Theorie van Concurrency

voorjaar 2010

`http://www.liacs.nl/home/kleijn/thvc-0910.html`

vierde college: 3 mrt 2010
5. Equivalences and Normal Forms:
5.4 en 5.5 (?)

vijfde college: 10 mrt 2009 **10.15-13.00 uur !!!**; β-banenmarkt
5 afmaken, 6. Processes

tweede werkgroep: 17 maart 2010
de opgaven bij 5. Equivalences and Normal Forms
en eventueel begin 6. Processes

Definition 40. An EN system M is *sequential* if $\#C = 1$ for all $C \in \mathbb{C}_M$.

Definition 44. An EN system M is *concurrency-free* if there do not exist $C \in \mathbb{C}_M$ and $t_1, t_2 \in T_M$ such that $\{t_1, t_2\}$ **con** C and $t_1 \neq t_2$

in other words

M is concurrency-free if it has no concurrent steps: there does not exist $C \in \mathbb{C}_M$ and $U \subseteq T_M$ such that U **con** C and $\#U \geq 2$.

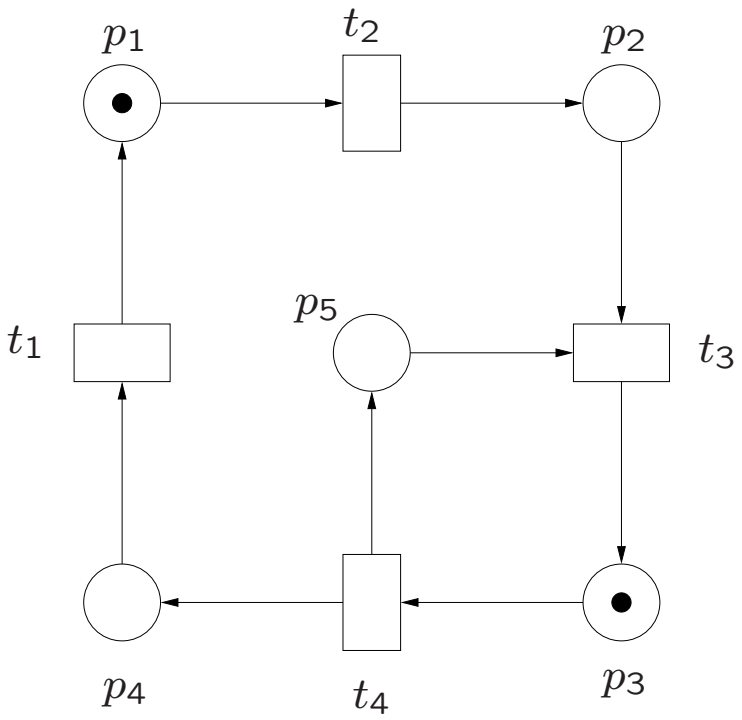


Fig. 39. An EN system with ... nontrivial subsystems ...

Definition 45. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be EN systems.

M' is a *subsystem of* M if:

(1) $P' \subseteq P$, $T' \subseteq T$, $F' \subseteq F$, $C'_{in} \subseteq C_{in}$,
(2) $\forall p \in P' : \ \mathbf{nbh}_M(p) \subseteq \mathbf{nbh}_{M'}(p)$, and
(3) $\forall p \in P' : \ \text{if } p \in C_{in}, \text{ then } p \in C'_{in}$.

If, moreover, M' is a sequential EN system, then M' is a *sequential component* of M.

Lemma 46. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be EN systems.

(1) M' is a subsystem of M iff
$P' \subseteq P$, $T' = \mathbf{nbh}_M(P')$,
$F' = F \cap ((P' \times T') \cup (T' \times P'))$, and
$C'_{in} = C_{in} \cap P'$.

(2) If M' is a subsystem of M then:
for every $t \in T'$, $({}^\bullet t)_{M'} = ({}^\bullet t)_M \cap P'$ and $(t^\bullet)_{M'} = (t^\bullet)_M \cap P'$,
for every $t \in T - T'$, $\mathbf{nbh}_M(t) \cap P' = \varnothing$,
for every $p \in P'$, $({}^\bullet p)_{M'} = ({}^\bullet p)_M$ and $(p^\bullet)_{M'} = (p^\bullet)_M$.

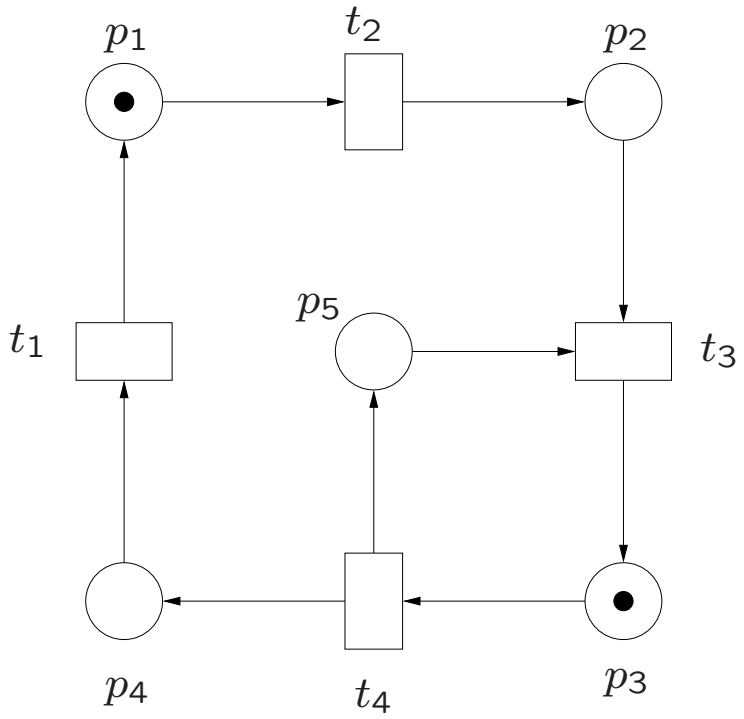


Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

Lemma 47. Let $M = (P, T, F, C_{in})$ be an EN system and let $S \subseteq P$.

