4 EN Systems

Exercise 4.1 The EN system of Figure 1 simulates the changing of the seasons.

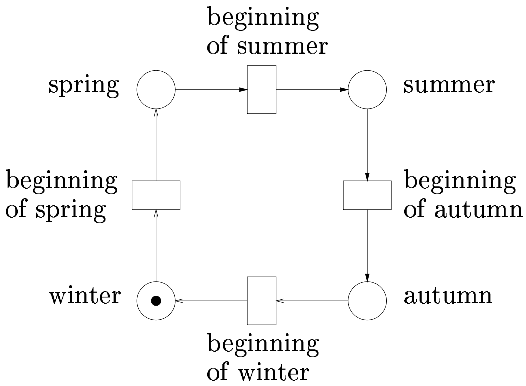

Fig. 1. The change of the seasons.

Add two places to the EN system; one that represents the state *neither winter nor spring* and one that represents the state *spring or autumn*.

Exercise 4.2 Let N be the net of Figure 2.

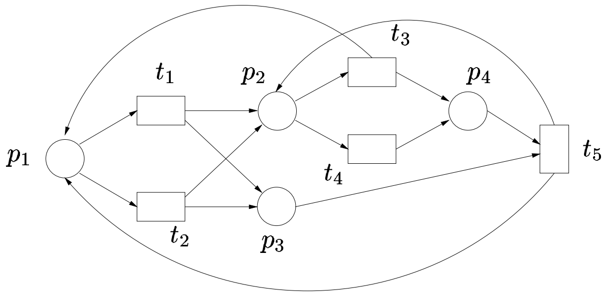

Fig. 2. The net N.

a. Determine

(i) ${}^{\bullet}p_2$
(ii) $({}^{\bullet}p_2)^{\bullet}$
(iii) $\{t_2, t_3\}^{\bullet}$
(iv) $(\{t_2, t_3\}^{\bullet})^{\bullet}$
(v) $\mathbf{nbh}(\{p_3, p_4\})$
(vi) $\mathbf{nbh}(\{p_2, t_3\})$
(vii) $F_N^+ \cap (T_N \times T_N)$

b. Is N
   (i) acyclic? (ii) P-simple? (iii) T-simple?

Exercise 4.3

a. Prove that, for an arbitrary net N, $P_N{}^\bullet = T_N$ and ${}^\bullet P_N = T_N$.
b. Prove that, for an arbitrary net N, $\mathbf{nbh}(T_N) = P_N$ if and only if N has no isolated places.
c. Prove that for all nets N and N' such that $N \equiv N'$, N is P-simple if and only if N' is P-simple.

Exercise 4.4 Give an example of a P-simple and T-simple net N in which there exists an $x \in X_N$ such that $({}^\bullet x)^\bullet \neq \{x\}$ and $(({}^\bullet x)^\bullet)^\bullet = x^\bullet$.

Exercise 4.5 Let M be the EN system for which $\mathbf{und}(M)$ is the net of Exercise 4.2 (depicted in Figure 2) and $(C_{in})_M = \{p_1\}$.

a. Determine all configurations C of $\mathbf{und}(M)$ such that t_3 **con** C. Determine for each of these C the configuration D of $\mathbf{und}(M)$ such that $C[t_3\rangle D$.
b. Determine configurations C_1, C_2 and C_3 of $\mathbf{und}(M)$ and transitions $u, v, w, x \in T_N$ such that $(C_{in})_M[u\rangle C_1[v\rangle C_2[w\rangle C_3[x\rangle\{p_1, p_4\}$.

Exercise 4.6 Consider the EN system M of Exercise 4.5 (see Figure 2). Determine $\mathrm{SCG}(M)$, $\mathrm{FS}(M)$, $\mathbb{C}_M$ and $\mathbf{use}(T_M)$. Does M contain live transitions?

Exercise 4.7 Let M be an EN system, let $s, u \in T_M$, and let C be a configuration of M. Prove that if sus **con** C, then $s^\bullet \subseteq {}^\bullet u$ and ${}^\bullet s \subseteq u^\bullet$.
(Hint: use Lemma 7.)
This is used in Theorem 39.

Exercise 4.8 Prove that a transition t of an EN system M is live if and only if $\forall x \in \mathrm{FS}(M) : \exists y \in T_M^* : xyt \in \mathrm{FS}(M)$.

Exercise 4.9 Give an example of an EN system M in which both live and non-live transitions appear. Give a regular expression for the language $\mathrm{FS}(M)$.

Exercise 4.10 Let M be the EN system of Figure 3.

a. Determine $\mathrm{SCG}(M)$.
b. Which transitions of M are live?
c. Determine all $U \subseteq T$ with $\#U \geq 2$ such that $\mathbf{disj}(U)$.
d. Determine $\mathrm{CG}(M)$.

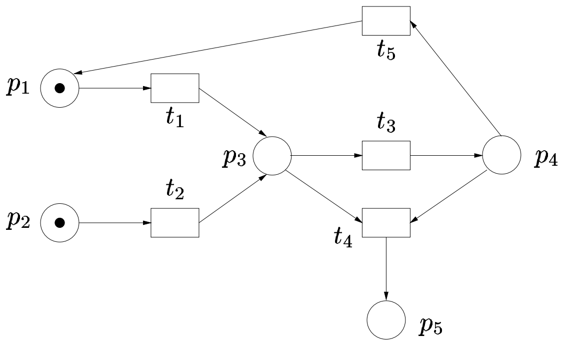

Fig. 3. The EN system M of Exercise 4.10.

Exercise 4.11 Let M be the EN system of Figure 4.

