Theorie van Concurrency

voorjaar 2010

`http://www.liacs.nl/home/kleijn/thvc-0910.html`

derde college: 24 feb 2010

5. Equivalences and Normal Forms

Definition 27. Two EN systems

$M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$

are *isomorphic*, denoted by $M \equiv M'$,

if there exist two bijections
$\alpha : P \to P'$ and $\beta : T \to T'$

such that $\mathbf{und}(M) \equiv^{\alpha}_{\beta} \mathbf{und}(M')$ and $\alpha(C_{in}) = C'_{in}$.

Definition 28. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.

Then M and M' are *configuration equivalent*,

denoted by $M \approx M'$,

if there exist two bijections
$\alpha : \mathbb{C}_M \to \mathbb{C}_{M'}$ and $\beta : \mathbf{use}_M(T) \to \mathbf{use}_{M'}(T')$

such that

(1) $\alpha(C_{in}) = C'_{in}$ and

(2) for all $C, D \in \mathbb{C}_M$ and $t \in \mathbf{use}_M(T)$, $C[t\rangle_M D$ iff $\alpha(C)[\beta(t)\rangle_{M'}\alpha(D)$.

Theorem 29. Let M and M' be two EN systems.

Then $M \approx M'$ iff

SCG(M) $\equiv$ SCG(M') iff

CG(M) $\equiv$ CG(M').