There exists a subsystem M' of M with $P_{M'} = S$
iff ${}^\bullet S = S^\bullet$.

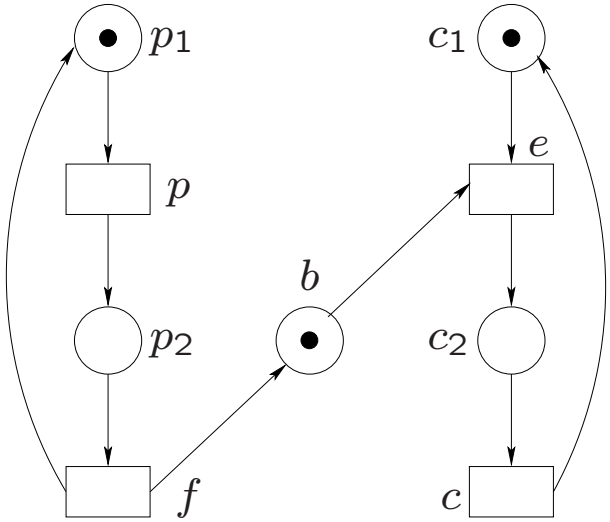
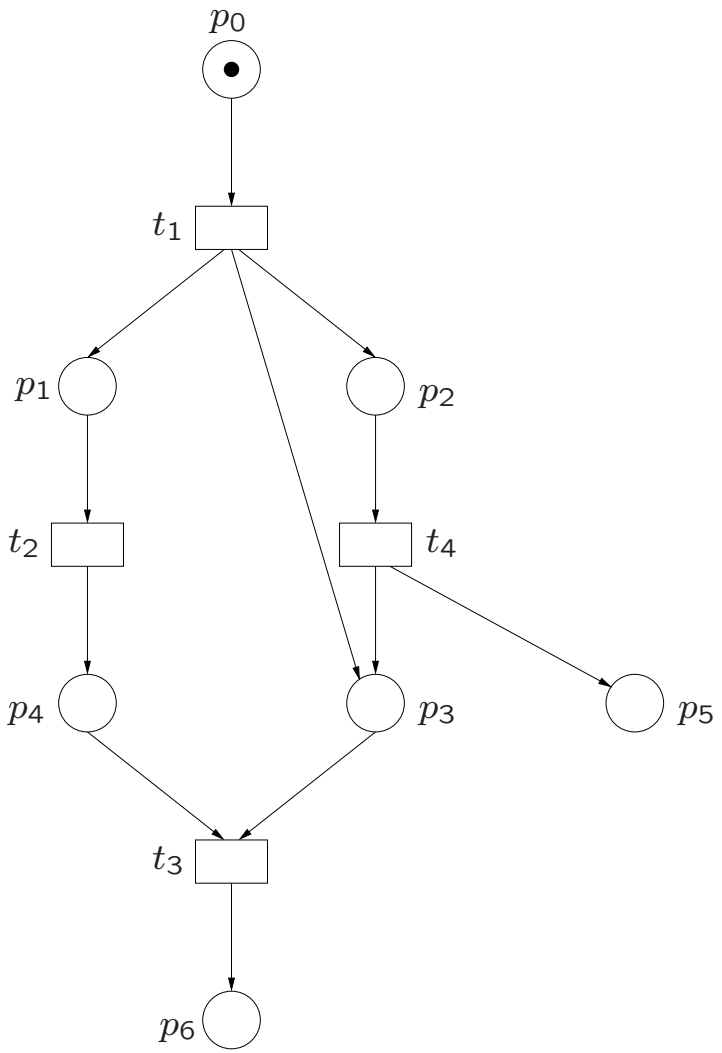


Fig. 12. Subsystems: $\{p_1, p_2\}$, the producer; $\{c_1, c_2\}$, the consumer; and $\{p_1, p_2, c_1, c_2\}$; otherwise trivial.
The buffer is NOT a subsystem

Lemma 48. Let $M' = (S, T', F', C'_{in})$ be a subsystem of an EN system $M = (P, T, F, C_{in})$.

(1) For all $C \subseteq P$, if $C \in \mathbb{C}_M$ then $C \cap S \in \mathbb{C}_{M'}$.

(2) For all $t \in T'$, if $t \in \mathbf{use}_M(T)$ then $t \in \mathbf{use}_{M'}(T')$.



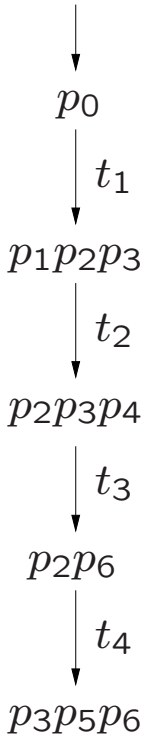


Fig. 40, 41. An EN system M and its configuration graph.