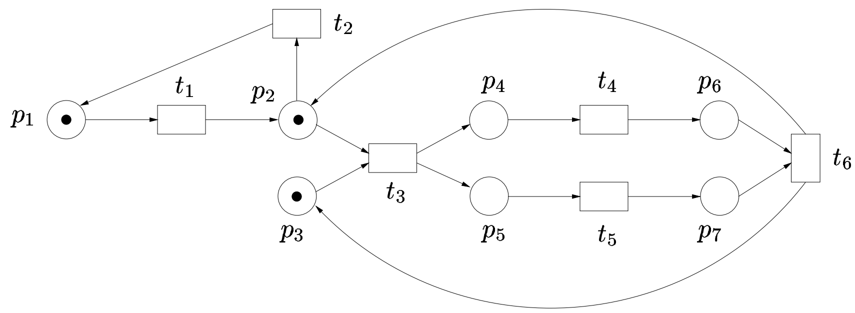

Fig. 4. The EN system M.

a. Determine SCG(M).
b. Which transitions of M are live?
c. Determine all $U \subseteq T$ with $\#U \geq 2$ for which there exist $C, D \in \mathbb{C}_M$ such that $C[U\rangle D$.

Exercise 4.12 Let $M = (P, T, F, C_{in})$ be a conflict-free EN system.

a. Prove that for all $C, D \in \mathbb{C}_M$ there exist $x, y \in T^*$ and $E \in \mathbb{C}_M$ such that $C[x\rangle E$ and $D[y\rangle E$.
(Hint: prove that for all $C, D, F \in \mathbb{C}_M$ and $u, v \in T^*$ with $F[u\rangle C$ and $F[v\rangle D$ there exist $x, y \in T^*$ and $E \in \mathbb{C}_M$ such that $C[x\rangle E$, $D[y\rangle E$, $|x| \leq |v|$ and $|y| \leq |u|$; do this by induction on $|u| + |v|$.)
b. Use a. to prove that M has no live transitions if and only if there exists $C \in \mathbb{C}_M$ in which no transition has concession.

Exercise 4.13 Let M be the EN system of Figure 5.

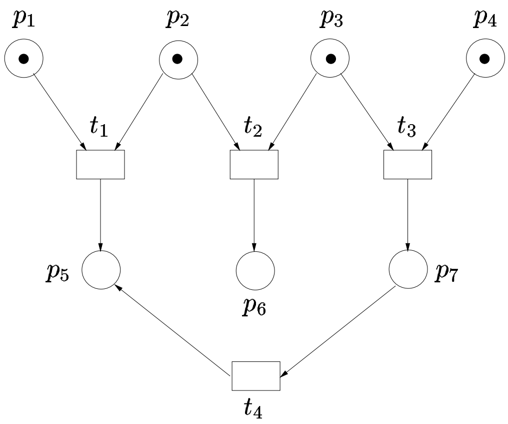

Fig. 5. The EN system M.

a. Determine $\mathbf{cfl}(t_1, (C_{in})_M)$.
   (i) Is $((C_{in})_M, t_1, t_4)$ a confusion?
   (ii) And $((C_{in})_M, t_1, t_3)$?
b. What type of confusions are the above (if they are confusions at all): cd, ci, symmetric, asymmetric?

Exercise 4.14 Do the EN systems of Exercise 4.10 and 4.11 (see Figures 3 and 4) contain confusions? If so, of which type?

5 Equivalences and Normal Forms

Exercise 5.1 Let $M = (P, T, F, C_{in})$ be the EN system of Figure 6.

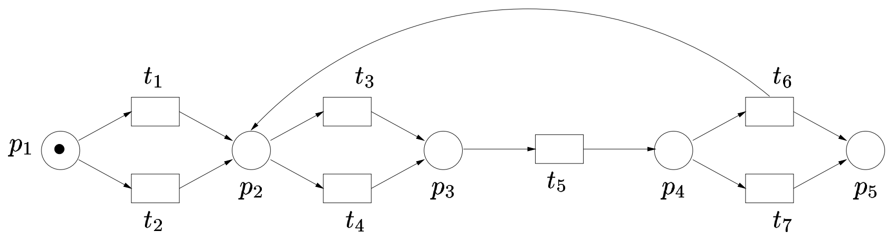

Fig. 6. The EN system M.

In the continuation of this exercise we will consider the EN systems M_1, M_2 and M_3, which are informally depicted in Figure 7.

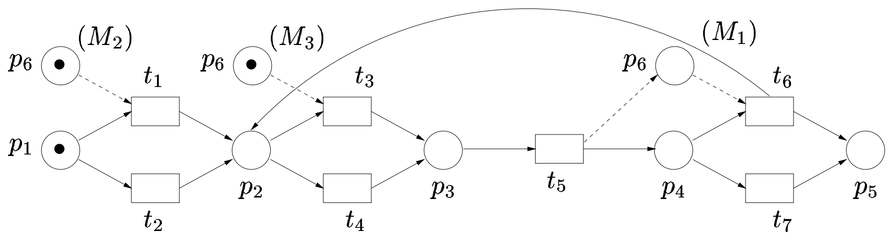

Fig. 7. The EN systems M_1, M_2 and M_3.

a. Consider the EN system $M_1 = (P_1, T_1, F_1, C_{in,1})$ with $P_1 = P \cup \{p_6\}$, $T_1 = T$, $F_1 = F \cup \{(t_5, p_6), (p_6, t_6)\}$ and $C_{in,1} = C_{in}$. Determine whether or not M and M_1 are configuration equivalent, weakly configuration equivalent, and firing sequence equivalent.

b. Consider the EN system $M_2 = (P_2, T_2, F_2, C_{in,2})$ with $P_2 = P \cup \{p_6\}$, $T_2 = T$, $F_2 = F \cup \{(p_6, t_1)\}$ and $C_{in,2} = C_{in} \cup \{p_6\}$. Determine whether or not M and M_2 are configuration equivalent, weakly configuration equivalent, and firing sequence equivalent.

c. Consider the EN system $M_3 = (P_3, T_3, F_3, C_{in,3})$ with $P_3 = P \cup \{p_6\}$, $T_3 = T$, $F_3 = F \cup \{(p_6, t_3)\}$ and $C_{in,3} = C_{in} \cup \{p_6\}$. Determine whether or not M and M_3 are configuration equivalent, weakly configuration equivalent, and firing sequence equivalent.