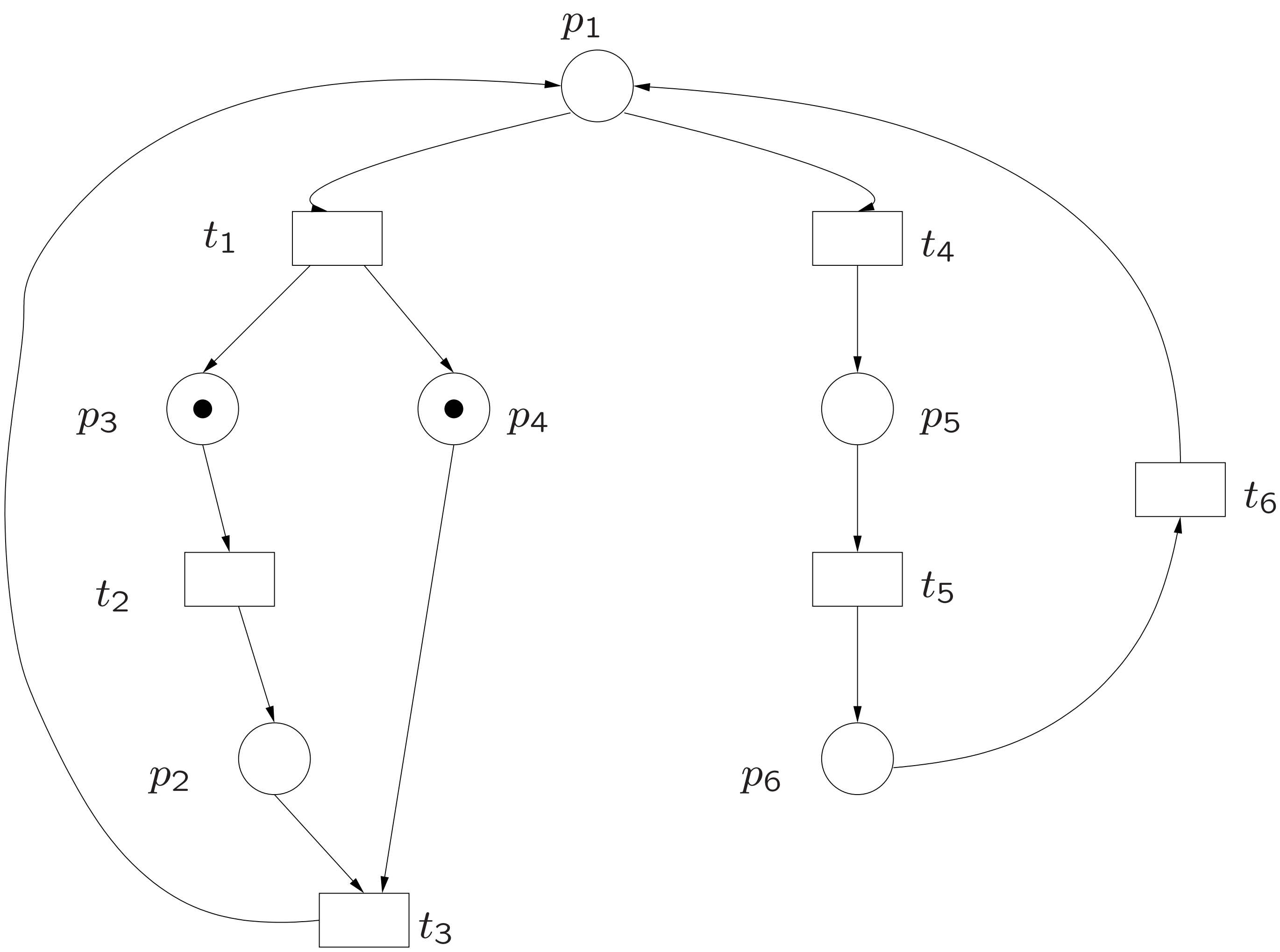


Fig. 29. An EN system M.

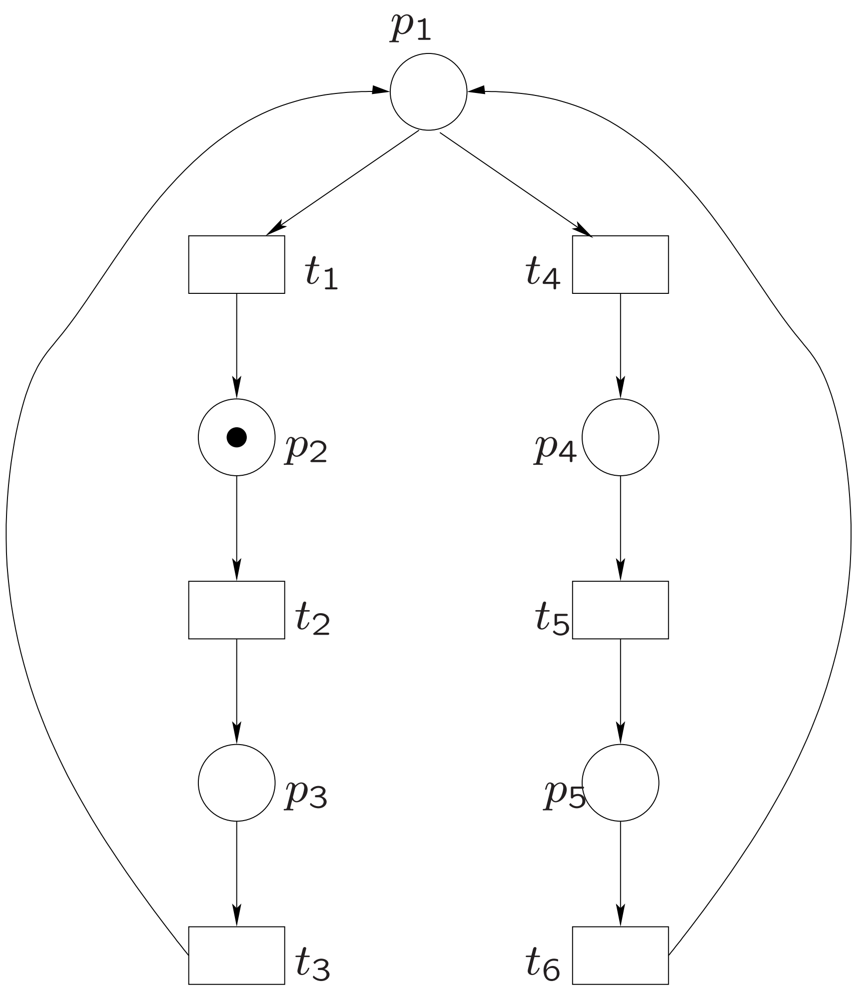


Fig. 30. An EN system M'.