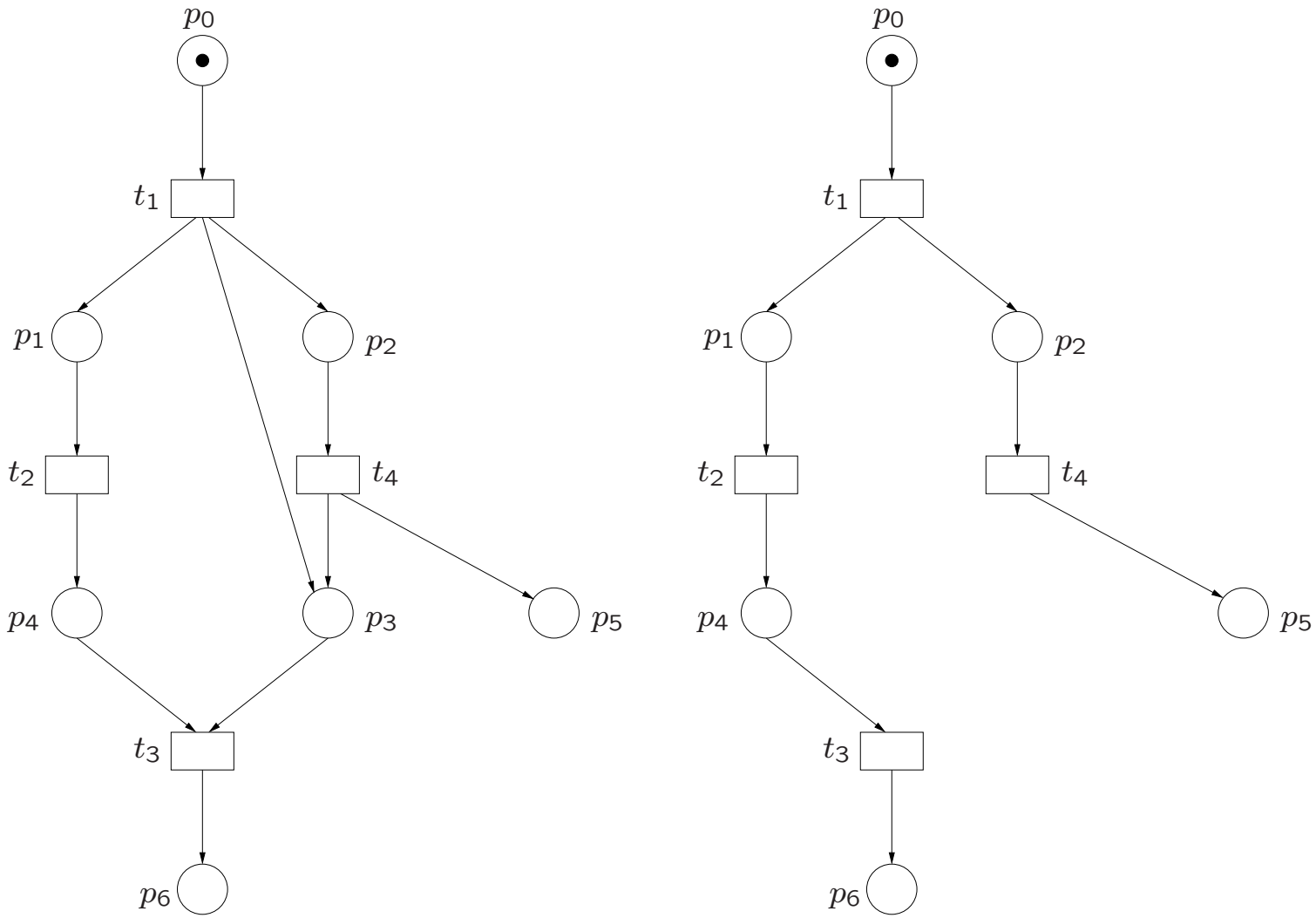


Fig. 40, 42. An EN system M and a subsystem M_1.

