Theorie van Concurrency

voorjaar 2010

`http://www.liacs.nl/home/kleijn/thvc-0910.html`

tweede college: 10 februari 2010

4.4 Concurrency

4.5 Fundamental Situations

eerste werkgroep: 17 februari 2010

alle opgaven bij 4. EN Systems

Definition 13. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) Let $U \subseteq T$. U is a *disjoint set of transitions*, notation $\mathbf{disj}(U)$, if **1.** $U \neq \varnothing$
and **2.** for all transitions $t_1 \neq t_2 \in U$: $\mathbf{nbh}(t_1) \cap \mathbf{nbh}(t_2) = \varnothing$.

(2) Let $U \subseteq T$ and let $C \subseteq P$. Then U *has concession in* C (or U *can be fired in* C, or U *is enabled in* C) if
1. $\mathbf{disj}(U)$, **2.** ${}^\bullet U \subseteq C$, and **3.** $U^\bullet \cap C = \varnothing$.
Notation: $U \ \mathbf{con} \ C$.

(3) Let $U \subseteq T$ and let $C, D \subseteq P$.
Then U *fires from* C *to* D, written as $C[U\rangle D$, if
1. $U \ \mathbf{con} \ C$ and **2.** $D = (C - {}^\bullet U) \cup U^\bullet$.
If $\#U \geq 2$, then U is a *concurrent step from* C *to* D.

Lemma 14. Let $M = (P, T, F, C_{in})$ be an EN system. Let $U \subseteq T$ and let $C, D \subseteq P$.
Then $C[U\rangle D$ holds iff $\mathbf{disj}(U)$, $C - D = {}^{\bullet}U$, and $D - C = U^{\bullet}$.

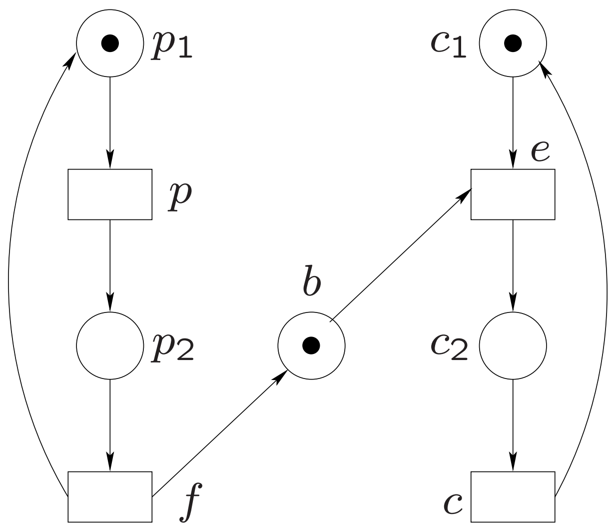

Fig. 12.

Lemma 15. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \subseteq P$ and let $U \subseteq T$ with $U \neq \varnothing$. Then U $\mathbf{con}$ C iff
(1) t $\mathbf{con}$ C for all $t \in U$, and
(2) for all $t_1 \neq t_2 \in U$, ${}^\bullet t_1 \cap {}^\bullet t_2 = \varnothing$ and $t_1{}^\bullet \cap t_2{}^\bullet = \varnothing$.

Lemma 16. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C, D \subseteq P$, and let $U \subseteq T$. Let $\{U_1, U_2\}$ be a partition of U.*
If $C[U\rangle D$, then there is $E \subseteq P$ such that $C[U_1\rangle E$ and $E[U_2\rangle D$.

* $U = U_1 \cup U_2$, $U_1 \cap U_2 = \varnothing$ and $U_1, U_2 \neq \varnothing$