Exercise 5.2 Provide the details of the proof of Theorem 33.

Exercise 5.3 Let M be the EN system of Figure 8.

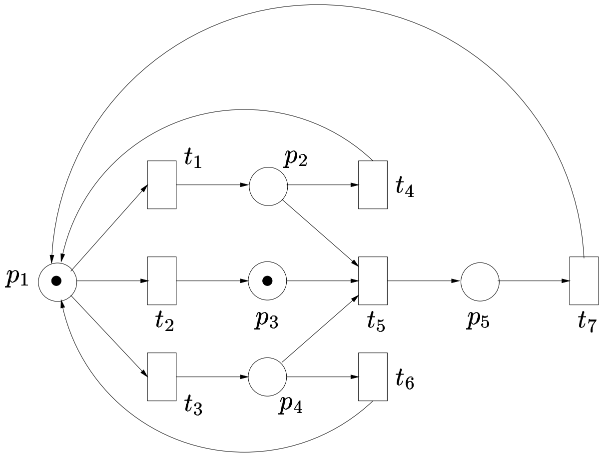

Fig. 8. The EN system M.

Determine a strongly reduced EN system that is configuration equivalent with M.

Exercise 5.4 Let M be a strongly reduced sequential EN system. Prove that M is live (i.e., that all transitions are live) if and only if $\mathbf{und}(M)$ is a strongly connected graph.

Exercise 5.5 Prove that an EN system M is configuration equivalent with a sequential EN system if and only if M has the following property:

$$\text{For all } C, D \in \mathbb{C}_M \text{ and } t \in T_M : \text{ if } t \textbf{ con } C \text{ and } t \textbf{ con } D, \text{ then } C = D.$$

Exercise 5.6 In a. and b. of this exercise we will show that an EN system M is firing sequence equivalent with a sequential EN system if and only if M has the following property:

$$\text{For all } C, D \in \mathbb{C}_M,\ t \in T_M \text{ and } x \in T_M^* : \\ \text{if } t \textbf{ con } C \text{ and } t \textbf{ con } D, \text{ then } (x \textbf{ con } C \Leftrightarrow x \textbf{ con } D).$$

a. Prove: if an EN system M is firing sequence equivalent with a sequential EN system, then M has the above property.

To prove that a. also holds the other way around, we now assume M to be an EN system having the above property. Define the relation $\alpha \subseteq \mathbb{C}_M \times \mathbb{C}_M$ by $\alpha = \{(C, D) \mid \forall x \in T_M^* : x \ \mathbf{con} \ C \Leftrightarrow x \ \mathbf{con} \ D\}$. Note that α is an equivalence relation on $\mathbb{C}_M$.

b. Construct a sequential EN system M' that is firing sequence equivalent with M.
   (Hint: let $P_{M'}$ be the set of equivalence classes of α and define a bisimulation α' such that $M \approx_w M'$.)
c. Prove that it is decidable whether or not an EN system M is firing sequence equivalent with a sequential EN system.
d. Determine whether or not the concurrency-free EN systems of Figure 9 are configuration and/or firing sequence equivalent with a sequential EN system.

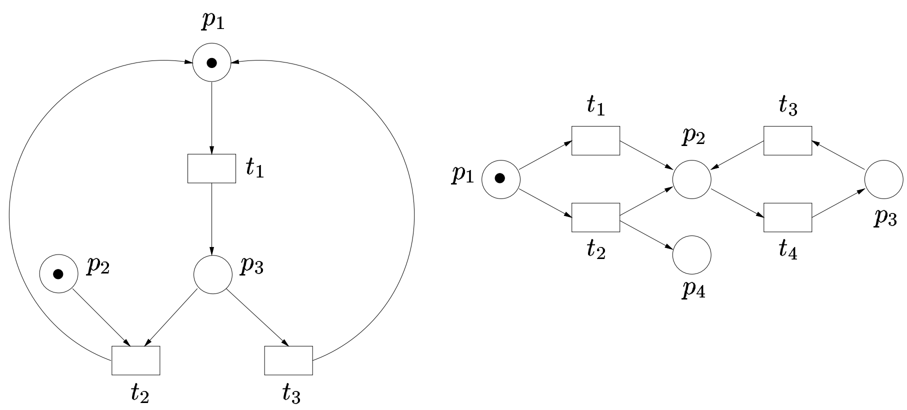

Fig. 9. Two concurrency-free EN systems.

Exercise 5.7 Use Theorem 59 to prove that the EN system of Figure 10 is contact-free.

Exercise 5.8 Let M be the EN system of Figure 11.

a. Determine SCG(M).
b. Determine all subsystems of M.
   Which of these subsystems are sequential?
c. Determine the contact-free EN system M' that is obtained by complementing those places of M that do not belong to a sequential component.
d. Does there exist a contact-free EN system M'' that is configuration equivalent with M and that contains less places than M' of c. does?
e. Let M_1 be the EN system with $\mathbf{und}(M_1) = \mathbf{und}(M)$ and $(C_{in})_{M_1} = \{p_1\}$. Show that M_1 is covered by sequential components (and is thus contact-free).