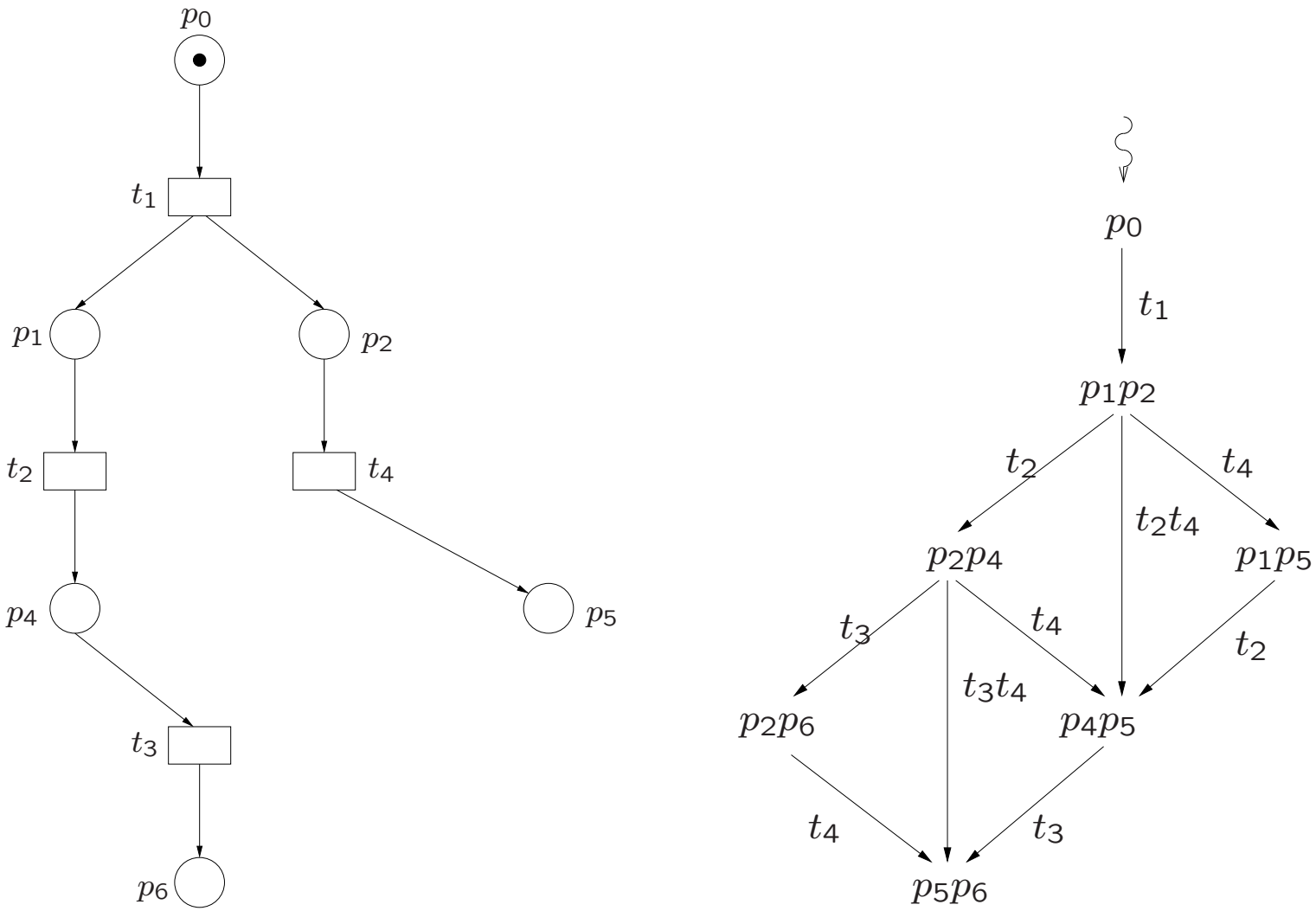


Fig. 42,43. M_1 and its configuration graph.

