Theorie van Concurrency

voorjaar 2010

http://www.liacs.nl/home/kleijn/thvc-0910.html

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Theorie van Concurrency — voorjaar 2010

`http://www.liacs.nl/home/kleijn/thvc-0910.html`

- hoorcollege/werkgroep $\sim$ 2/1
woensdag 3 februari - 28 april, zaal 412
13.45 -16.30, **10 maart: 10.15-13.00**

- dictaat en opgavenbundel
tutorial article (recommended)

- **eerste wg 17 februari:** laptop, Java platform, ...
inlichtingen: Wouter

- tentamendata: vrijdag 28 mei; vrijdag 13 augustus

- bachelorprojecten ...

Petri Net C.A. Petri 1962
model voor *concurrente/parallelle* systemen
d.w.z. niet sequentiele systemen,
vaak bestaand uit parallelle componenten:
samenwerkend, concurrerend, communicerend
complex!!!
toepassingen:
operating systems
distributed algorithms
manufacturing systems (industrial production)
computer supported cooperative work (cscw)
protocol specification (e.g. telecom)
semantics of parallel programming languages
hardware design (asynchronous circuits, multi core chips)
biomodeling ...
Dit college: niet modelleren, maar **theorie** van het model

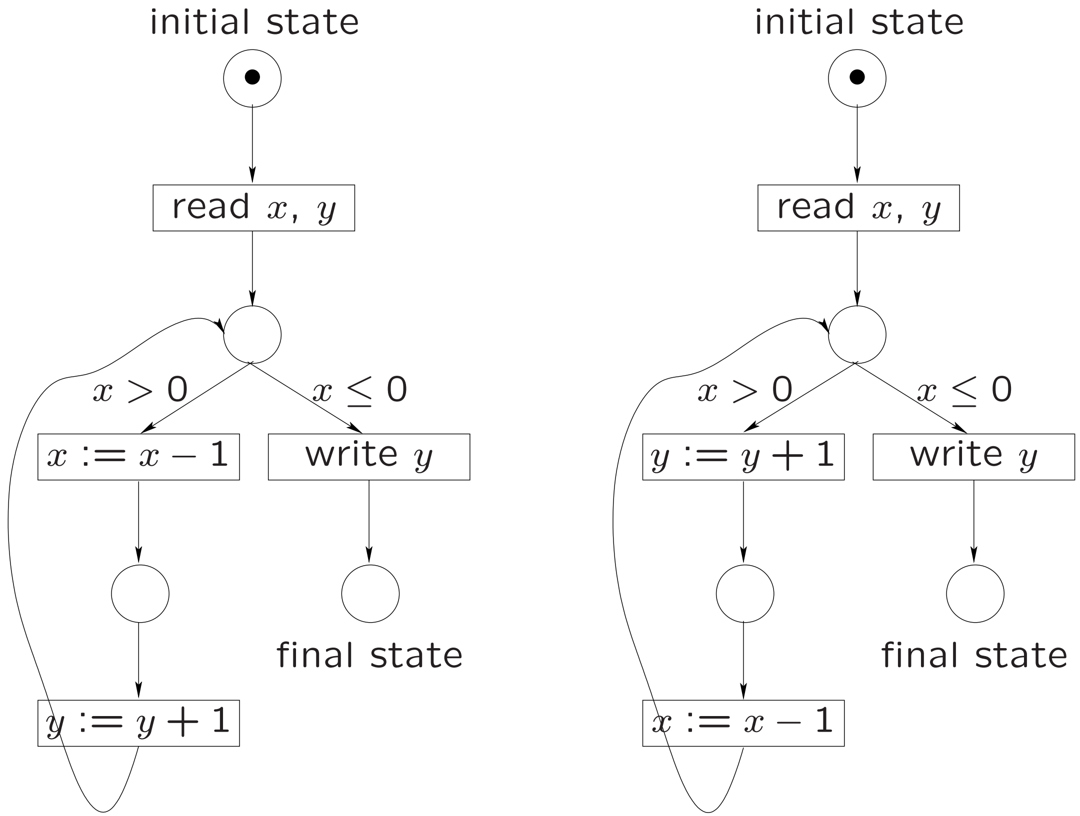


Fig. 1. Sequential addition programs.

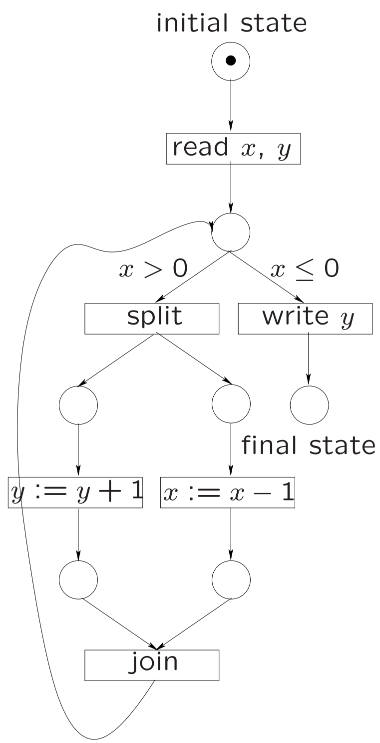


Fig. 2. A concurrent addition program.

