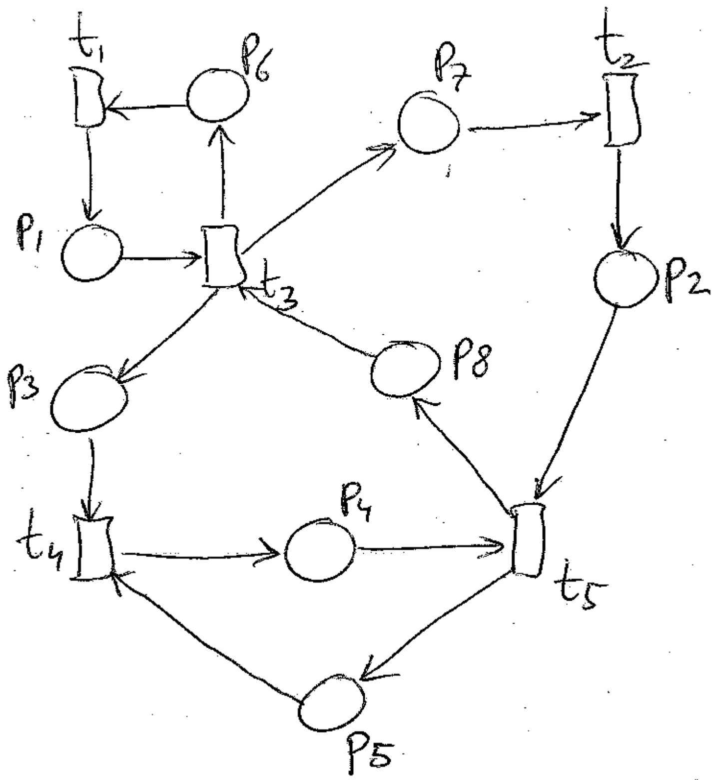
5 d The multinet:



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The cycles of this multinet are:

~~P1 P6~~
~~P2 P8 P7~~

(p1, p6)
(p2, p8, p7)
(p3, p4, p8)
(p4, p5).

Choose the initial configuration C_{in} = ~~{p1, p5, p8}~~ (1,0,0,0,1,0,0,1)

(so, p1, p5 and p8 have 1 token, the others none).

Then all cycles have value 1, and all places occur in at least one cycle. } (*)

(continued)

The resulting marked graph then is:

- live: since all cycles have value > 0
- safe: since it is live, and all places occur in a cycle with value 1 (follows from (*)).