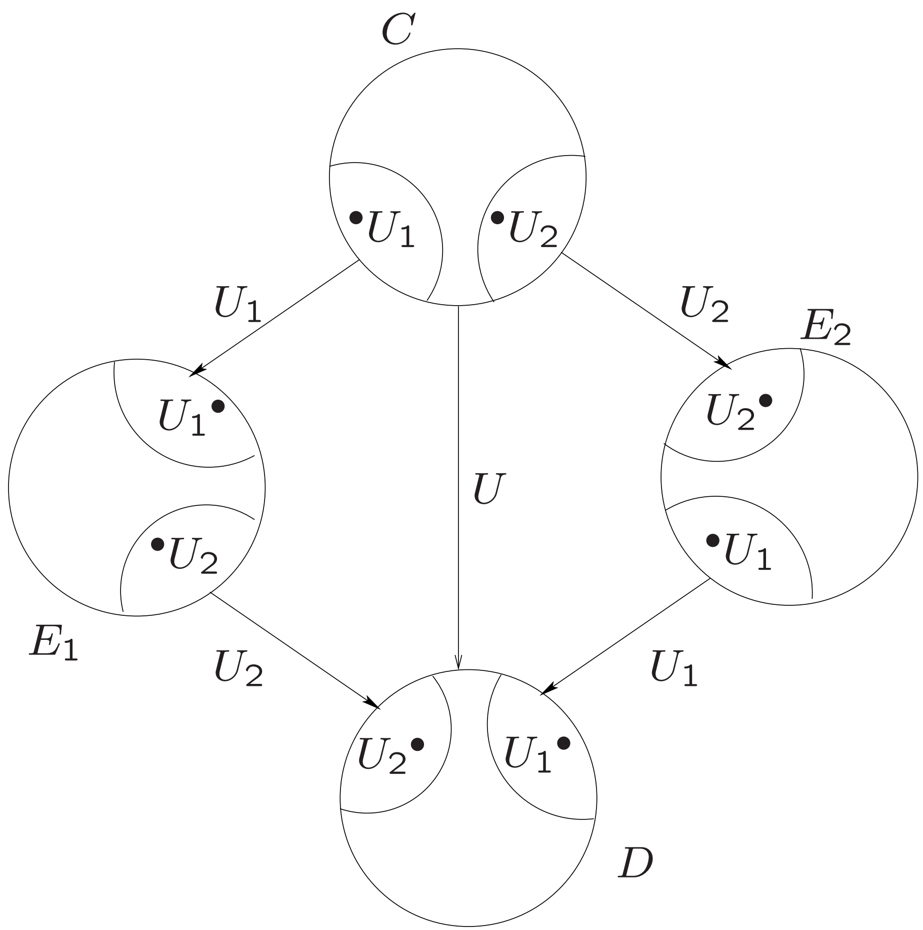

Fig. 17. A diamond.

Lemma 17. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C, D \subseteq P$ and let $U \subseteq T$.
If $C[U\rangle D$, then $C[t_1 \cdots t_n\rangle D$ for each ordering $(t_1, \ldots, t_n)$ of the elements of U.

Definition 18. Let M be an EN system. The *configuration graph of* M, denoted by CG(M), is the edge-labelled graph $(V, \Gamma, \Sigma, v_{in})$, where
$V = \mathbb{C}_M$,
$v_{in} = (C_{in})_M$,
$\Sigma = \mathbf{use}(T_M)$, and
$\Gamma = \{(C, U, D) \mid C, D \in \mathbb{C}_M,\ U \subseteq T_M,\ C[U\rangle_M D\}$.

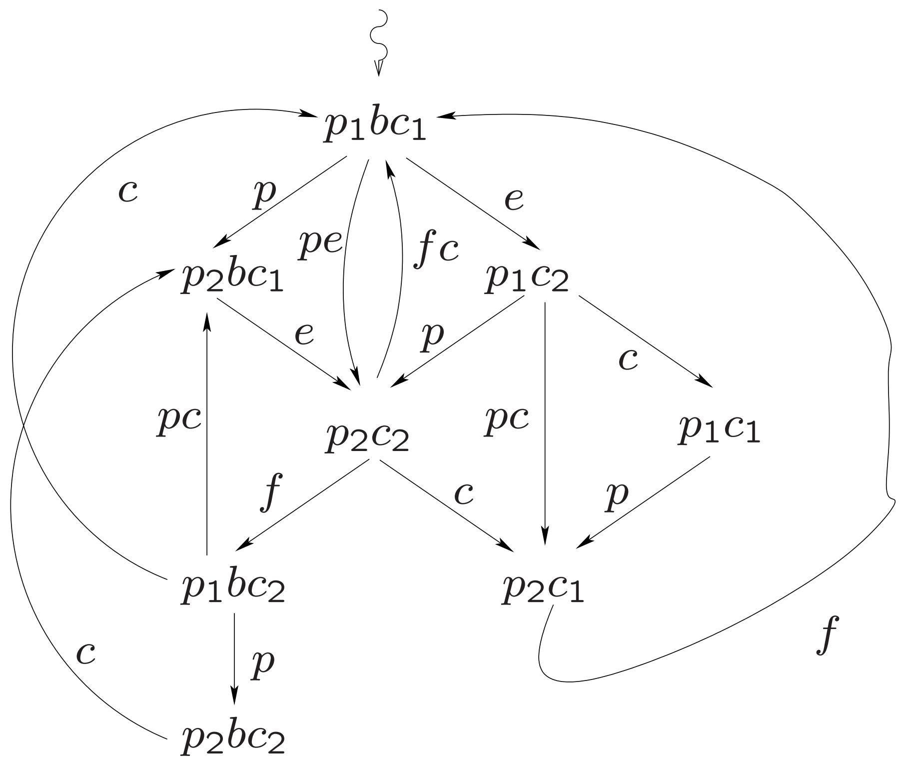

Fig. 18. A configuration graph.