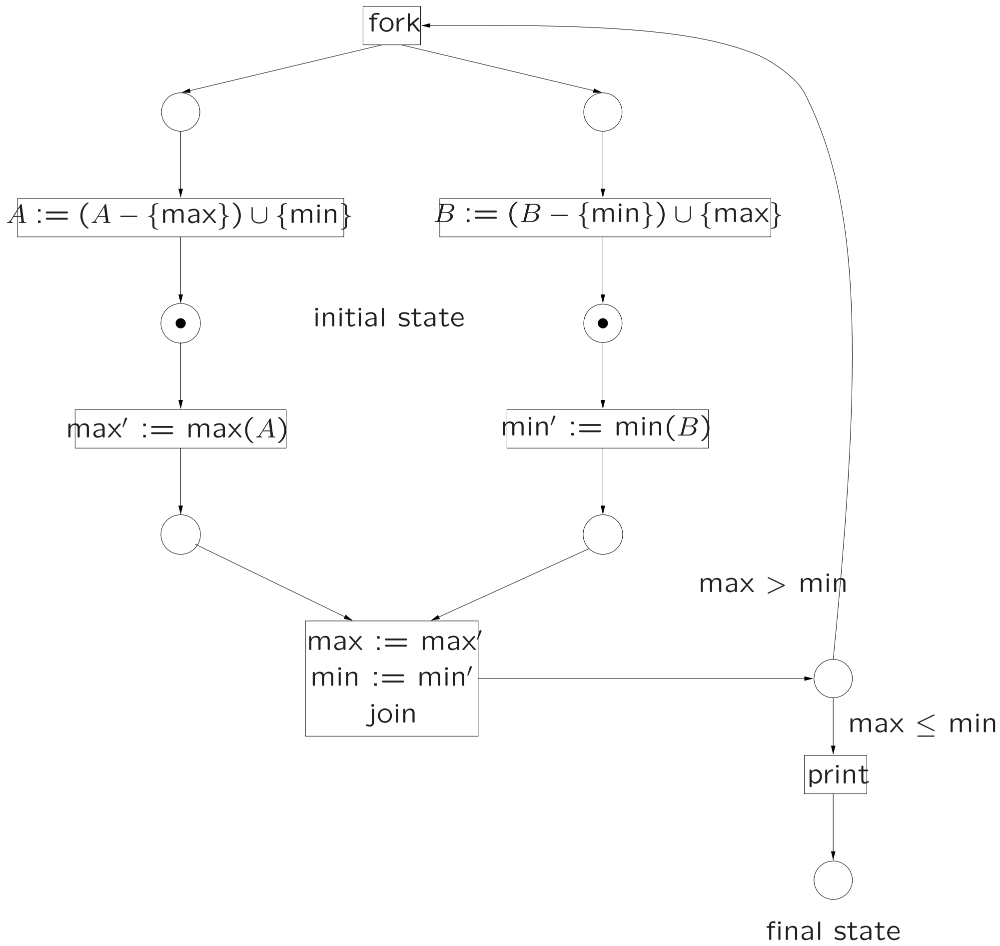


Fig. 3. Concurrent operations on sets.

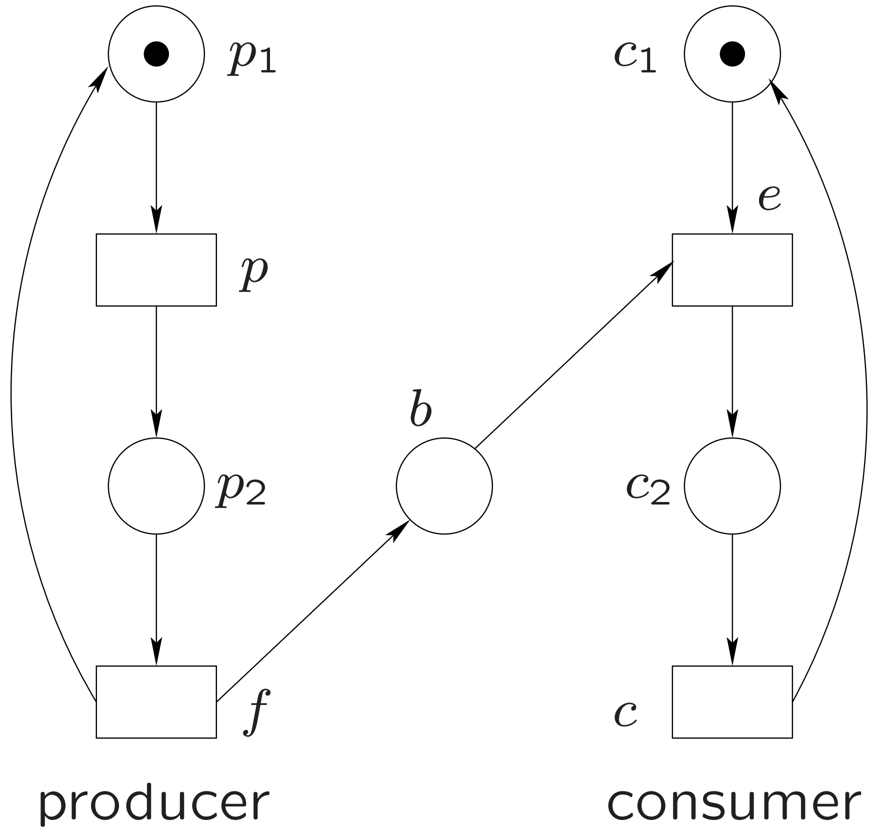


Fig. 4. The producer/consumer problem.

