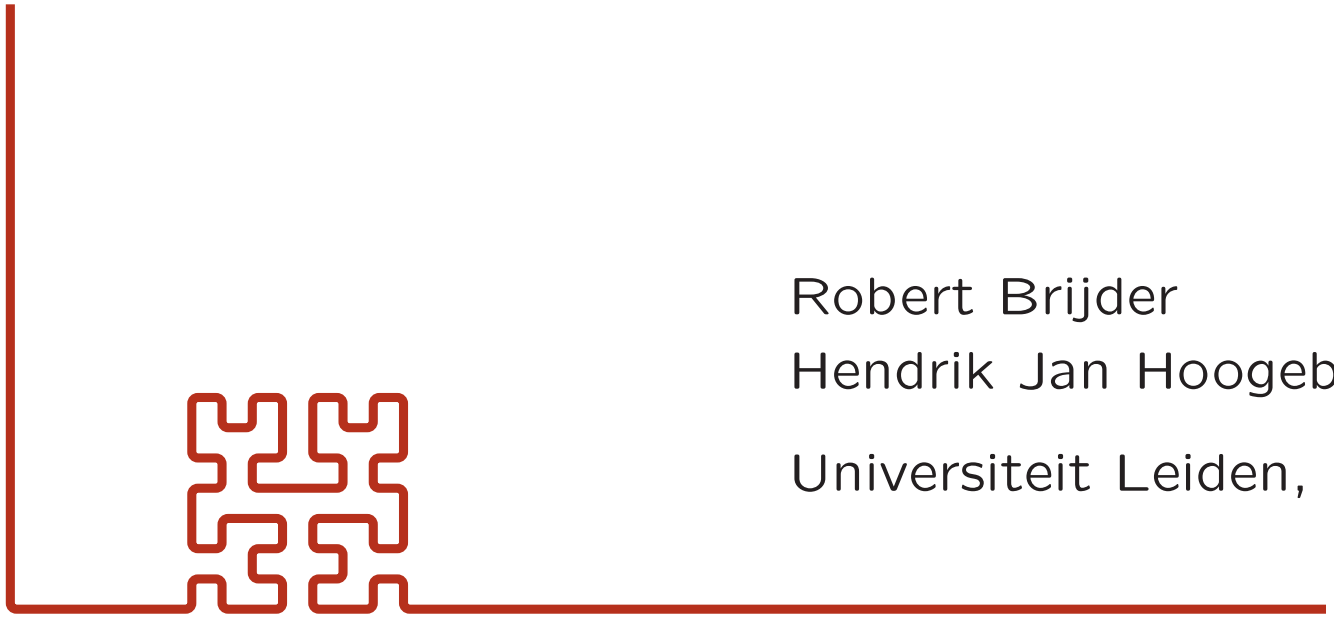
Perfectly Quilted Rectangular Snake Tilings



Robert Brijder
Hendrik Jan Hoogeboom