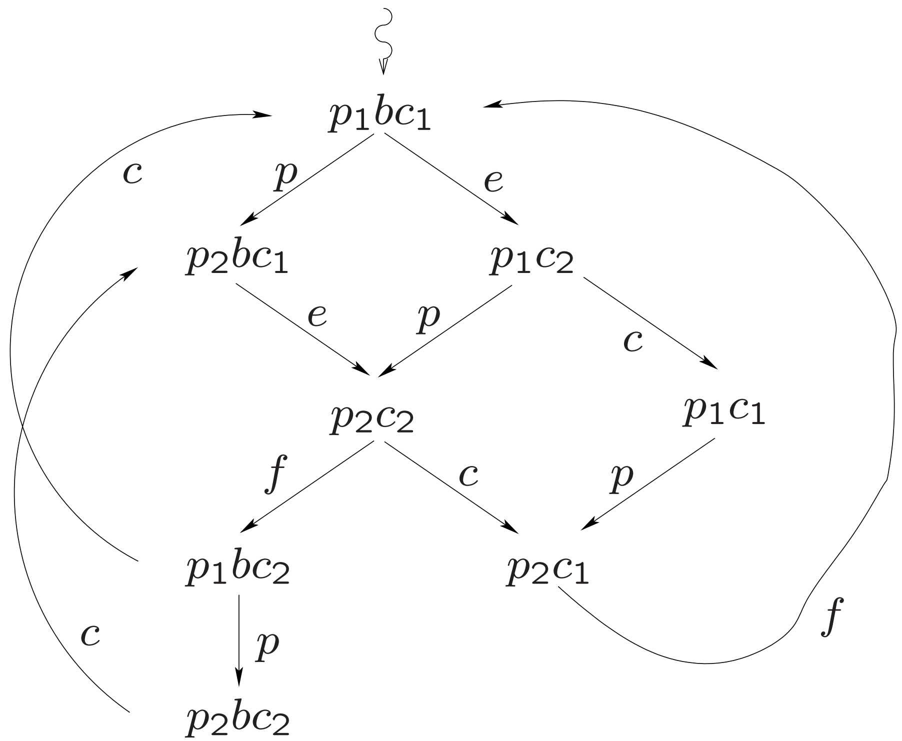

Fig. 16. A sequential configuration graph.

Lemma 19. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \subseteq P$ and let $s, t \in T$.
If st **con** C and t **con** C, then $\{s, t\}$ **con** C.

Theorem 20. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C, D \subseteq P$ and let $U \subseteq T$ with $U \neq \varnothing$. Then

(1) U $\mathbf{con}$ C iff $t_1 \cdots t_n$ $\mathbf{con}$ C for every ordering $(t_1, \ldots, t_n)$ of the elements of U, and

(2) $C[U\rangle D$ iff $C[t_1 \cdots t_n\rangle D$ for every ordering $(t_1, \ldots, t_n)$ of the elements of U.

Theorem 21. For EN systems M and M', $\mathsf{SCG}(M) \equiv \mathsf{SCG}(M')$ iff $\mathsf{CG}(M) \equiv \mathsf{CG}(M')$.