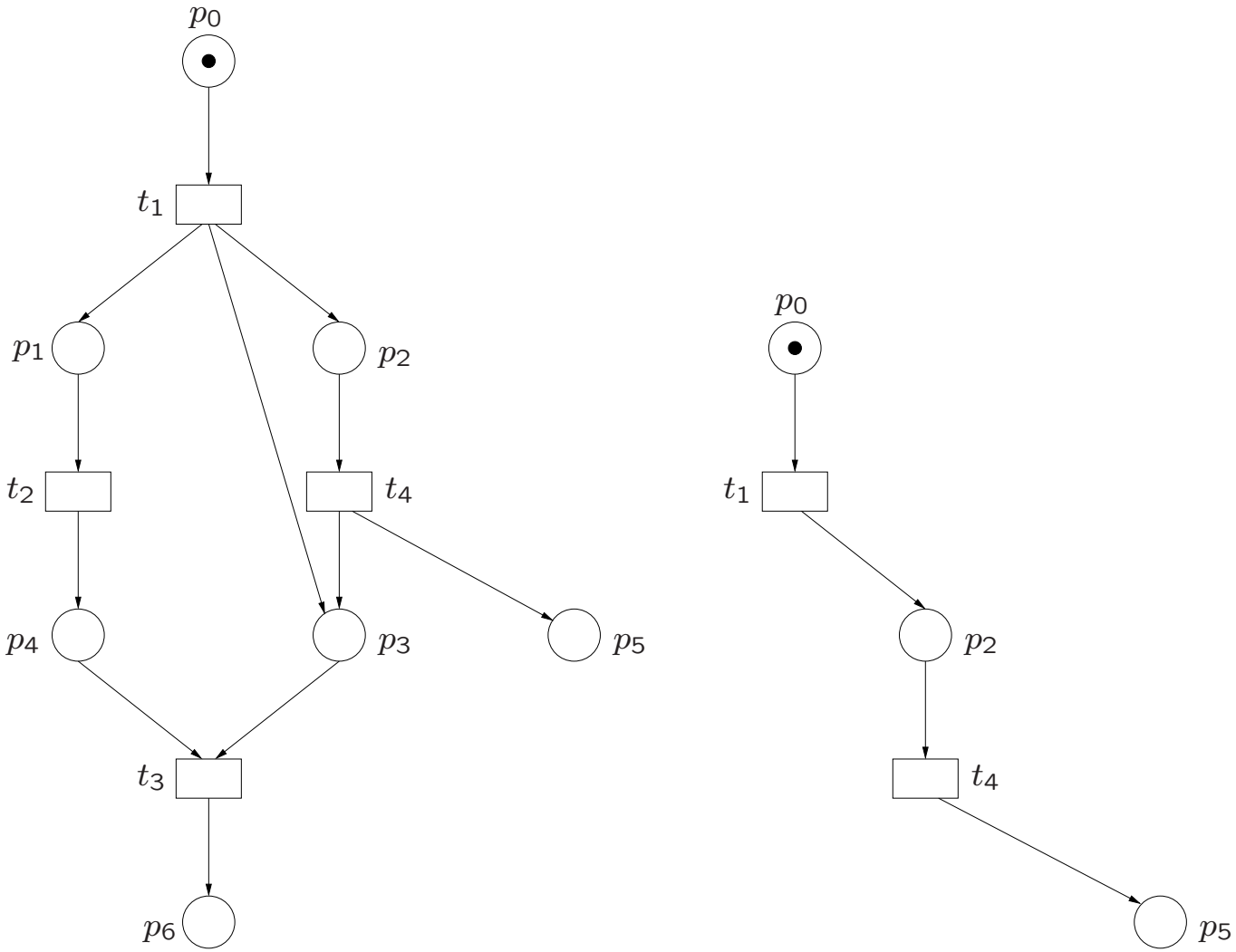


Fig. 40, 44. An EN system M and a subsystem M_2.

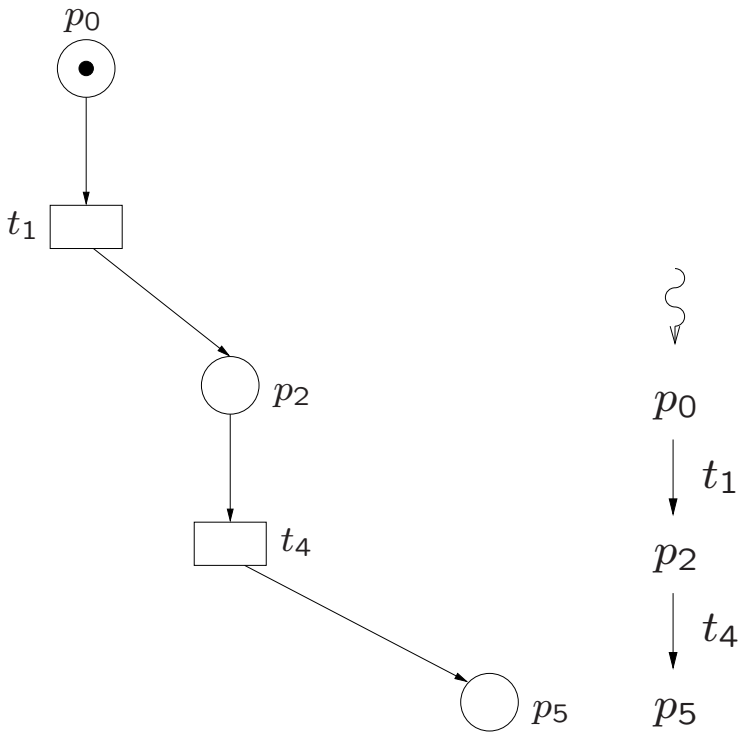


Fig. 44,45. M_2 and its configuration graph.

Theorem 49. Let $M = (P, T, F, C_{in})$ be a reduced EN system and let $S \subseteq P$.
Then the following statements are equivalent.

(1) There is a sequential component M' of M with $P_{M'} = S$,

(2) $\#(C \cap S) = 1$ for all $C \in \mathbb{C}_M$,

(3) (i) $\#(C_{in} \cap S) = 1$, and
(ii) $\forall t \in T$:
$\#({}^\bullet t \cap S) = \#(t^\bullet \cap S) = 1$ or $\#({}^\bullet t \cap S) = \#(t^\bullet \cap S) = 0$.

Lemma 50. Let M be a strongly reduced sequential EN system and let M' be a subsystem of M.

Then M' is trivial.

Theorem 51. Let M be a strongly reduced EN system, and let M' be a sequential component of M.

Then M' has no nontrivial subsystems.