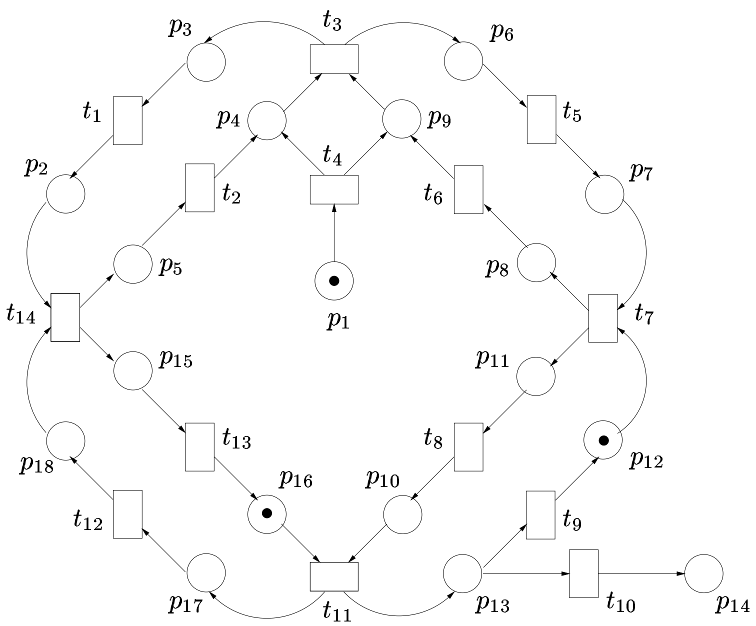

Fig. 10. The EN system of Exercise 5.7.

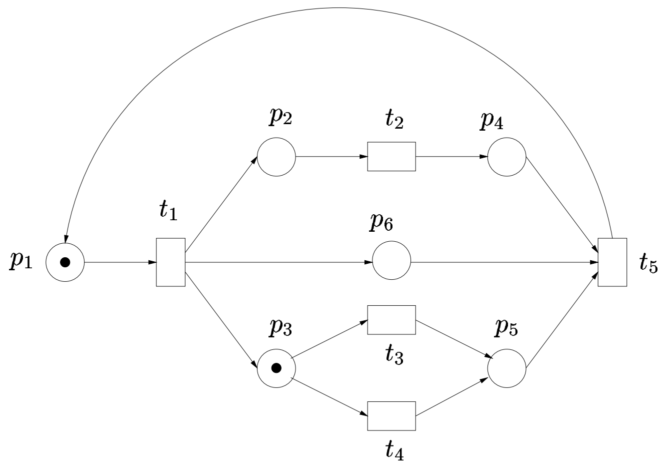

Fig. 11. The EN system M of Exercise 5.8.

Exercise 5.9 Let M be the EN system of Figure 12.

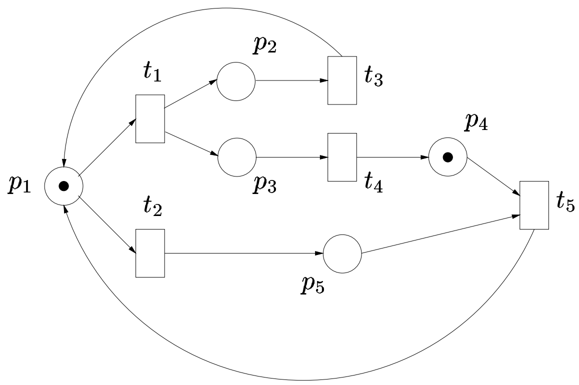

Fig. 12. The EN system M.

a. Determine SCG(M).
b. Determine CG(M).
c. Does M contain live transitions?
d. Determine all subsystems of M.
e. Determine a contact-free EN system M' with 7 places that is configuration equivalent with M.
f. Determine CG(M').

6 Processes

Exercise 6.1 Figure 13 represents a partial order ρ on $\{a, b, c, d, e, f, g\}$ (with $x \ \rho \ y$ if there is a directed path from x to y).

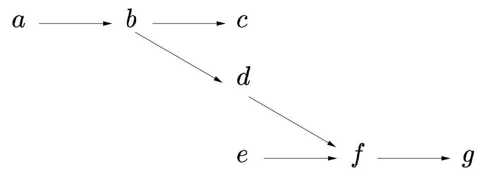

Fig. 13. A partial order ρ on $\{a, b, c, d, e, f, g\}$.

Determine $\mathbf{li}_\rho$ and $\mathbf{co}_\rho$.

Exercise 6.2 Determine the σ-cliques of the reflexive symmetric relations σ of Figure 14(i), (ii) and (iii), where $x \ \sigma \ y$ if $x = y$ or there is an edge between x and y.
Which σ-cliques are maximal?

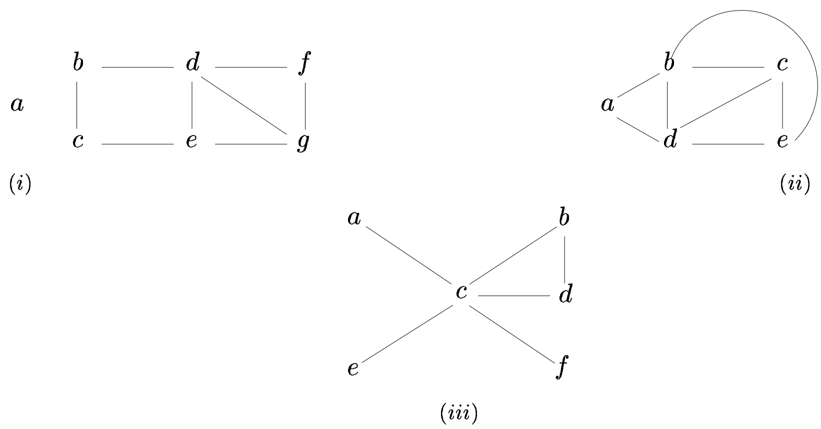

Fig. 14. Three reflexive symmetric relations σ.