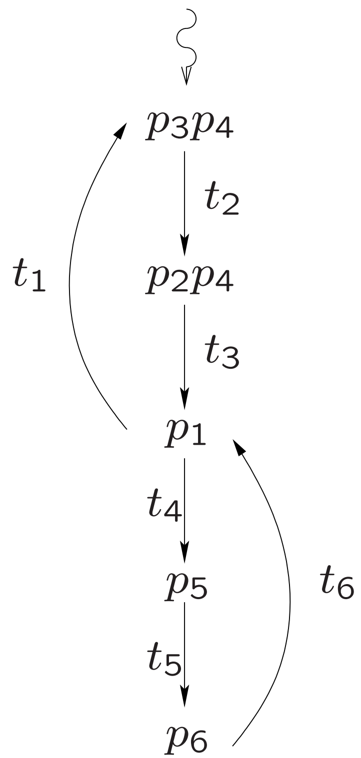


Fig. 31. The configuration graph of M of Fig. 29.

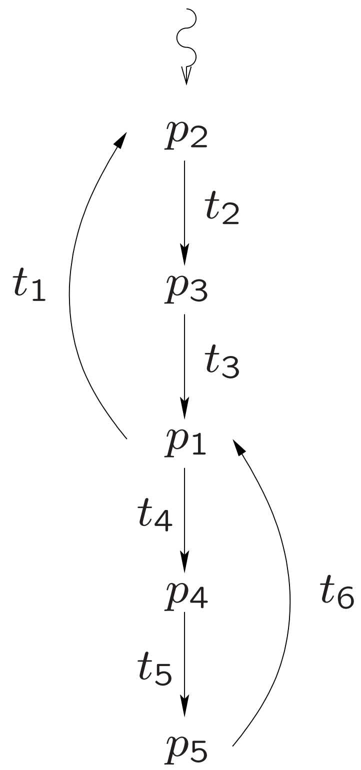


Fig. 32. The configuration graph of M' of Fig. 30.

Lemma 30. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.

If α is an injective function, $\alpha : \mathbb{C}_M \to \mathcal{P}(P')$,
and β is a bijective function, $\beta : \mathbf{use}_M(T) \to T'$, such that

(1) $\alpha(C_{in}) = C'_{in}$ and
(2) for all $C, D \in \mathbb{C}_M$ and $t \in \mathbf{use}_M(T)$,
$C[t\rangle_M D$ implies $\alpha(C)[\beta(t)\rangle_{M'}\alpha(D)$, and
$\beta(t)\ \mathbf{con}_{M'}\ \alpha(C)$ implies $t\ \mathbf{con}_M\ C$,

then $M \approx^{\alpha}_{\beta} M'$.

Definition 31. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.
M and M' are *weakly configuration equivalent*,
denoted by $M \approx_w M'$,
if there exists a relation $\alpha \subseteq \mathbb{C}_M \times \mathbb{C}_{M'}$
and a bijection $\beta : \mathbf{use}(T) \rightarrow \mathbf{use}(T')$, such that

(1) $(C_{in}, C'_{in}) \in \alpha$,

(2) for all $C, D \in \mathbb{C}_M$, $C' \in \mathbb{C}_{M'}$, and $t \in \mathbf{use}(T)$:
if $C[t\rangle_M D$ and $(C, C') \in \alpha$, then
there is a $D' \in \mathbb{C}_{M'}$ such that $C'[\beta(t)\rangle_{M'} D'$ and $(D, D') \in \alpha$, and

(3) for all $C', D' \in \mathbb{C}_{M'}$, $C \in \mathbb{C}_M$, and $t' \in \mathbf{use}(T')$:
if $C'[t'\rangle_{M'} D'$ and $(C, C') \in \alpha$, then
there is a $D \in \mathbb{C}_M$ such that $C[\beta^{-1}(t')\rangle_M D$ and $(D, D') \in \alpha$.