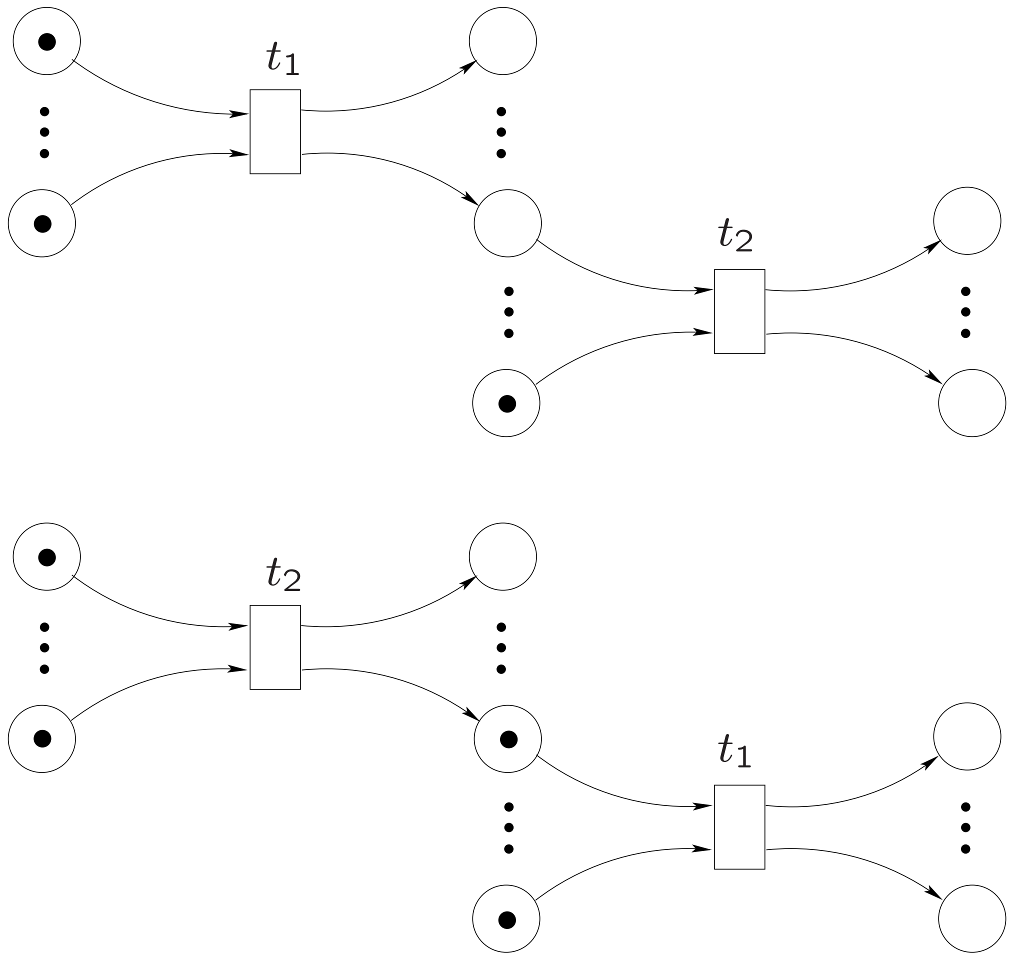

Fig. 19, **20.** Causality.

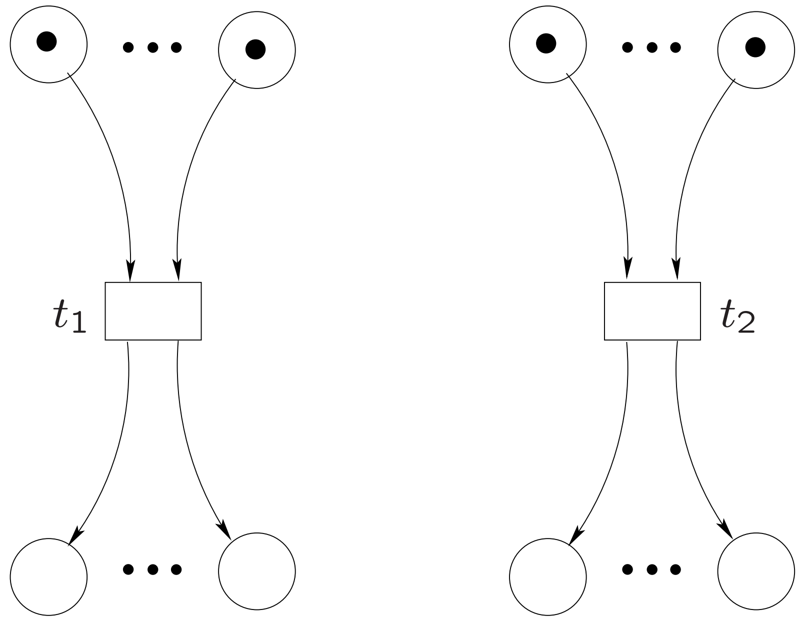

Fig. 21. Concurrency.