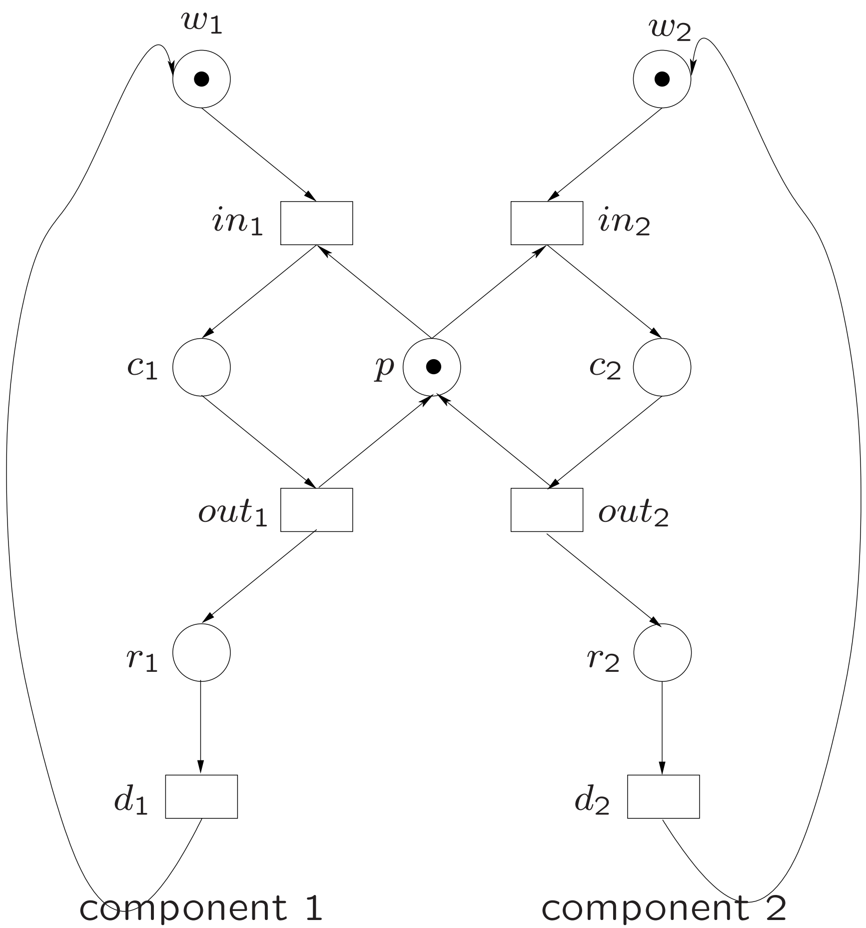
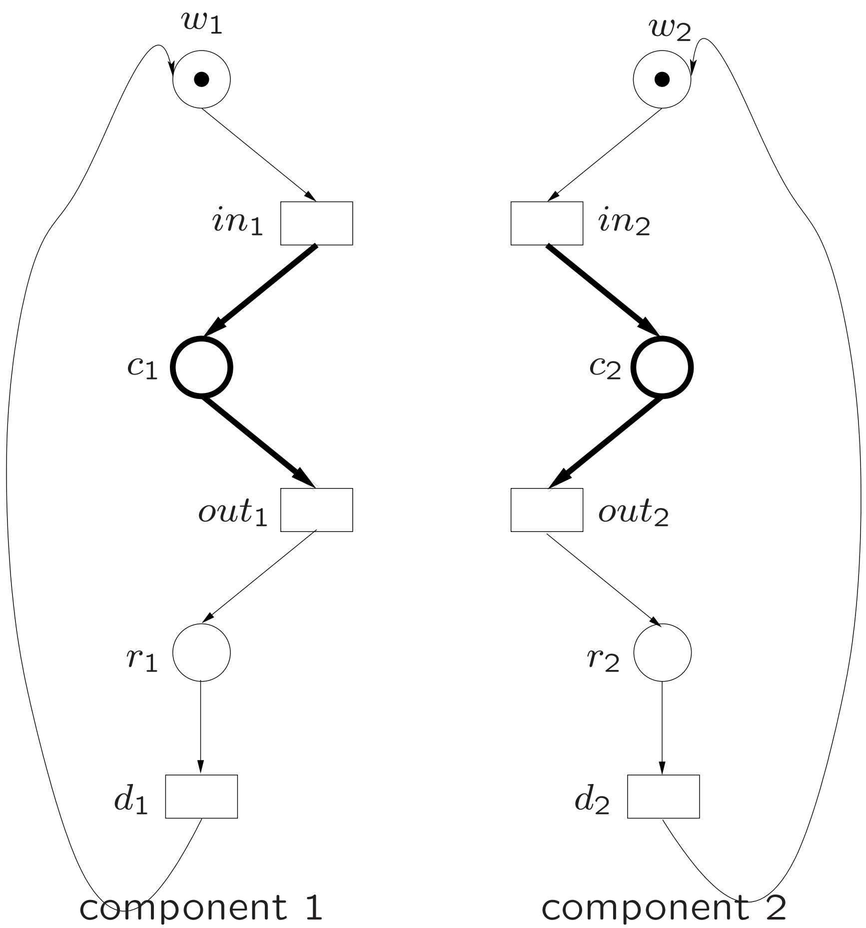
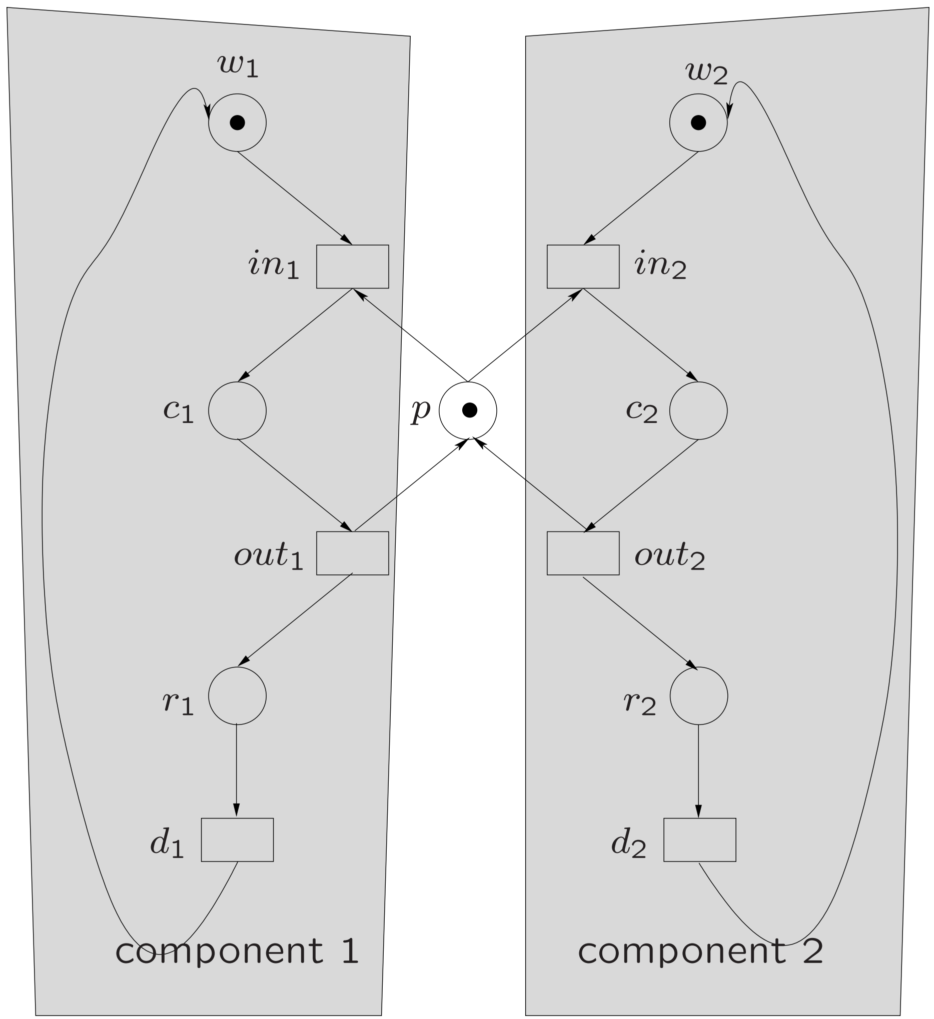