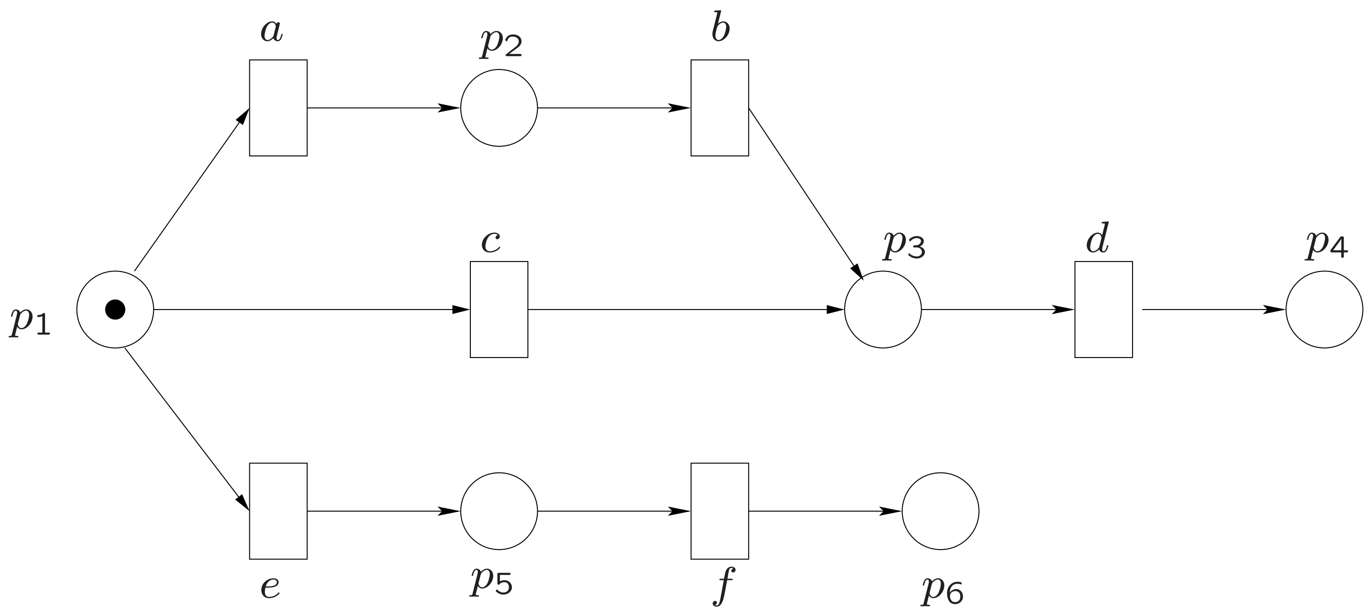


Fig. 33. M.

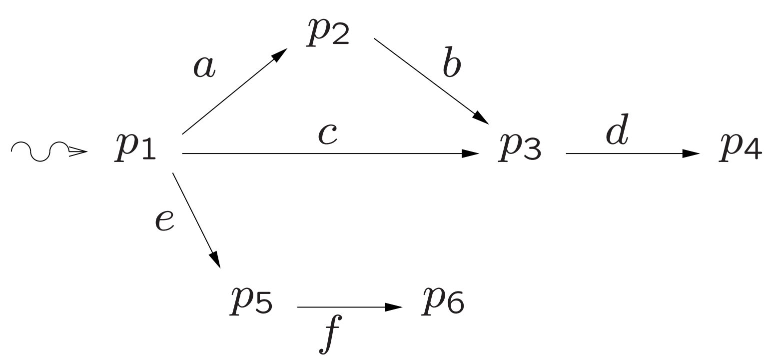


Fig. 34. The configuration graph of M of Fig. 33.