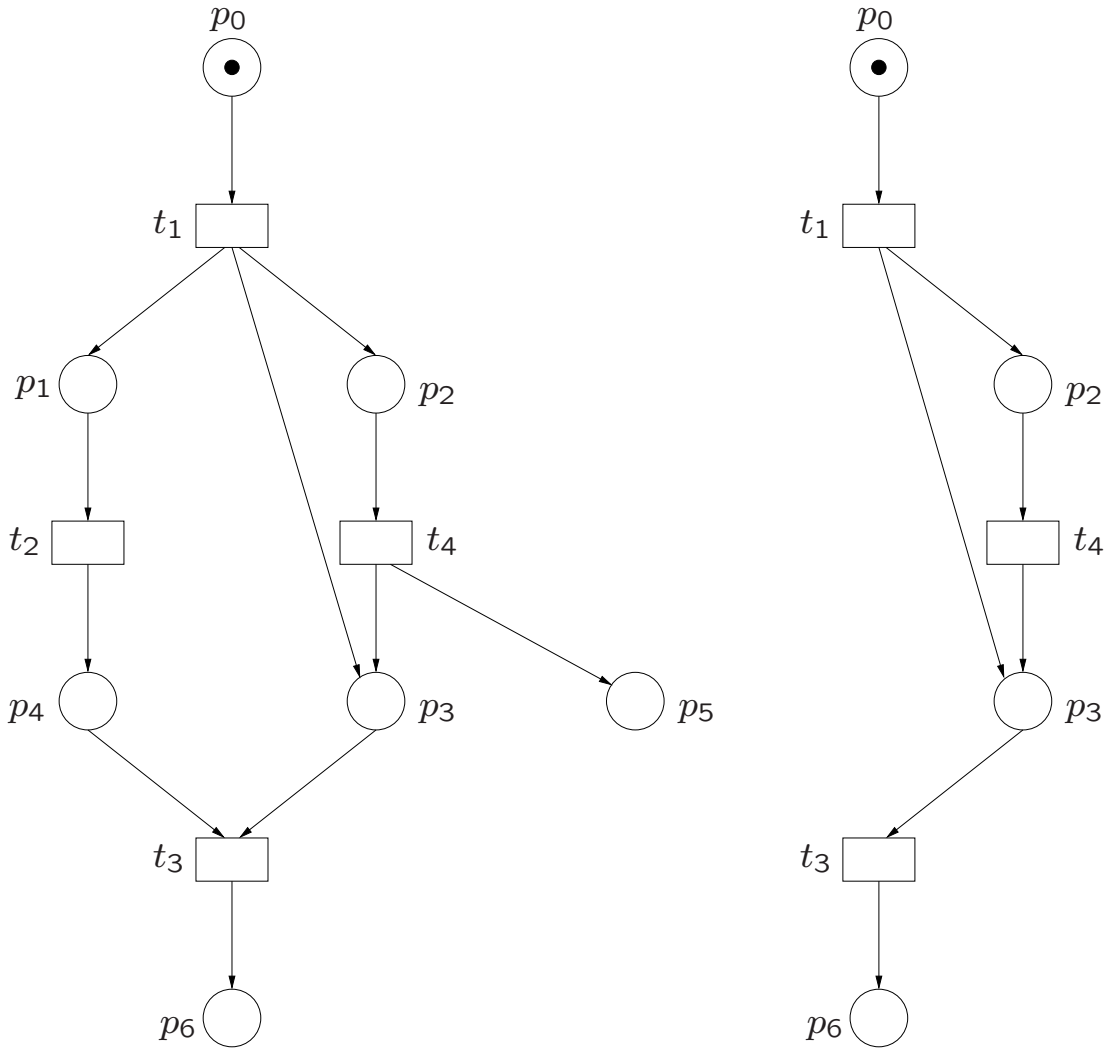


Fig. 40, 46. An EN system M and a subsystem M_3.

Definition 52. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) A set $\{M_1, \ldots, M_n\}$ of subsystems of M, $n \geq 0$,
with $M_i = (S_i, T_i, F_i, (C_{in})_i)$ for $1 \leq i \leq n$,
is a *covering* of M if
$P = \bigcup_{i=1}^{n} S_i$,
$T = \bigcup_{i=1}^{n} T_i$,
$F = \bigcup_{i=1}^{n} F_i$, and
$C_{in} = \bigcup_{i=1}^{n} (C_{in})_i$.

(2) M is *covered by sequential components*
if there exists a covering $\{M_1, \ldots, M_n\}$, $n \geq 0$, of M
such that M_i is a sequential component of M for every $1 \leq i \leq n$.

Lemma 53. Let $M = (P, T, F, C_{in})$ be an EN system, and let, for every $1 \leq i \leq n$ (with $n \geq 0$), $M_i = (S_i, T_i, F_i, (C_{in})_i)$ be a subsystem of M.

Then $\{M_1, \ldots, M_n\}$ is a covering of M
iff $P = \bigcup_{i=1}^{n} S_i$.

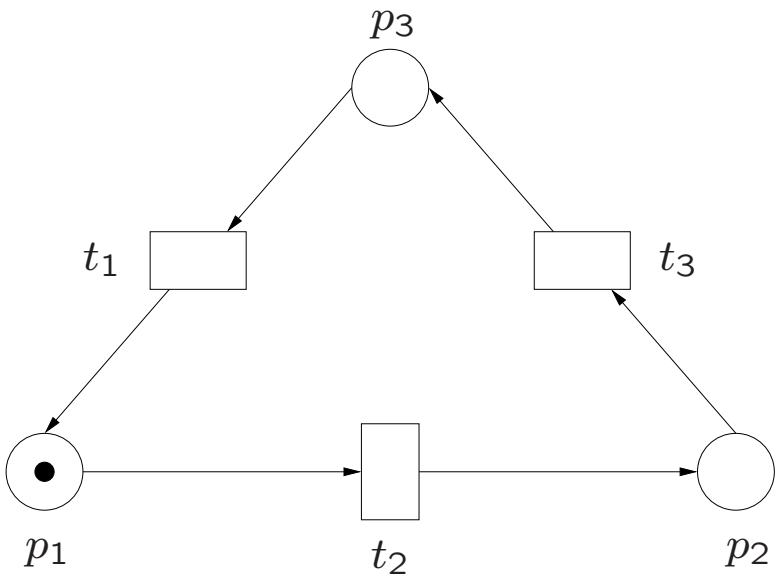


Fig. 47. A sequential EN system.