Fig. 5. The mutual exclusion problem.



mutual exclusion



competition

Definition 1. A *net* is a triple $N = (P, T, F)$, where:

(1) P and T are finite sets with $P \cap T = \varnothing$,

(2) $F \subseteq (P \times T) \cup (T \times P)$,

(3) for every $t \in T$ there exist $p, q \in P$ such that $(p, t), (t, q) \in F$, and

(4) for every $t \in T$ and $p, q \in P$,
if $(p, t), (t, q) \in F$, then $p \neq q$.

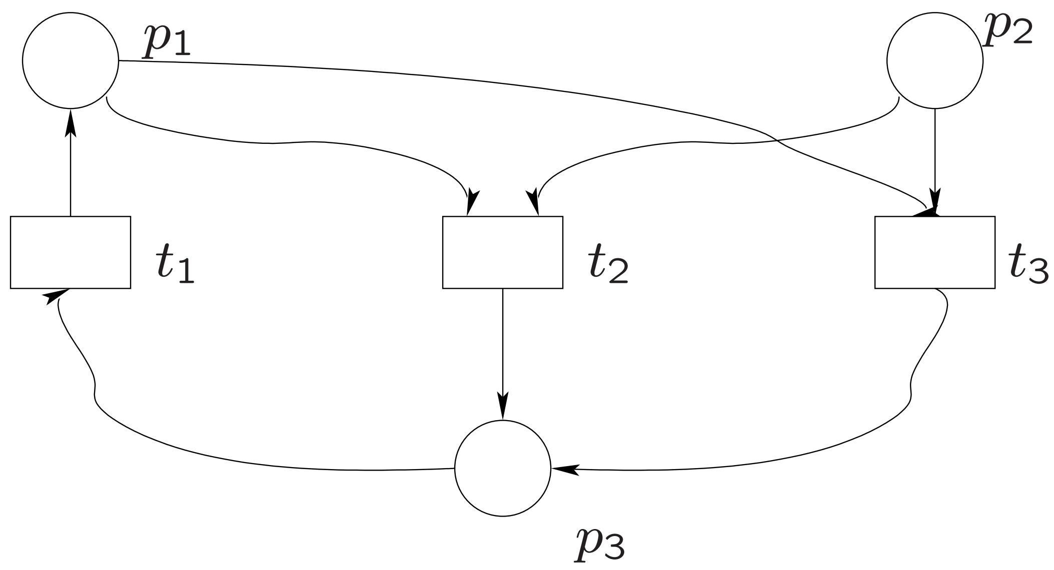


Fig. 6. A net.

Definition 2. A net $N = (P, T, F)$

(1) is *acyclic* if, for every $x \in X$, $(x, x) \notin F^+$,

(2) is *P-simple* if, for all $p, q \in P$,
$({}^\bullet p = {}^\bullet q$ and $p^\bullet = q^\bullet)$ implies $p = q$,

(3) is *T-simple* if, for all $s, t \in T$,
$({}^\bullet s = {}^\bullet t$ and $s^\bullet = t^\bullet)$ implies $s = t$,

(4) has *no isolated places* if, for all $p \in P$, $\mathbf{nbh}(p) \neq \varnothing$.