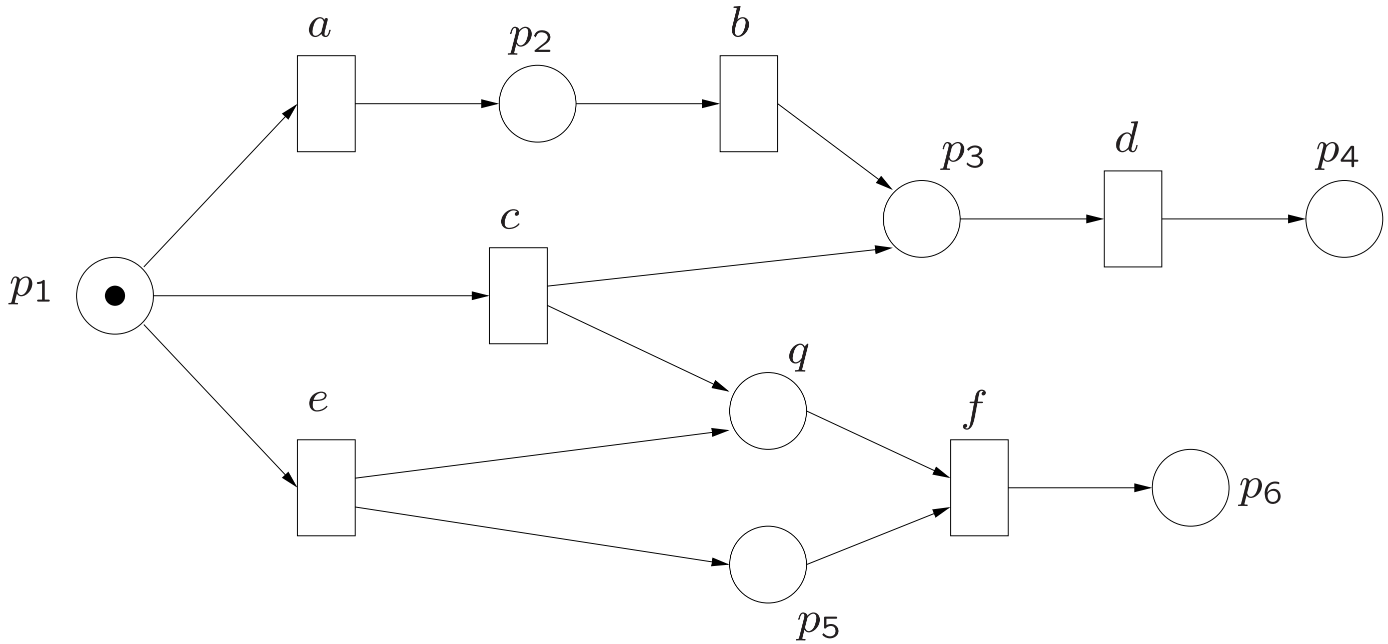


Fig. 35. M', weakly equivalent with M of Fig. 33.

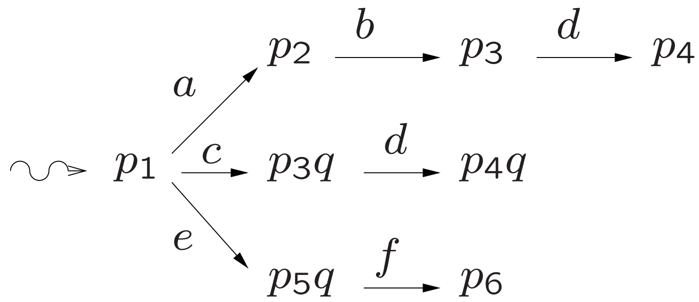
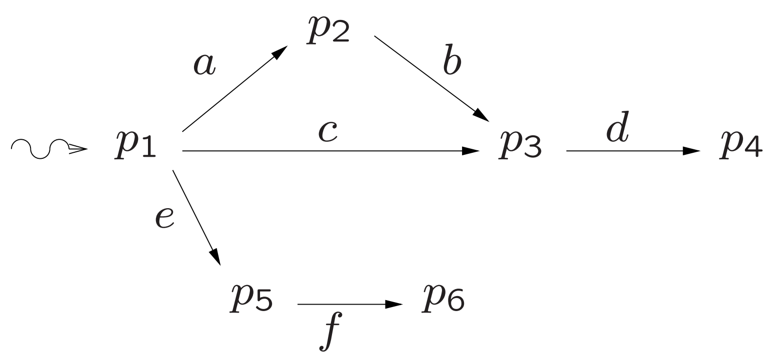


Fig. 36. The configuration graph of M' of Fig. 35.