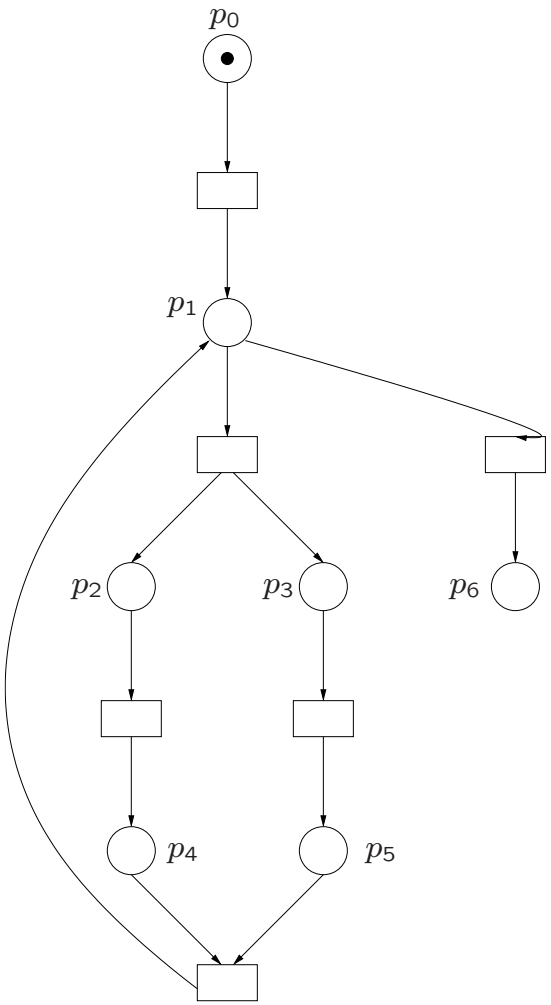


Fig. 15.