Exercise 6.3 Determine the lines and cuts of ρ of Exercise 6.1 (depicted in Figure 13). Is ρ dense?

Exercise 6.4 Let Figure 15 represent the partial order ρ on $\{a, b, c, d, e, f\}$.

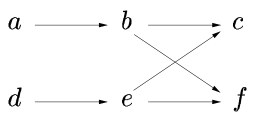

Fig. 15. A partial order ρ on $\{a, b, c, d, e, f\}$.

a. Determine $\mathbf{li}_\rho$ and $\mathbf{co}_\rho$.
b. Determine the lines and cuts of ρ. Is ρ dense?

Exercise 6.5 Let ρ be the partial order of Exercise 6.1 (depicted in Figure 13). Determine

a. $(\{d\}^{\rightarrow})_\rho$
b. $(^{\rightarrow}\{d\})_\rho$
c. $(^{\rightarrow}\{a\})_\rho$
d. $(^{\rightarrow}\{c\})_\rho$
e. $(\{b, d, e, f\}^{\circ})_\rho$
f. $(^{\circ}\{b, d, e, f\})_\rho$

Exercise 6.6 Let ρ be the partial order of Exercise 6.4 (depicted in Figure 15). Determine

a. $(\{c, e\}^{\rightarrow})_\rho$
b. $(^{\rightarrow}\{b, d\})_\rho$
c. $(\{b, d\}^{\rightarrow})_\rho$
d. $(\{b, c, d\}^{\circ})_\rho$
e. $(\{a, b, c, d, e, f\}^{\circ})_\rho$
f. $(^{\circ}\{a, b, c, d, e, f\})_\rho$

Exercise 6.7 Determine the lines, cuts and slices of the process net N of Figure 16.

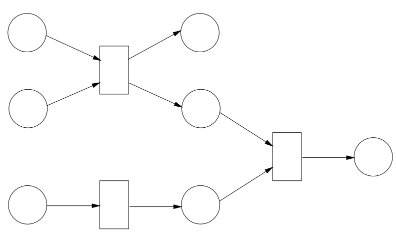

Fig. 16. The process net N.

Determine all reachable configurations and all sequential components of this process net. Check that N is dense.

Exercise 6.8 Prove Theorem 71.

Exercise 6.9 Let $M = (P, T, F, C_{in})$ be the contact-free EN system of Figure 17 and let $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ be the labelled process net of Figure 18.

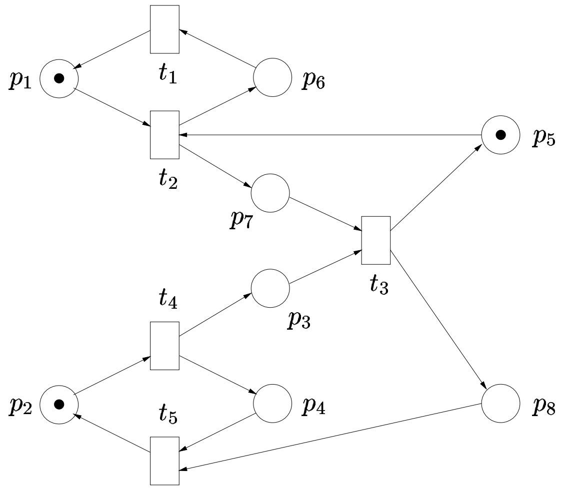

Fig. 17. The contact-free EN system M.

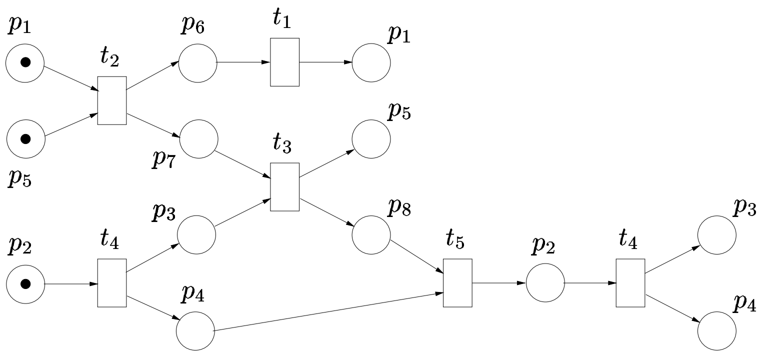

Fig. 18. The labelled process net N.

a. Check that N is a process of M.
b. Determine a firing sequence $x \in T_N^*$ such that $^\circ N[x\rangle N^\circ$.
c. Determine $\mathbf{ctr}(N)$, $\mathbf{pru}(\mathbf{ctr}(N))$ and $\mathbf{tra}(\mathbf{ctr}(N))$.
d. Determine a (different) process N of M with a firing sequence $x \in T_N^*$ such that $^\circ N[x\rangle N^\circ$ and $\phi(x) = t_2 t_1$. Do the same for $\phi(x) = t_2 t_1 t_4 t_3 t_2$.

Exercise 6.10 Let M be the contact-free EN system of Figure 19.

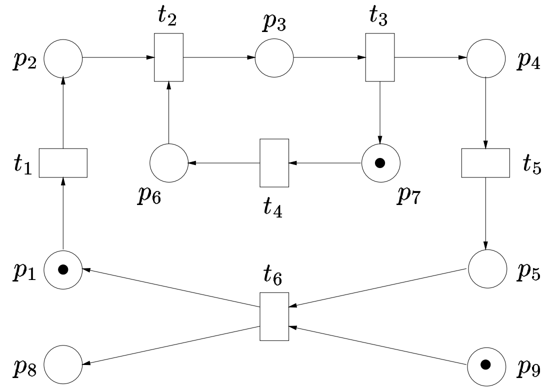

Fig. 19. The contact-free EN system M.

a. Find a process $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ of M such that for every $t \in T_M$ there is exactly one $s \in T_N$ with $\phi_2(s) = t$.
b. Determine all sequential components of N.
c. Determine $\mathbf{ctr}(N)$ and $\mathbf{pru}(\mathbf{ctr}(N))$.