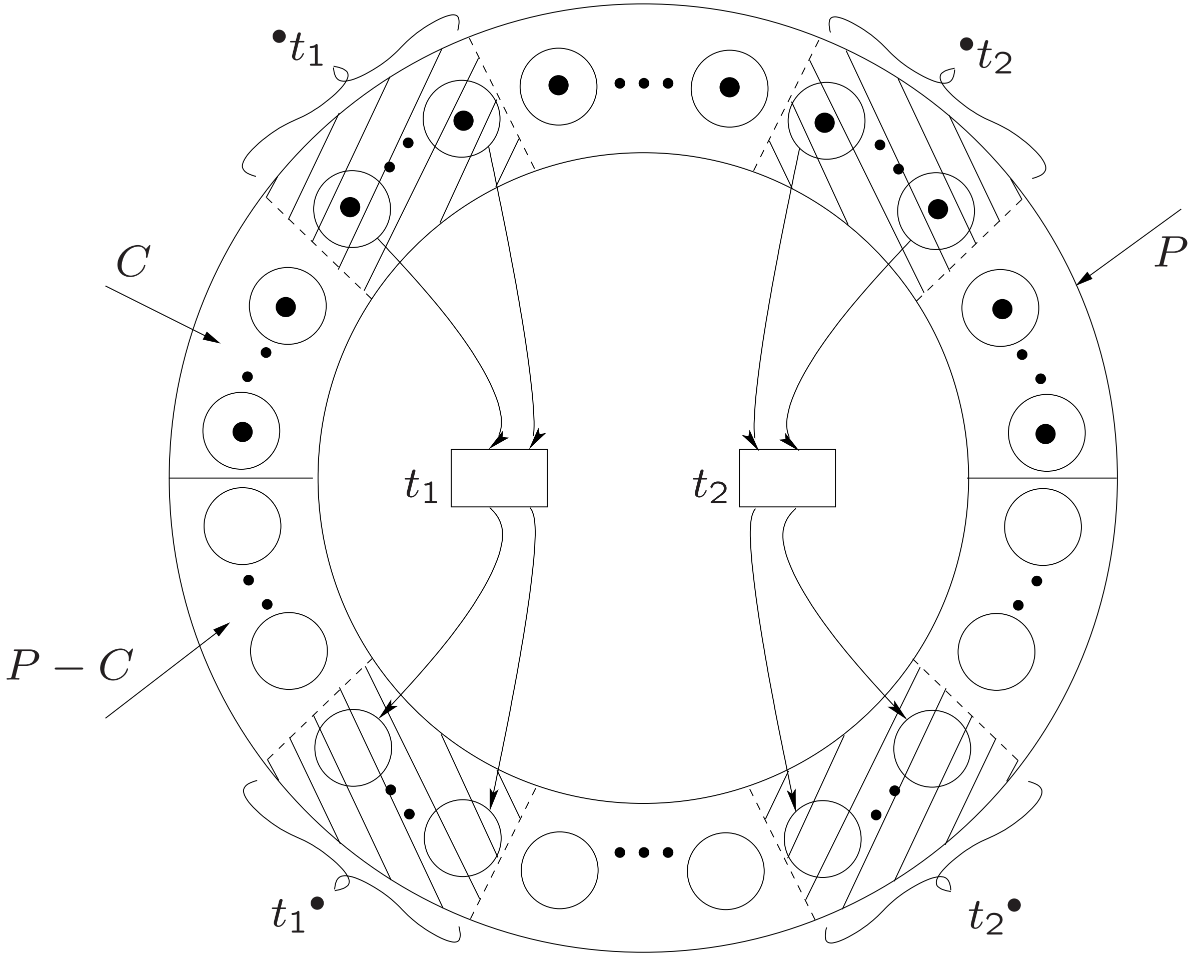

Fig. 22. Concurrency, the complete picture.