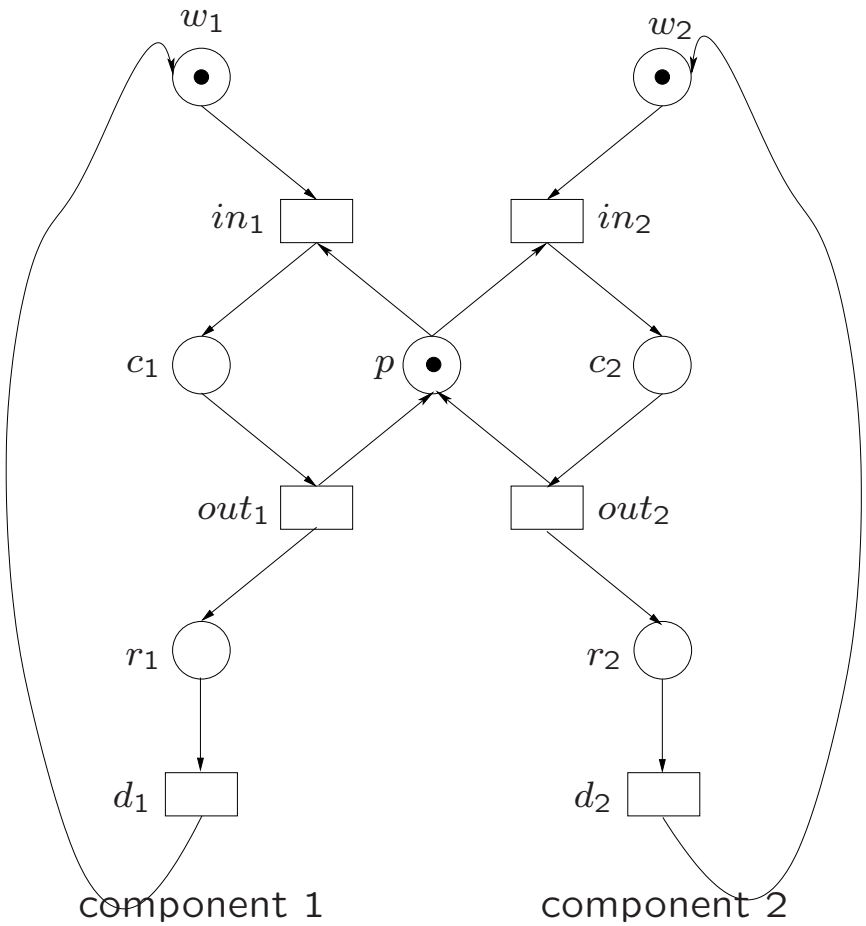

w_1
w_2
in_1
in_2
c_1
p
c_2
out_1
out_2
r_1
r_2
d_1
d_2
component 1
component 2

Fig. 5. Mutex

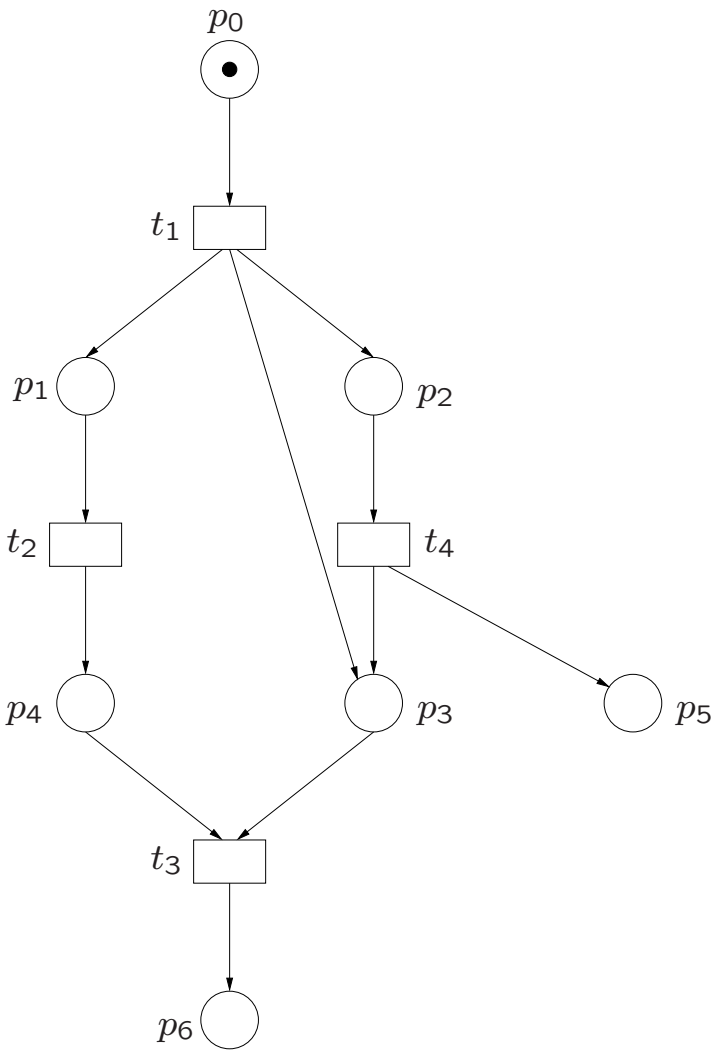


Fig. 40.

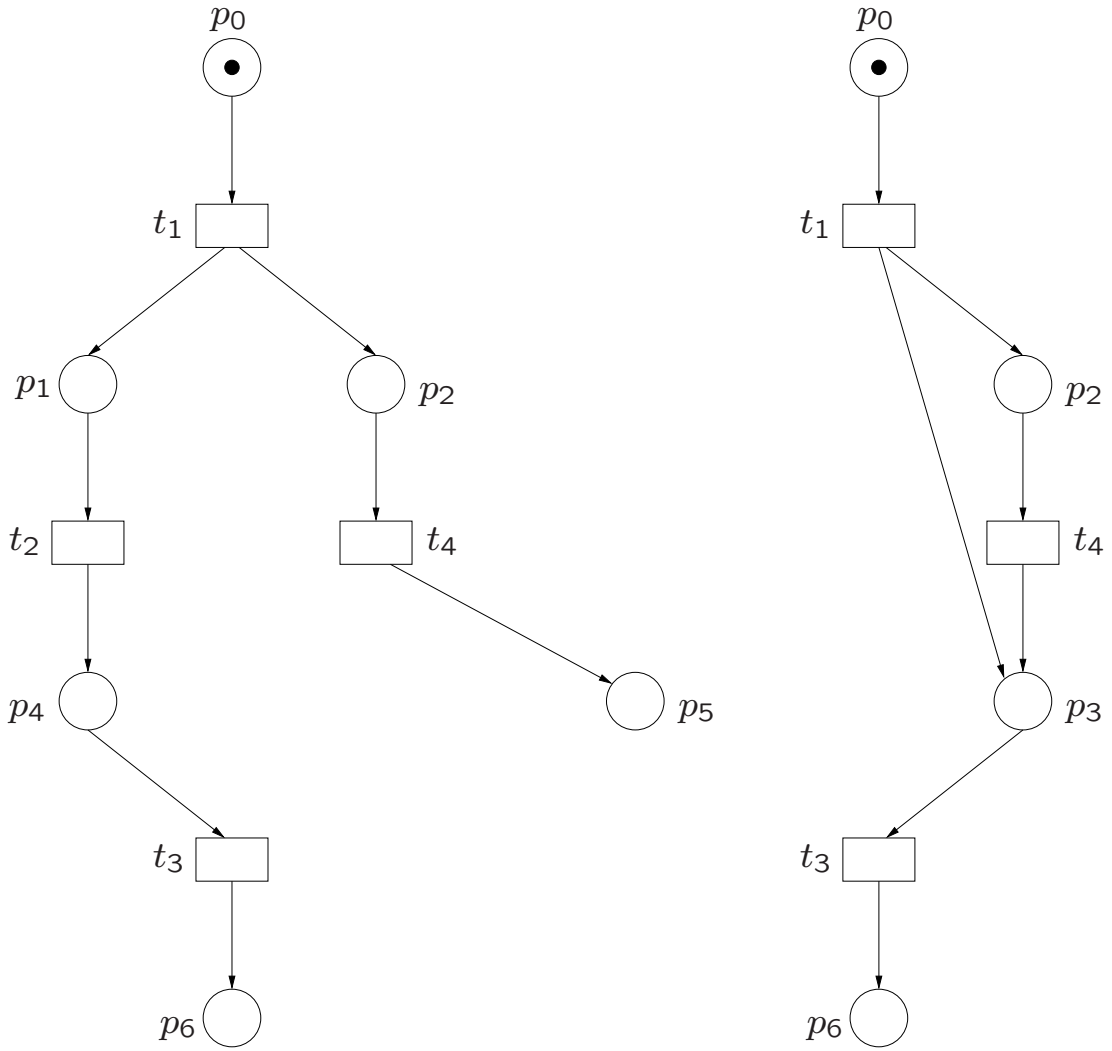


Fig. 42, 46.

AIM

Theorem 54. For every EN system M there exists
a reduced EN system M'
that is configuration equivalent with M and
that is covered by at most $\#P_M$ sequential components.

Definition 55. Let M be an EN system and let $p, q \in P_M$.

Then p and q are *complementary*, denoted by $p \ \mathbf{com} \ q$, if

$p^\bullet = {}^\bullet q$ and ${}^\bullet p = q^\bullet$.

Lemma 56. Let $M = (P, T, F, C_{in})$ be a reduced EN system. For all $p, q \in P$,

$\{p, q\}$ is a sequential component of M

iff

$\#(C_{in} \cap \{p, q\}) = 1$ and p **com** q.