Exercise 6.11 Let M be the EN system of Figure 20.

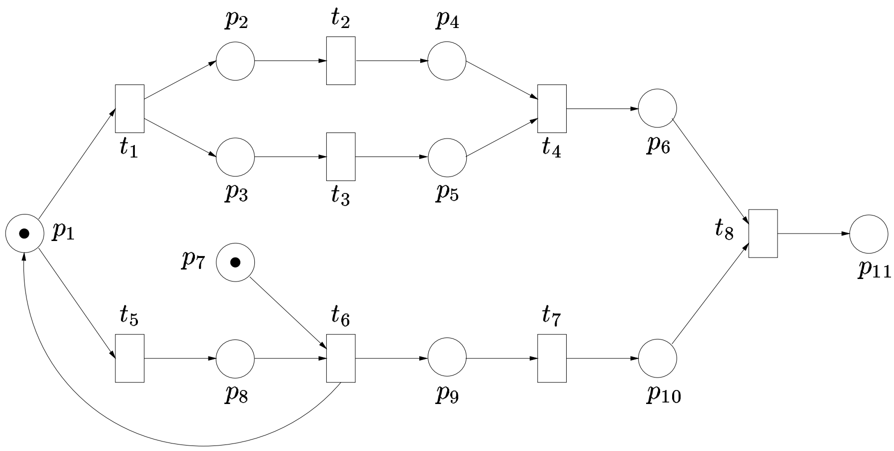

Fig. 20. The EN system M.

a. Show that M is contact-free by providing a covering by sequential components.
b. Find a process $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ of M such that for every $t \in T_M$ there is exactly one $s \in T_N$ with $\phi_2(s) = t$.
c. Determine $\mathbf{ctr}(N)$ and $\mathbf{pru}(\mathbf{ctr}(N))$.

7 Comparison of Partial and Linear Order

Exercise 7.1 Prove Lemma 111. Prove that for every acyclic labelled graph G,

$$\mathbf{words}(\mathbf{pru}(G)) = \mathbf{words}(\mathbf{tra}(G)) = \mathbf{words}(G).$$

Exercise 7.2

a. Determine $\mathbf{words}(\mathbf{pru}(\mathbf{ctr}(N)))$ for the process N of Exercise 6.11.b. Check for some elements of $\mathbf{words}(\mathbf{pru}(\mathbf{ctr}(N)))$ that they are firing sequences of M (see Theorem 114).
b. Give for $t_1t_2t_3t_4 \in \mathrm{FS}(M)$ a graph $G \in \mathrm{LPO}(M)$ such that $t_1t_2t_3t_4 \in \mathbf{words}(G)$.

Exercise 7.3 Let M be the contact-free EN system of Figure 21.

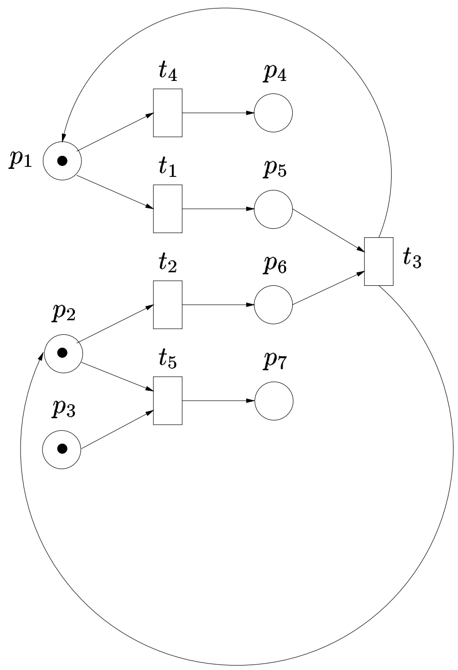

Fig. 21. The contact-free EN system M.

a. Determine $\mathbf{ind}(M)$, the independency relation of M.
b. Determine $\mathbf{dep}_M(t_1t_2t_3t_4t_2)$ and $\mathbf{pru}(\mathbf{dep}_M(t_1t_2t_3t_4t_2))$.
c. Give a process N of M for which $\mathbf{pru}(\mathbf{ctr}(N)) \equiv \mathbf{pru}(\mathbf{dep}_M(t_1t_2t_3t_4t_2))$ (see Theorem 122).

Exercise 7.4 Show that Theorem 123 follows from Theorems 92 and 122.

Exercise 7.5 Let M, M_1, M_2 and M_3 be the EN systems of Exercise 5.1 (depicted in Figures 6 and 7). Check for $i = 1, 2, 3$ that M and M_i are lpo-equivalent.

Exercise 7.6 Let $\Sigma = \{a, b, c, d\}$ and let $I = \{\{a, b\}, \{b, c\}, \{c, d\}, \{b, d\}\}$ be an independency relation over Σ, in which the unordered pair $\{e, e'\}$ is a shorthand notation for the ordered pairs (e, e') and (e', e).

a. Determine $[bcda]_I$.
b. Determine $\mathbf{dep}_I(bcadbda)$.
c. Determine $\mathbf{pru}(\mathbf{dep}_I(bcadbda))$.
d. Determine $\mathbf{words}(\mathbf{dep}_I(bcadbda))$.

