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Tetris?

- Tetris is NP complete !!
- what configurations ?
- undecidable Tetris
- the AI of Tetris

www.liacs.nl/home/kosters/tetris/
How hard is Tetris?

- Basic Rules
- **Offline Tetris**
- Complexity
- Reduction

History

1985  Alexey Pazhitnov (Алексей Пажитнов) invents Tetris inspired by ‘pentominoes’.

1989  Nintendo released Tetris on 8-bit console and Game-Boy

Now  Many other versions of Tetris are still sold, played and loved.
Other versions ...
Basic Rules of Tetris

- History
- Basic Rules
- Offline Tetris
- Complexity
- Reduction
- Conclusion

7 different pieces, 4 blocks each

- left / right
- rotate: 90 degrees
- drop
- one block look-ahead
Basic Rules of Tetris

Full lines are deleted
Full lines are deleted
Basic Rules of Tetris

- History
- Basic Rules
- Offline Tetris
- Complexity
- Reduction
- Conclusion

Full lines are deleted
Basic Rules of Tetris

Full lines are deleted
Basic Rules of Tetris

Full lines are deleted
... and may leave ‘floating’ blocks
‘Offline’ Tetris

- Partially filled board.
- All pieces are known at the beginning.

“Given an initial game board and a sequence of pieces, can the board be cleared?”
‘Offline’ Tetris

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“Given an initial game board and a sequence of pieces, can the board be cleared?”

“yes”
Problem groups:

**NP**
- solution *checkable* within ‘reasonable’ time

**P**
- problem *solvable* within ‘reasonable’ time

**NP-complete**
- problem is NP and
- algorithm for this problem can be ‘translated’ to any other NP problem

big question: $P = NP$? -- $1.000.000$

http://www.claymath.org/millennium/P_vs_NP/
Floortje has bought a new floor, the salesman told no sawing was required: “every row can be filled with three tiles from the pack”.

**Question:** Was salesman telling the truth?
Complexity

Floor tile example:

12 floor tiles (in centimeters):
26, 26, 28, 30, 31, 32, 33, 34, 36, 36, 40, 48

4 rows in hallway:

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```

100 100 100 100
# Complexity

## Floor tile example:

<table>
<thead>
<tr>
<th>26, 26, 28, 30, 31, 32, 33, 34, 36, 36, 40, 48</th>
</tr>
</thead>
<tbody>
<tr>
<td>28, 31, 26, 30</td>
</tr>
<tr>
<td>32, 33, 26, 34</td>
</tr>
<tr>
<td>40, 36, 48, 36</td>
</tr>
<tr>
<td>100, 100, 100, 100</td>
</tr>
</tbody>
</table>
Complexity

Number of possible configurations:

- 9 tiles \(\rightarrow\) 1,680
- 12 tiles \(\rightarrow\) 369,600
- 15 tiles \(\rightarrow\) 168,168,000

Grows exponentially: NP ... intuitively ...

*3-partitioning problem* proven to be NP-complete

so ... any NP problem can be solved using the algorithm for 3-partitioning
Reduction

now:
*translate the floor tiling problem into a Tetris problem*

if we can solve Tetris
then we can solve floor tiling
then we can solve every NP-problem

Tetris itself is NP complete
Reduction

‘Hallway’ in Tetris
Reduction

Floor tile in Tetris:

- End
- Length (= 3)
- Begin
Reduction

... some details on the formalities ...

check: filling the game board is equivalent to filling a hallway:

- floor tiles only fit in one row each.
- the lines can not be cleared before all the floor tiles have been laid.

... “yes” in floor tile problem ⇔ “yes” in Tetris.

→ Tetris is NP-complete
Even if there is a finite number of pieces and their order is known, it is very hard (NP-complete) to compute whether a given initial game board can be cleared.

**In other words:**

If you find an algorithm that plays Tetris optimal within reasonable time, you have proven that $P = NP$ and you become famous, ... and rich.
• Tetris is NP complete !!
• what configurations ?
• undecidable Tetris
• the AI of Tetris
Configurations

restrictions:
• even number blocks
  add 4 blocks
  delete 10 blocks
• empty & full rows

NP complete
Configurations
Undecidable
AI of Tetris

10
Configurations

- NP complete
- Configurations
- Undecidable
- AI of Tetris

nieuwe rij

platform
Configurations

- NP complete
- Configurations
- Undecidable
- AI of Tetris
Configurations

- NP complete
- Configurations
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- AI of Tetris

overflow
nieuwe rij
platform
AI of Tetris

Mathematical proof:
no optimal stacking of ‘S’ and ‘Z’

What is the best move?
(using single block look-ahead)
thank you...
Questions

- NP complete
- Configurations
- Undecidable
- AI of Tetris