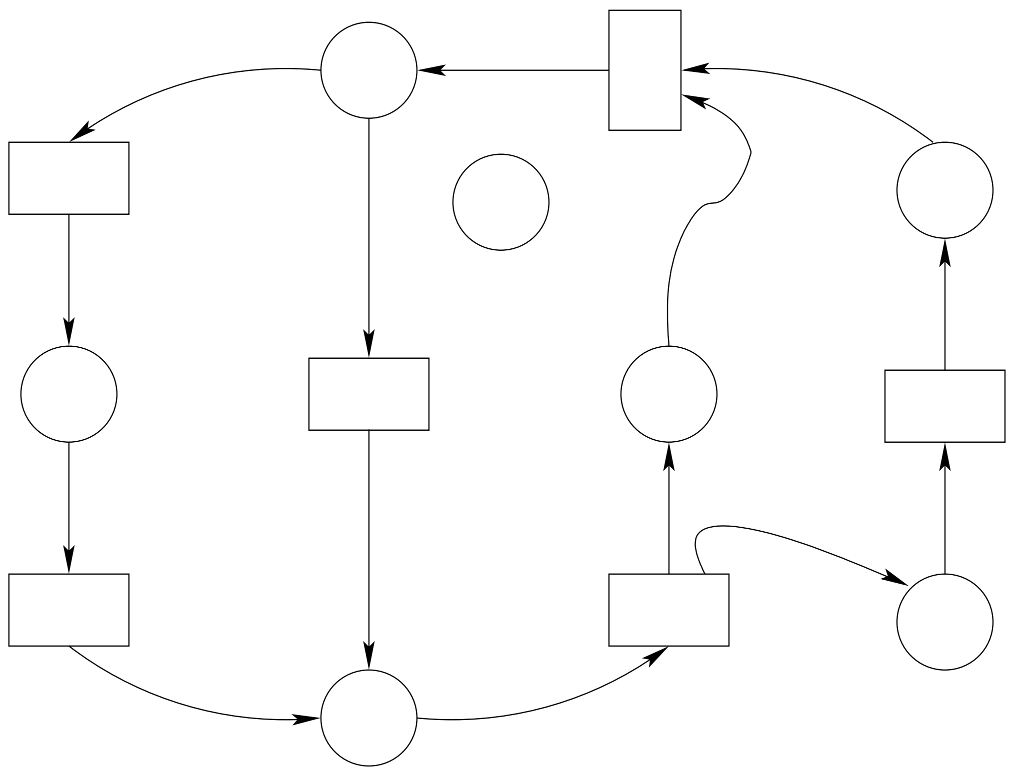
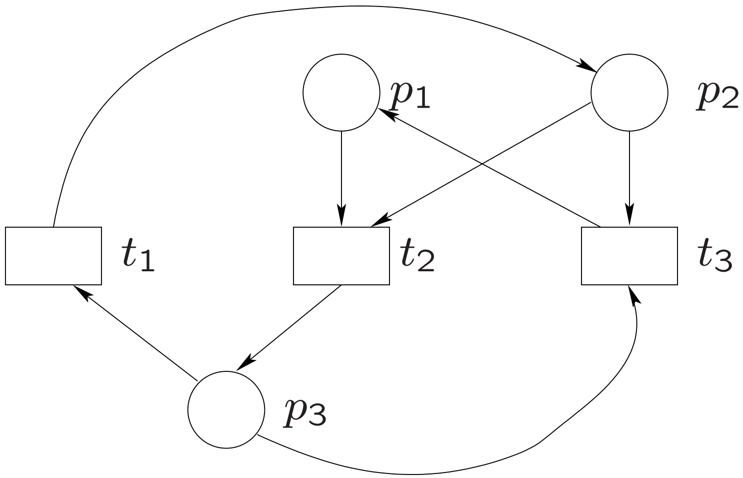


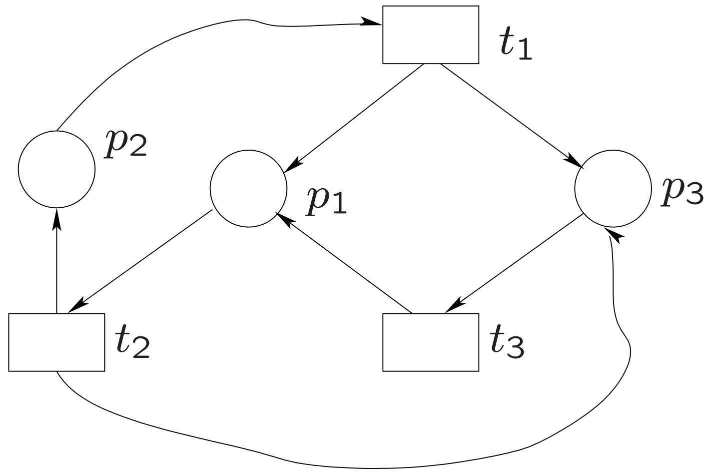
Fig. 7. A net that is P-simple and T-simple.

Definition 3. Two nets
$N = (P, T, F)$ and $N' = (P', T', F')$
are *isomorphic*, denoted by $N \equiv N'$,

if there exist two bijections
$\alpha : P \rightarrow P'$ and $\beta : T \rightarrow T'$,

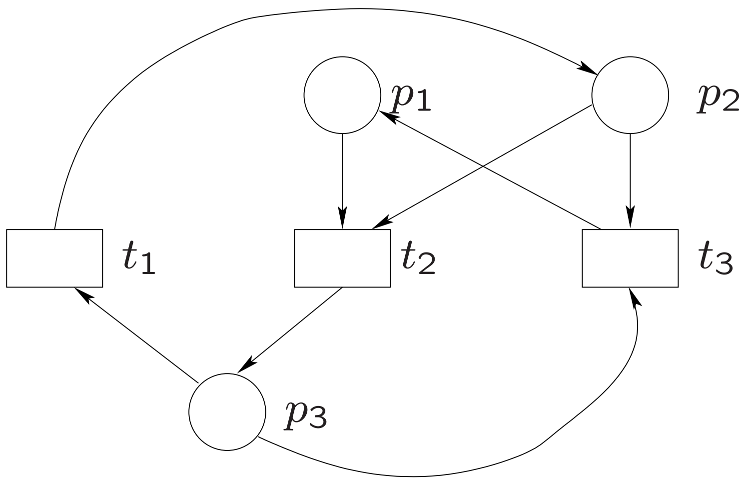
such that for every $p \in P$ and $t \in T$,
$(p, t) \in F$ iff $(\alpha(p), \beta(t)) \in F'$
and
$(t, p) \in F$ iff $(\beta(t), \alpha(p)) \in F'$.

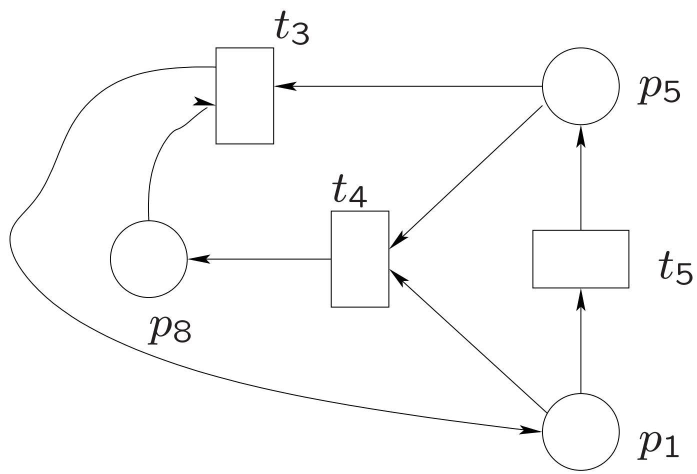




A net N **Fig. 8** and a net N' **Fig. 9**, not isomorphic.