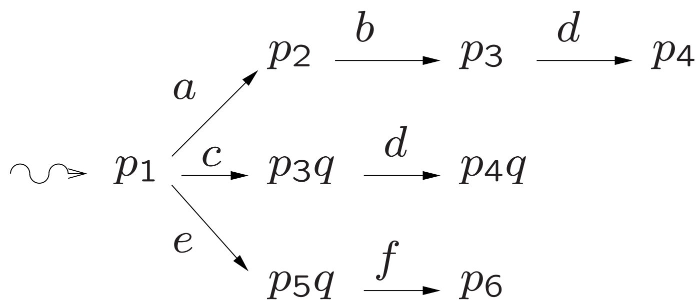


Fig. 34., **Fig. 36.** The configuration graphs of M of Fig. 33 and of M' of Fig. 35.

Definition 32. Let M and M' be two EN systems.

M and M' are *firing sequence equivalent*, denoted by $M \approx_{fs} M'$, if there exists a bijection $\beta : \mathbf{use}(T_M) \to \mathbf{use}(T_{M'})$ such that $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$.

Theorem 33. Two EN systems are firing sequence equivalent iff they are weakly configuration equivalent.

Corollary 34 If two EN systems are configuration equivalent, then they are also firing sequence equivalent.

Definition 35 An EN system M
is *reduced* if all transitions of M are useful.
M is *strongly reduced* if M is reduced and has no isolated places.

Theorem 36. For every EN system M
there exists a reduced EN system M' such that $M \approx M'$.

This theorem can be strengthened as follows.

Theorem 37. For every EN system M
there exists a strongly reduced EN system M' such that $M \approx M'$.