Exercise 7.7 Let $\Sigma = \{a, b, c, d, e\}$ and let $I = \{\{a, b\}, \{b, c\}, \{c, d\}, \{b, e\}, \{c, e\}\}$ be an independency relation over Σ, in which again the unordered pair $\{e, e'\}$ is used as a shorthand notation for the ordered pairs (e, e') and (e', e).

a. Determine $[abecd]_I$.
b. Determine $\mathbf{dep}_I(abecd)$.
c. Determine $\mathbf{pru}(\mathbf{dep}_I(abecd))$.
d. Determine $\mathbf{words}(\mathbf{dep}_I(abecd))$ and $\mathbf{words}(\mathbf{pru}(\mathbf{dep}_I(abecd)))$.

Exercise 7.8 Let M be the EN system of Exercise 5.9 (depicted in Figure 12) and let $x = t_2 t_5 t_1 t_3 t_4 t_2$.

a. Determine $\mathbf{ind}(M)$.
b. Determine $\mathbf{dep}_M(x)$ and $\mathbf{pru}(\mathbf{dep}_M(x))$.
c. Determine $[x]_{\mathbf{ind}(M)}$.

8 Branching Processes

Exercise 8.1 Let N be the b-process net of Figure 22.

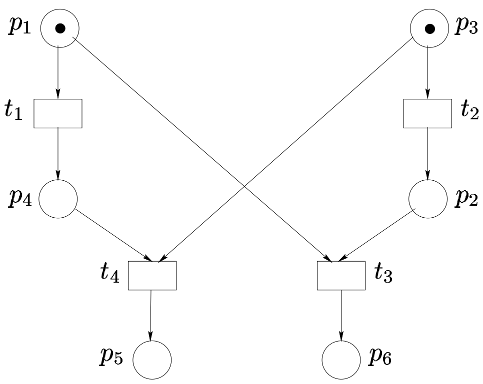

Fig. 22. The b-process net N.

a. Determine the conflict relation $\otimes_N$.
b. Give all slices of N.

Exercise 8.2 Let M be the EN system of Exercise 6.9 (depicted in Figure 17). Prove that all b-processes of M are processes of M.

Exercise 8.3 Let N be the underlying net of the EN system M of Exercise 6.11 (depicted in Figure 20).

a. Is N a b-process net?
   And if p_1 is removed from $t_6{}^\bullet$?
   And if, moreover, p_6 is removed from ${}^\bullet t_8$?
b. Find a b-process N' of M that is not a process of M, such that for every $t \in T_M$, excluding t_8, there is exactly one $s \in T_{N'}$ with $\phi_{N'}(s) = t$.
c. M has only finitely many b-processes.
   Determine the largest b-process N'' of M.
   Determine $\mathbf{pru}(\mathbf{ctr}(N''))$.

9 P/T Systems

Exercise 9.1 Let M be the P/T system of Figure 23.

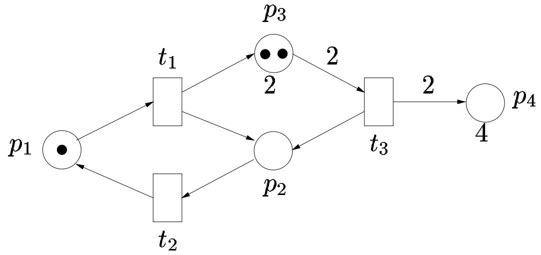

Fig. 23. The P/T system M.

a. For which $t \in T_M$ does t **con** $(C_{in})_M$ hold?
b. Make M contact-free by complementing those places $p \in P_M$ for which $K_M(p) \in \mathbb{N}_+$.
c. Determine a contact-free P/T system M' which is configuration equivalent with M and for which $K_{M'}(p) = \omega$ for all $p \in P_{M'}$.

Exercise 9.2 A place p of a P/T system M is *bounded* if there exists a $k \in \mathbb{N}$ such that $C(p) \leq k$ for all $C \in \mathbb{C}_M$ (Definition 181).
Use Lemma 161 to show that the P/T system of Figure 24 contains no bounded places.

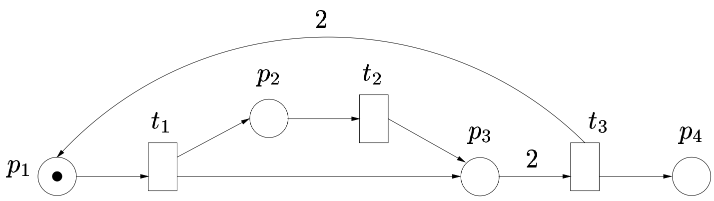

Fig. 24. The P/T system of Exercise 9.2.

Exercise 9.3 Let M be the P/T system of Figure 25.

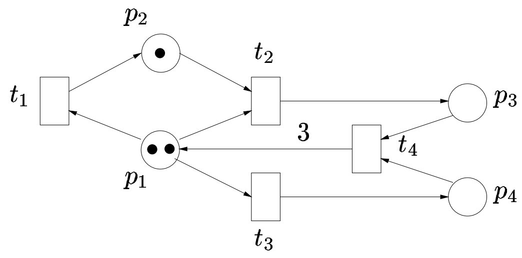

Fig. 25. The P/T system M.

Use the algorithm in the proof of Theorem 167 to check whether or not $\mathbb{C}_M$ is finite.
Now let M' be the P/T system that is obtained from M by setting $W(t_4, p_1) = 4$. Check whether or not $\mathbb{C}_{M'}$ is finite.

Exercise 9.4 Determine the p-invariants of the P/T system of Exercise 9.2 (depicted in Figure 24).

Exercise 9.5 Determine the p-invariants of the P/T systems M and M' of Exercise 9.3 (M is depicted in Figure 25).

Exercise 9.6 Let M be the P/T system of Figure 26.