G_N and $G_{N'}$ are isomorphic graphs!





A net N **Fig. 8** and a net N'' **Fig. 10**, isomorphic.

Definition 4. A *configuration* of a net $N = (P, T, F)$ is a subset of P.

Definition 5. An *elementary net system*, EN system for short, is a quadruple $M = (P, T, F, C_{in})$, where:

(1) (P, T, F) is a net and

(2) $C_{in} \subseteq P$ is the *initial configuration*.

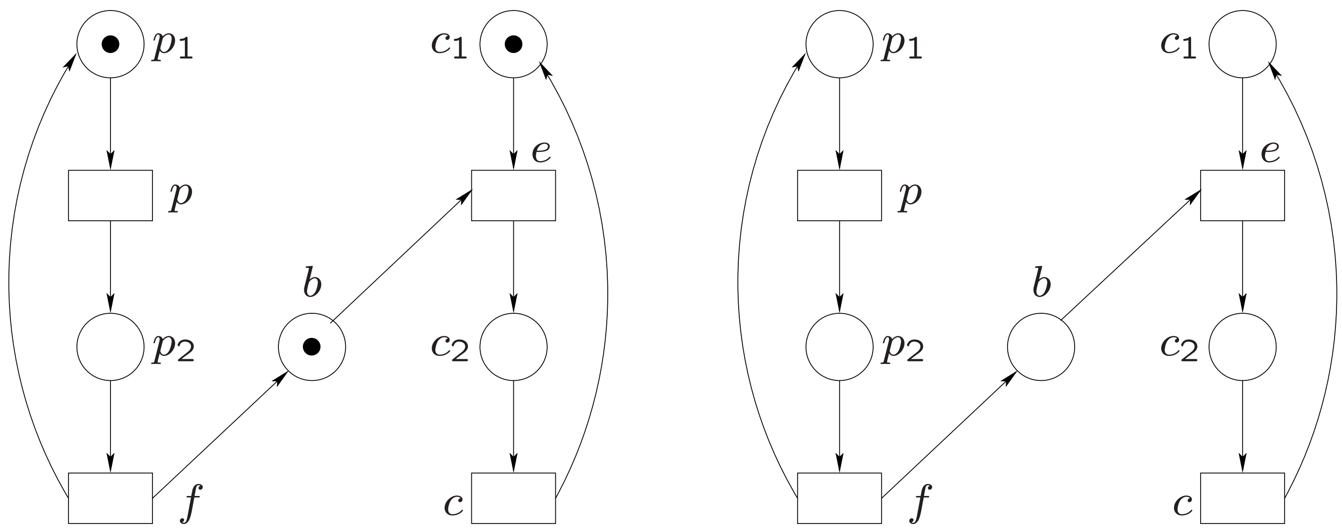


Fig. 12. An EN system and its underlying net **Fig. 11.**

Definition 6. Let $M = (P, T, F, C_{in})$ be an EN system and let $t \in T$.

(1) Let $C \subseteq P$ be a configuration.
Then t *has concession in* C
(or t *can be fired in* C, or t *is enabled in* C)
if ${}^\bullet t \subseteq C$ and $t^\bullet \cap C = \varnothing$, written as $t \ \mathbf{con} \ C$.

(2) Let $C, D \subseteq P$. Then t *fires from* C *to* D if $t \ \mathbf{con} \ C$ and $D = (C - {}^\bullet t) \cup t^\bullet$, written as $C[t\rangle D$;
t is also called a *sequential step from* C *to* D.