Universiteit Leiden, NL

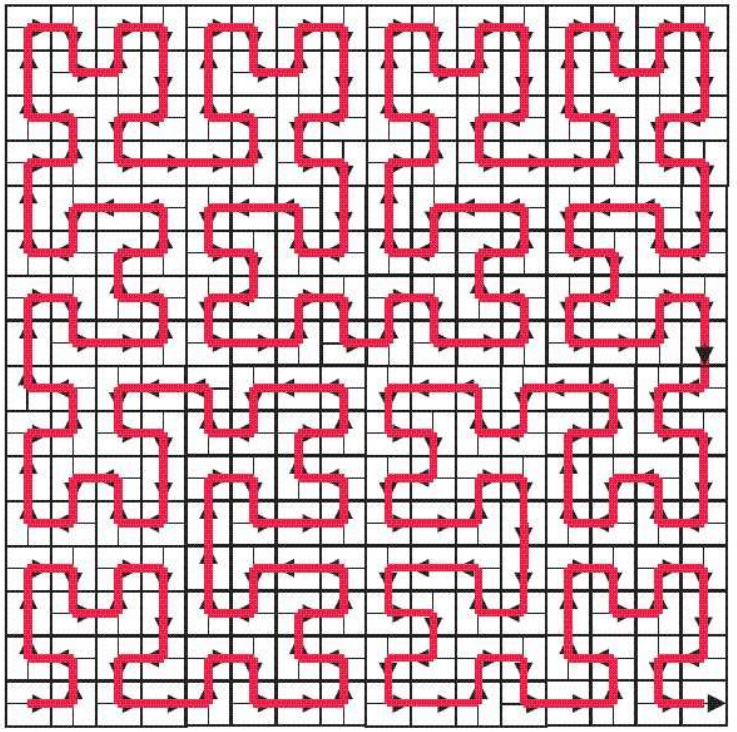
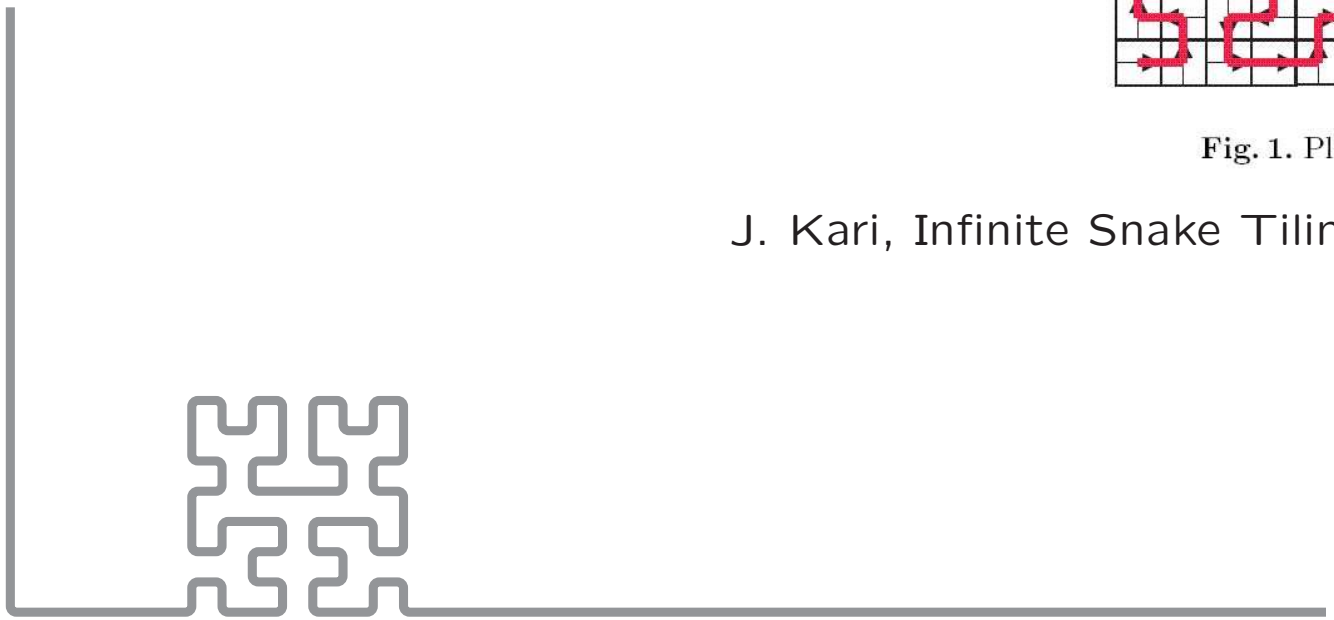