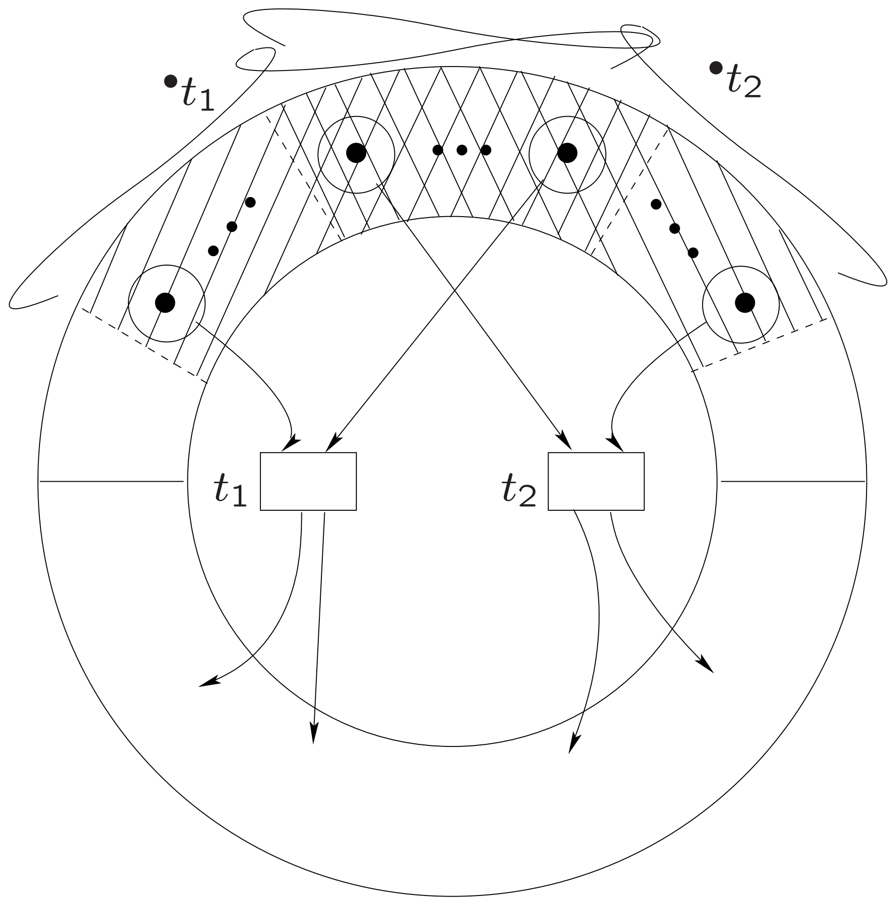

Fig. 23. Input-conflict.

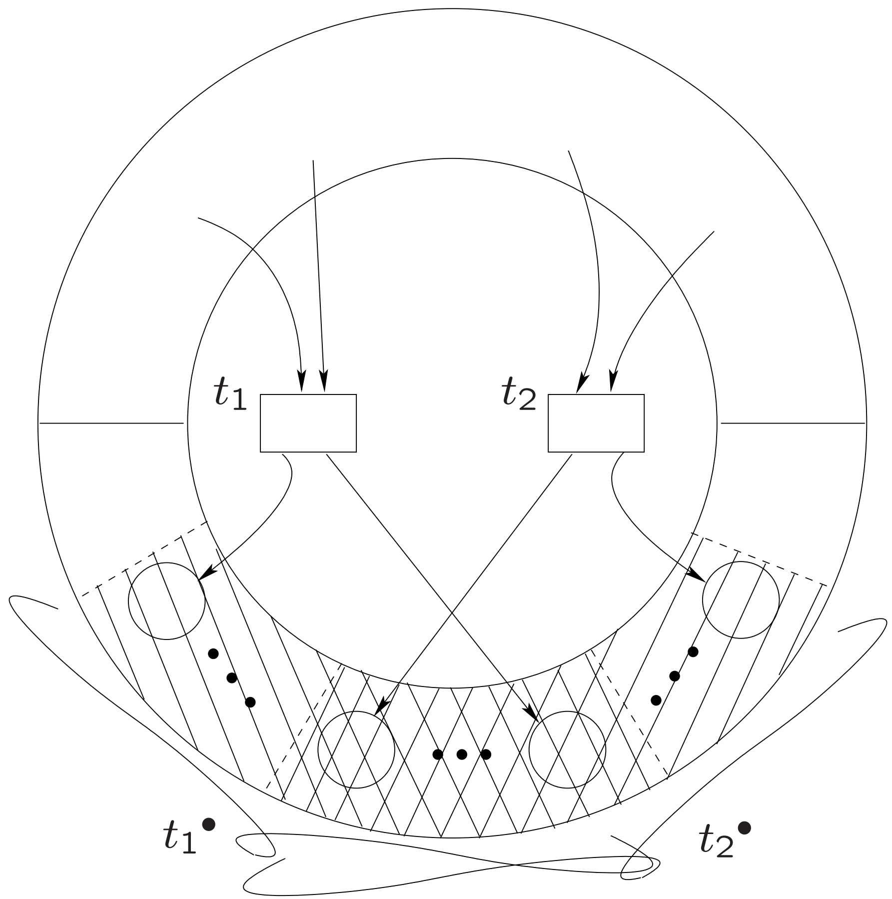

Fig. 24. Output-conflict.

Definition 22. An EN system $M = (P, T, F, C_{in})$ is *conflict-free* if, for every $C \in \mathbb{C}_M$ and all transitions $t_1, t_2 \in T$: $\{t_1, t_2\}$ $\mathbf{con}$ C whenever t_1 $\mathbf{con}$ C and t_2 $\mathbf{con}$ C.