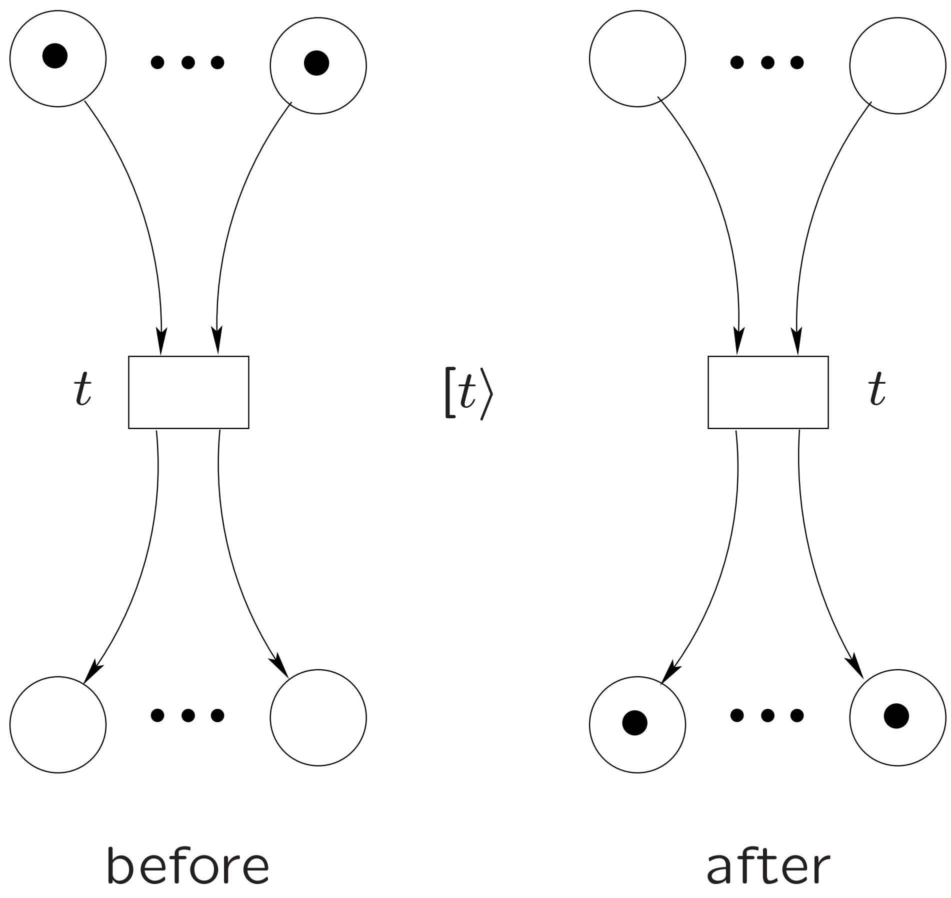
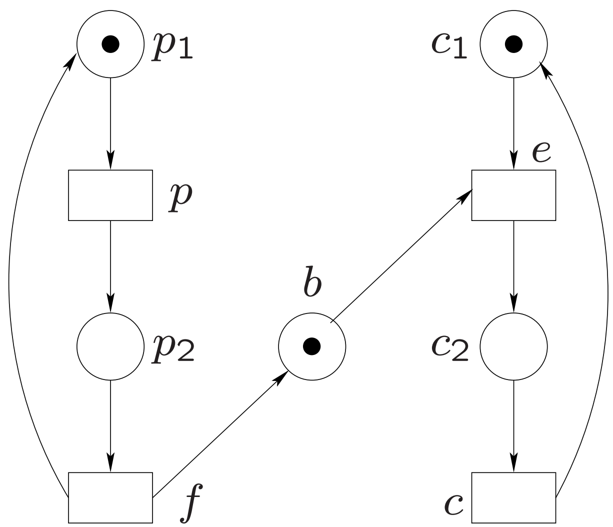
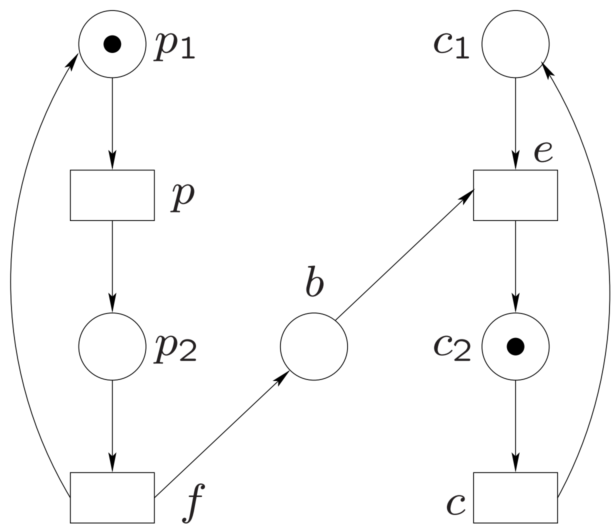


Fig. 13. Firing of transition t.





Before **Fig. 12.** and after **Fig. 14.** firing transition e.

Lemma 7. Let $M = (P, T, F, C_{in})$ be an EN system. Let $t \in T$ and let $C, D \subseteq P$.

Then $C[t\rangle D$ iff $C - D = {}^\bullet t$ and $D - C = t^\bullet$.

Definition 8. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) Let $t_1 \cdots t_n \in T^*$, with $n \geq 0$ and $t_1, \ldots, t_n \in T$. Let $C, D \subseteq P$. Then $t_1 \cdots t_n$ *fires from* C *to* D if there exist configurations $C_0, C_1, \ldots, C_n \subseteq P$ with $C_0 = C$, $C_n = D$ and $C_{i-1}[t_i\rangle C_i$ for all $1 \leq i \leq n$, written as $C[t_1 \cdots t_n\rangle D$.

(2) Let $x \in T^*$ and $C \subseteq P$.
Then x *has concession in* C
(or x *can be fired in* C, or x *is enabled in* C)
if there exists a $D \subseteq P$ such that $C[x\rangle D$, written as $x \ \mathbf{con} \ C$.

(3) $x \in T^*$ is a *firing sequence of* M if $x \ \mathbf{con} \ C_{in}$. The set of all firing sequences of M is denoted by $\mathsf{FS}(M)$.

Definition 8 Ctd. Let $M = (P, T, F, C_{in})$ be an EN system.

(4) $C \subseteq P$ is a *reachable configuration of* M
if there exists an $x \in \mathsf{FS}(M)$ with $C_{in}[x\rangle C$.
The set of all reachable configurations of M is denoted by $\mathbb{C}_M$.

(5) $t \in T$ is a *useful transition of* M
if there exists a reachable configuration C of M such that $t \ \mathbf{con}\ C$.
The set of useful transitions of M is denoted by $\mathbf{use}_M(T)$, or just $\mathbf{use}(T)$ when M is clear from the context.

(6) $t \in T$ is a *live transition of* M
if for each $C \in \mathbb{C}_M$ there exists an $x \in T^*$ with $xt \ \mathbf{con}\ C$.

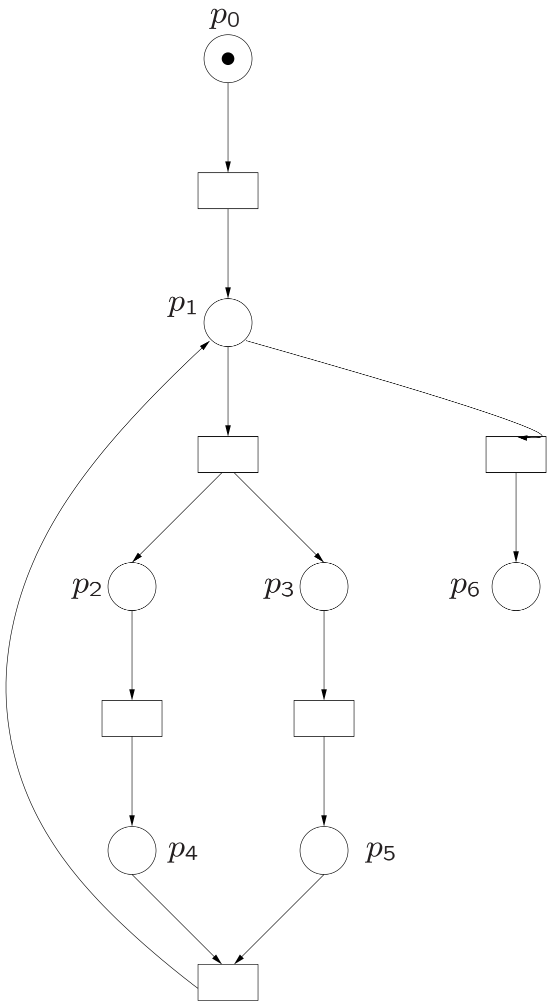


Fig. 15. All transitions are useful, but none is live.