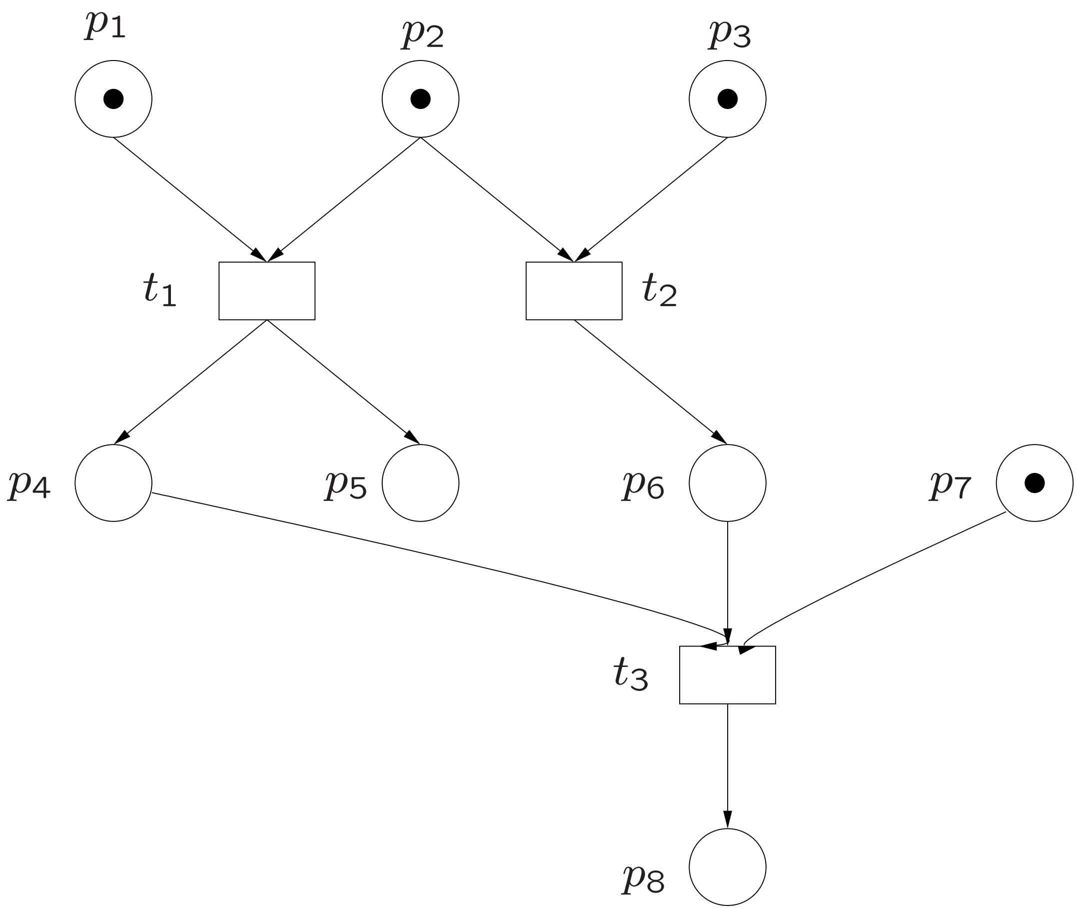


Fig. 37. An EN system with useless transition t_3.

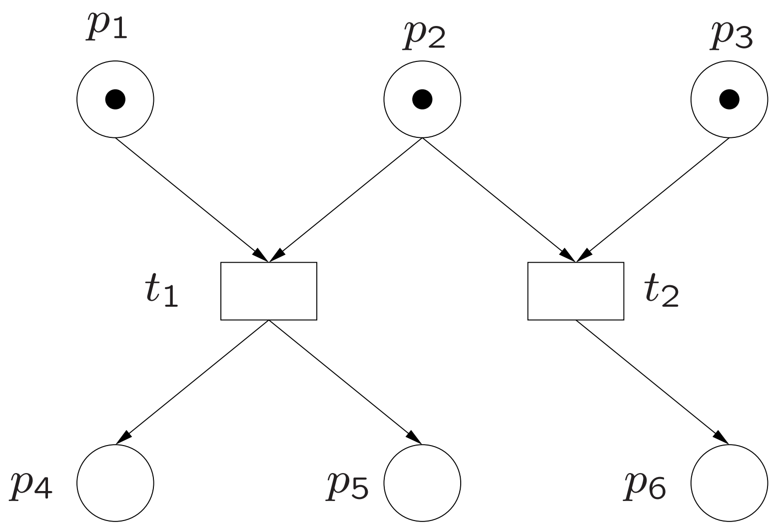


Fig. 38. A strongly reduced EN system, configuration equivalent with the EN system of Fig. 37.

Theorem 38. Let $M = (P, T, F, C_{in})$ be a strongly reduced EN system.

For every $p \in P$
there exist configurations $C, D \in \mathbb{C}_M$ such that

$p \in C$ and $p \notin D$.

Theorem 39. There exists an EN system M such that for every EN system M':

if $M' \approx_{fs} M$, then M' is not T-simple.

Definition 40. An EN system M is *sequential*

if $\#C = 1$ for all $C \in \mathbb{C}_M$.

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Lemma 41. If $M = (P, T, F, C_{in})$ is an EN system for which
(1) $\#C_{in} = 1$, and
(2) $\#({}^{\bullet}t) = \#(t^{\bullet}) = 1$ for all $t \in T$,

then M is sequential.

Lemma 41. If $M = (P, T, F, C_{in})$ is an EN system for which
(1) $\#C_{in} = 1$, and
(2) $\#(^\bullet t) = \#(t^\bullet) = 1$ for all $t \in T$,
then M is sequential.

Lemma 42. Let $M = (P, T, F, C_{in})$ be a reduced EN system.

(1) M is sequential iff
  (i) $\#C_{in} = 1$, and
  (ii) $\#(^\bullet t) = \#(t^\bullet) = 1$ for all $t \in T$.

(2) If M is strongly reduced and sequential,
then $\mathbb{C}_M = \{\{p\} \mid p \in P\}$.

Theorem 43. Let M and M' be two strongly reduced sequential EN systems. Then

$M \approx M'$ iff $M \equiv M'$.

Definition 44. An EN system M is *concurrency-free* if there do not exist $C \in \mathbb{C}_M$ and *different!* $t_1, t_2 \in T_M$ *such that* $\{t_1, t_2\}$ $\mathbf{con}$ C.