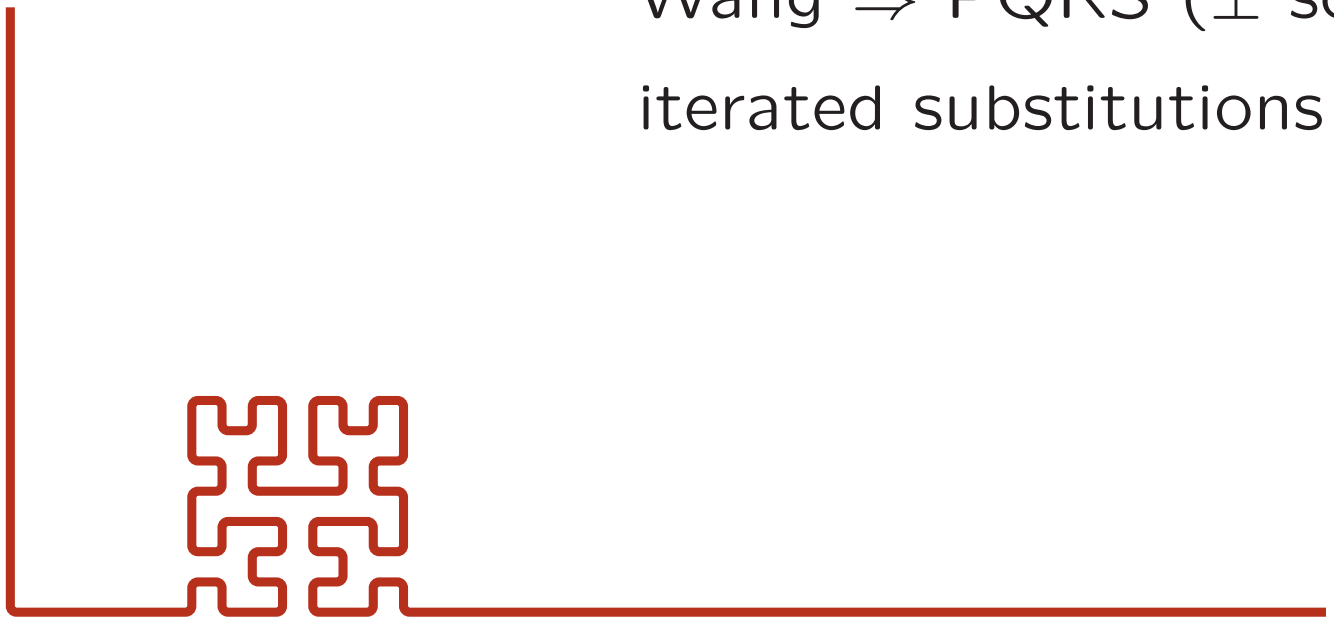


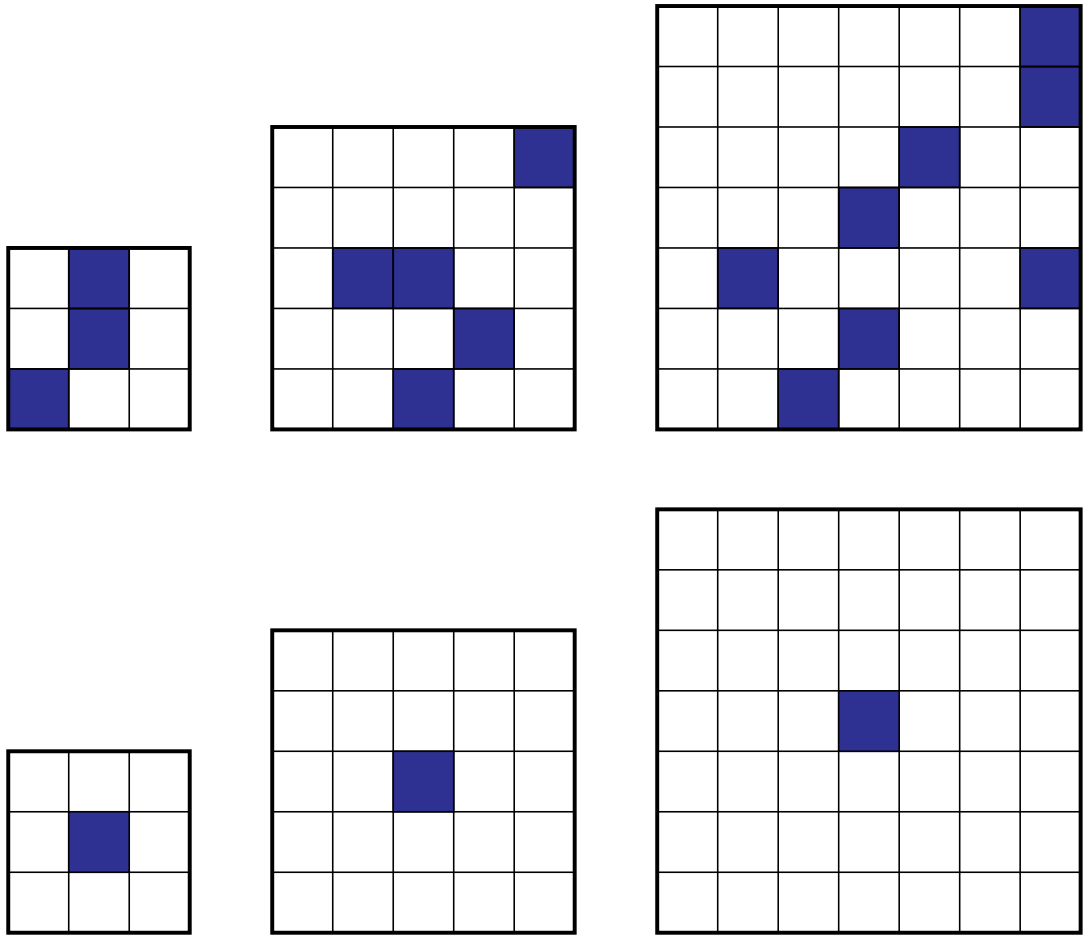
Fig. 1. Plane filling snake forced by D_0.