Definition 9. An (initialized) *edge-labelled graph* is a quadruple $(V, \Gamma, \Sigma, v_{in})$, where

V is a finite set of *nodes*,
$v_{in} \in V$ is the *initial node*,
Σ is a finite set of *(edge-) labels*, and
$\Gamma \subseteq V \times \mathcal{P}(\Sigma) \times V$ is a set of *(labelled) edges*.

Definition 10. Let $G_1 = (V_1, \Gamma_1, \Sigma_1, v_1)$ and $G_2 = (V_2, \Gamma_2, \Sigma_2, v_2)$ be edge-labelled graphs.
Then G_1 and G_2 are *isomorphic*,
denoted by $G_1 \equiv G_2$, if there exist

two bijections $\alpha : V_1 \to V_2$ and $\beta : \Sigma_1 \to \Sigma_2$

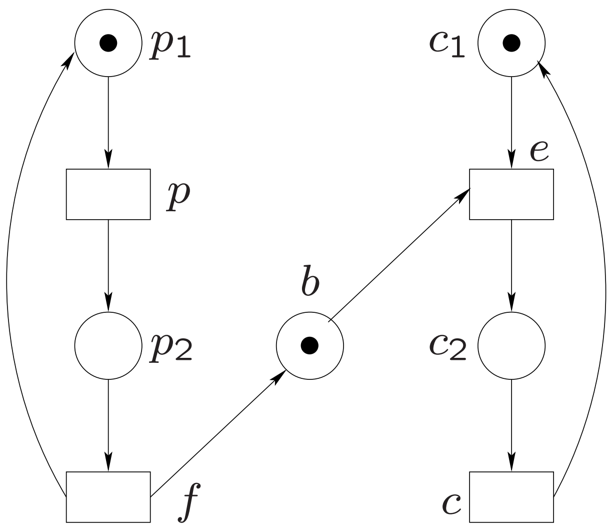
such that $\alpha(v_1) = v_2$ and,

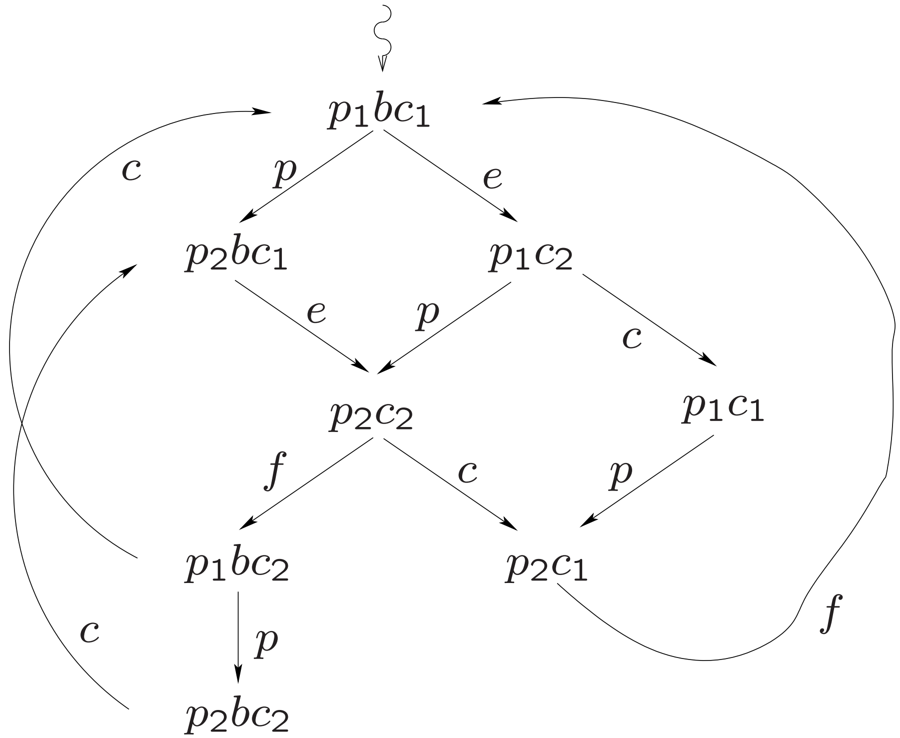
for all $v, w \in V_1$ and all $U \subseteq \Sigma_1$,
$(v, U, w) \in \Gamma_1$ iff $(\alpha(v), \beta(U), \alpha(w)) \in \Gamma_2$.

Definition 11. Let M be an EN system. The *sequential configuration graph of* M,
denoted by SCG(M),
is the edge-labelled graph $(V, \Gamma, \Sigma, v_{in})$, where

$V = \mathbb{C}_M$, $v_{in} = (C_{in})_M$, $\Sigma = \mathbf{use}(T_M)$, and

$\Gamma = \{(C, t, D) \mid C, D \in \mathbb{C}_M,\ t \in T_M,\ C[t\rangle_M D\}$.





An EN system **Fig. 12.** and its sequential configuration graph **Fig. 16.**

Theorem 12. For every EN system M, FS(M) is a regular language.

behandeld eerste college:

1. Preface, 2. Introduction
(3. Preliminaries — zelf lezen)
4. EN systems: 4.1, 4.2, 4.3

tweede college: 10 februari 2010
4.4 Concurrency
4.5 Fundamental Situations

eerste werkgroep: 17 februari 2010
alle opgaven bij 4. EN Systems