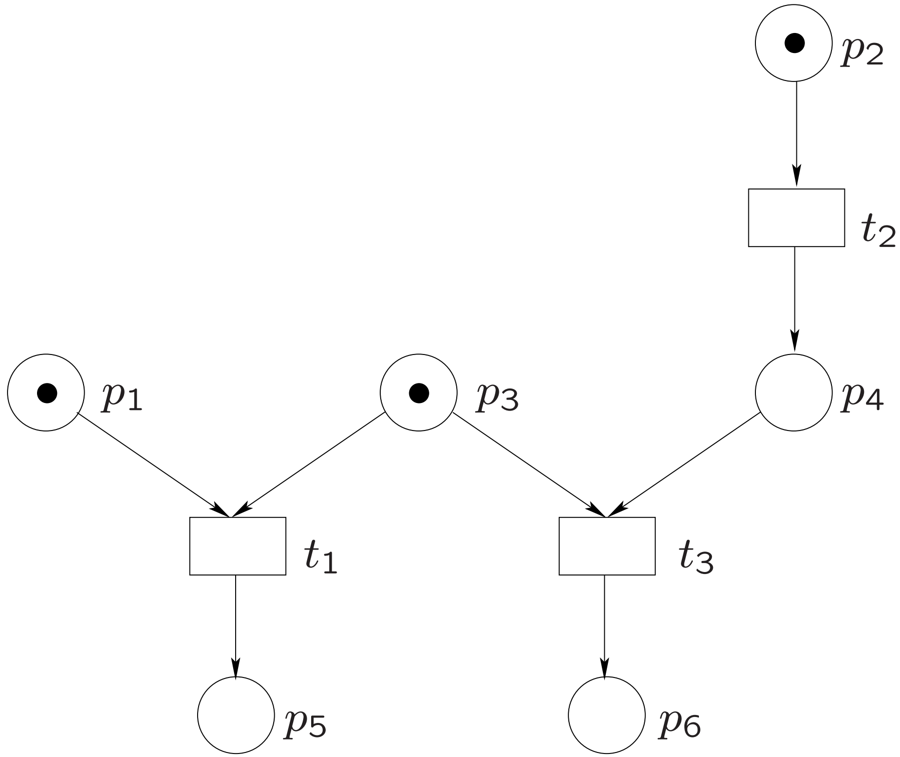

Fig. 25. A conflict-increasing confusion.

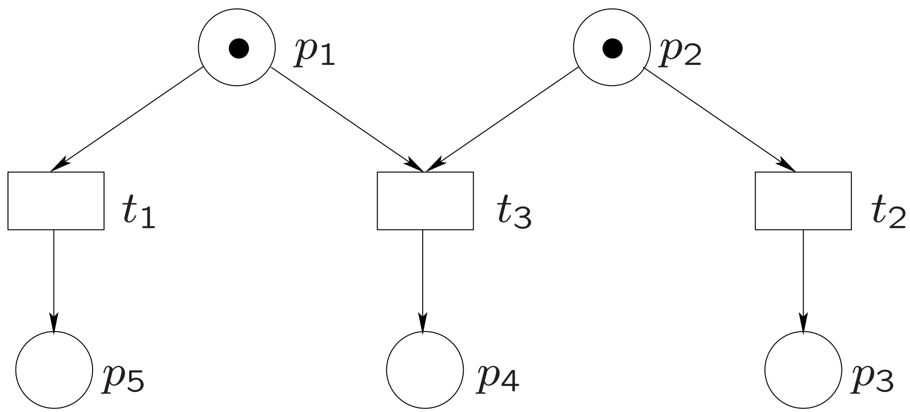

Fig. 26. A conflict-decreasing confusion.

Definition 23. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$, and let $t \in T$ be such that t **con** C.
Then $\mathbf{cfl}(t, C) = \{t' \in T \mid t' \ \mathbf{con}\ C$ and $\neg\ \{t, t'\}\ \mathbf{con}\ C\}$ is the *conflict set of* t *in* C.

Definition 24. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$, and let $t_1, t_2 \in T$.
The triple (C, t_1, t_2) is called a *confusion* (*in* C) if

1. $t_1 \neq t_2$,
2. $\{t_1, t_2\}$ $\mathbf{con}$ C, and
3. $\mathbf{cfl}(t_1, C) \neq \mathbf{cfl}(t_1, D)$, where $C[t_2\rangle D$.

M *is confused in* C if there is a confusion in C.