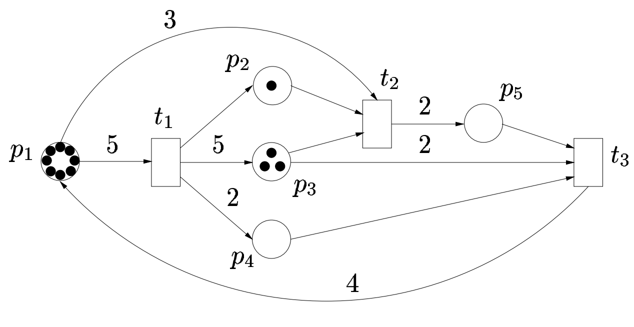

Fig. 26. The P/T system M.

a. Determine the p-invariants of M.
b. Is M covered by positive p-invariants?
   Is M bounded?
c. Does there exist a $C \in \mathbb{C}_M$ for which $C(p) = 2$ for all $p \in P_M$?

Exercise 9.7 Let M be the P/T system of Figure 27.

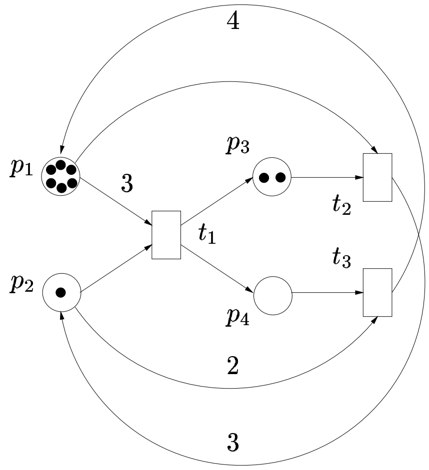

Fig. 27. The P/T system M.

a. Determine the p-invariants of M.
   Does there exist a covering of M by positive p-invariants?
b. Use the answer to a. to prove that $C(p_3) + C(p_4) < 6$ for all $C \in \mathbb{C}_M$.
   Show then that $C(p_4) \leq 1$ for all $C \in \mathbb{C}_M$.
c. Determine SCG(M).

Exercise 9.8 Prove Lemma 195.

Exercise 9.9 Let M be the P/T system of Figure 28.

a. Determine the characteristic p-invariants and the sequential components of M.
b. Prove that $\sum_{p \in P_M} C(p) = 2$ for all $C \in \mathbb{C}_M$.

Exercise 9.10 Let $M = (P, T, F, W, C_{in})$ be the marked graph of Figure 29.

a. Determine all cycles of M.
b. Use the answer to a. to determine whether or not M is live and whether or not M is safe.
c. Does there exist a configuration C of (P, T, F, W) for which $C(p_3) = C(p_4) = 1$ and (P, T, F, W, C) is live and safe?

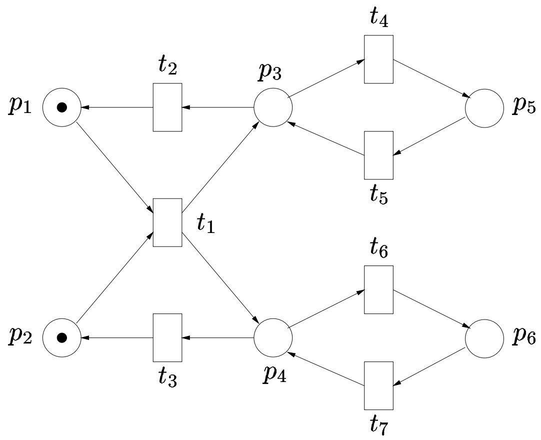

Fig. 28. The P/T system M of Exercise 9.9.

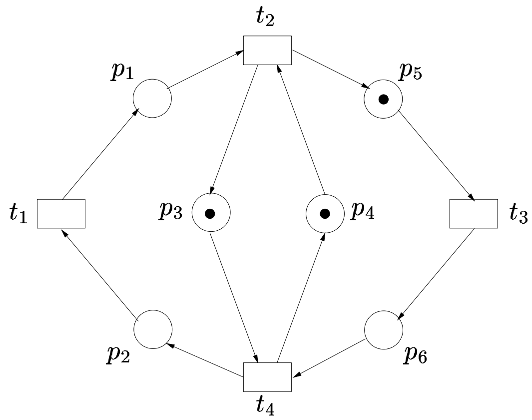

Fig. 29. The marked graph M of Exercise 9.10.

Exercise 9.11 Let M be the free-choice system of Exercise 9.9 (depicted in Figure 28).
Determine the siphons and traps of M. Determine whether M is live and/or safe.

Exercise 9.12 Let $M = (P, T, F, W, C_{in})$ be the free-choice system of Figure 30.

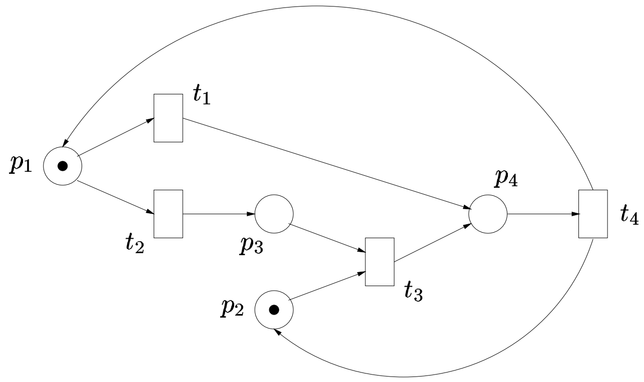

Fig. 30. The free-choice system M.

a. Determine the siphons and traps of M.
b. Use the answer to a. to determine whether M is live and/or safe.
c. Let C be the configuration of (P, T, F, W) for which $C(p_1) = 1$ and $C(p) = 0$ for all $p \in P - \{p_1\}$. Use the answer to a. to determine whether or not the free-choice system (P, T, F, W, C) is live.