Definition 25 Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$ and $t_1, t_2 \in T$.
Let $\gamma = (C, t_1, t_2)$ be a confusion and $C[t_2\rangle D$.

(1) γ is a *conflict-increasing confusion*, *ci confusion* for short, if $\mathbf{cfl}(t_1, D) \supsetneq \mathbf{cfl}(t_1, C)$.

(2) γ is a *conflict-decreasing confusion*, *cd confusion* for short, if $\mathbf{cfl}(t_1, D) \subsetneq \mathbf{cfl}(t_1, C)$.

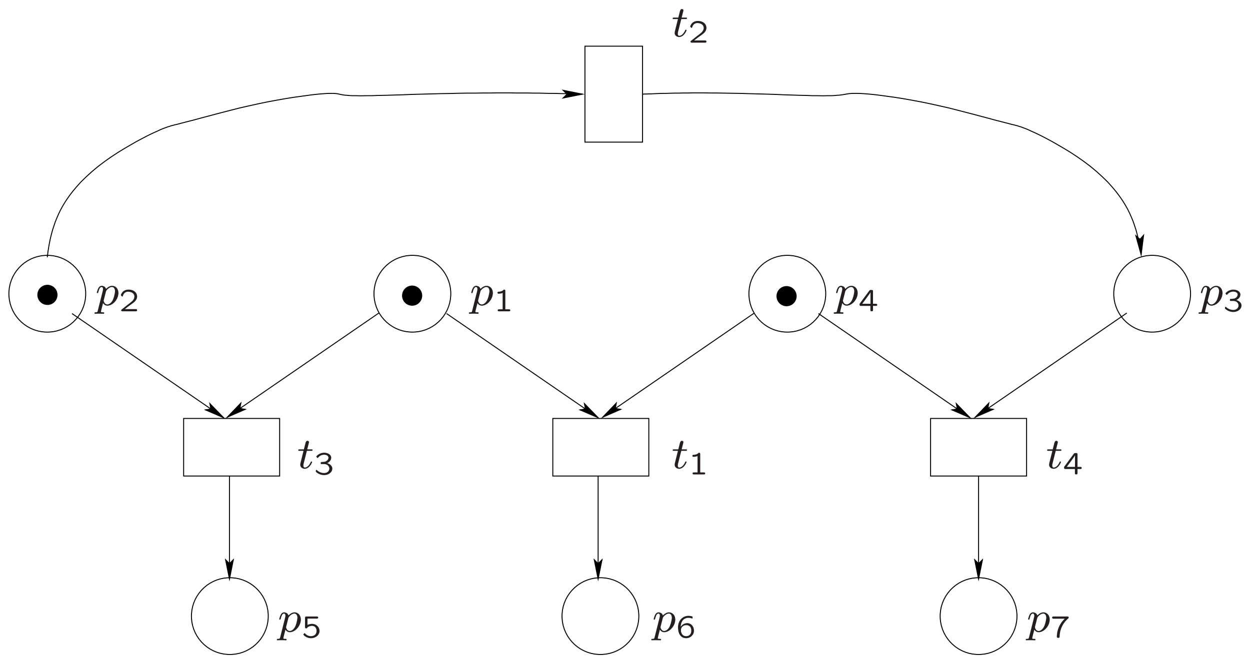

Fig. 27. A confusion which is neither ci nor cd.

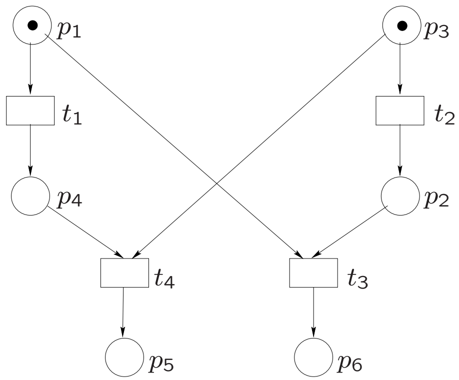

Fig. 28. A symmetric confusion.

Definition 26. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$ and $t_1, t_2 \in T$. Let $\gamma = (C, t_1, t_2)$ be a confusion.

γ is *symmetric* if (C, t_2, t_1) is also a confusion, otherwise γ is *asymmetric*.

Consider the EN system Mutex (Figure 5).
Give CG(Mutex) and
determine all confusions (C, t_1, t_2) with $C \in \mathbb{C}_{\text{Mutex}}$.

Give - if possible - examples of confusions which are conflict-increasing, conflict-decreasing, neither and in addition (a)symmetric.

Prove: every confusion which is not ci is symmetric.