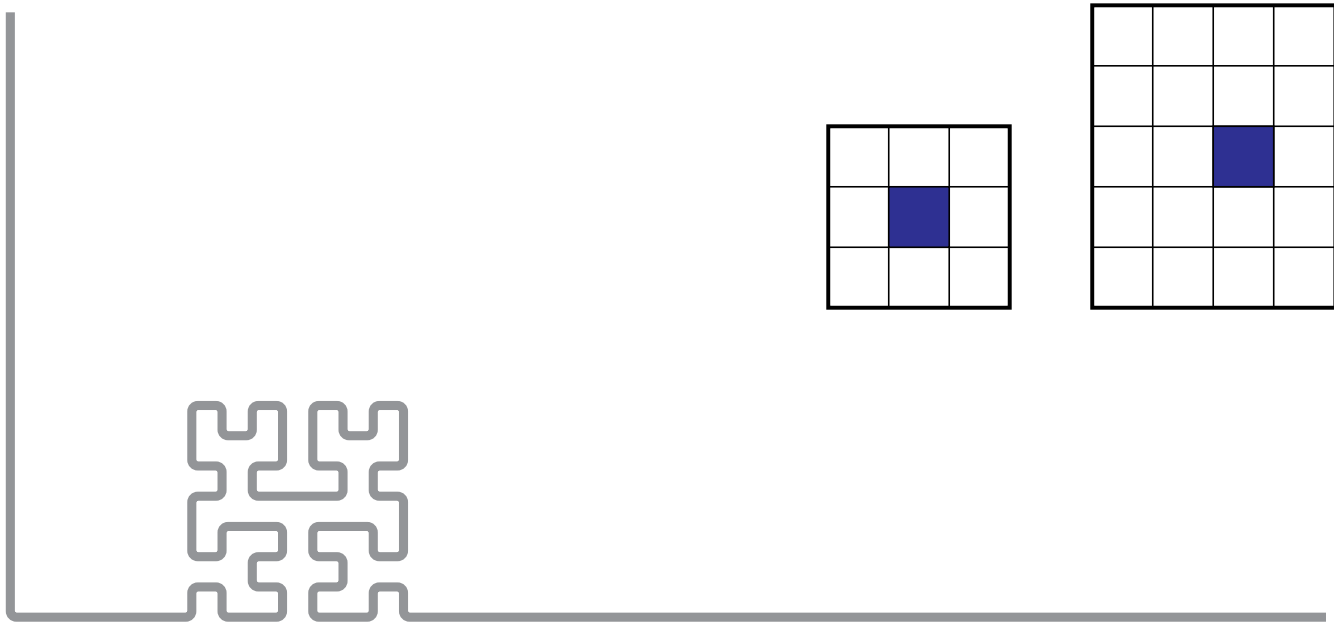
J. Kari, Infinite Snake Tiling Problems, DLT2002



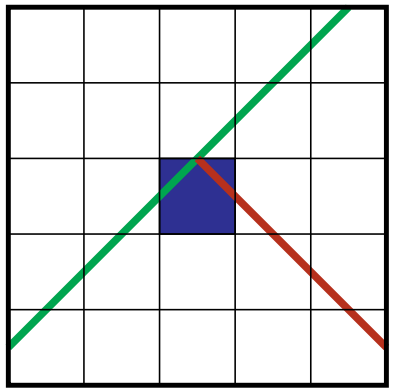
- two families
- PQRS $\Rightarrow$ Wang
- undecidability
- Wang $\Rightarrow$ PQRS ($\pm$ scaling)
- iterated substitutions

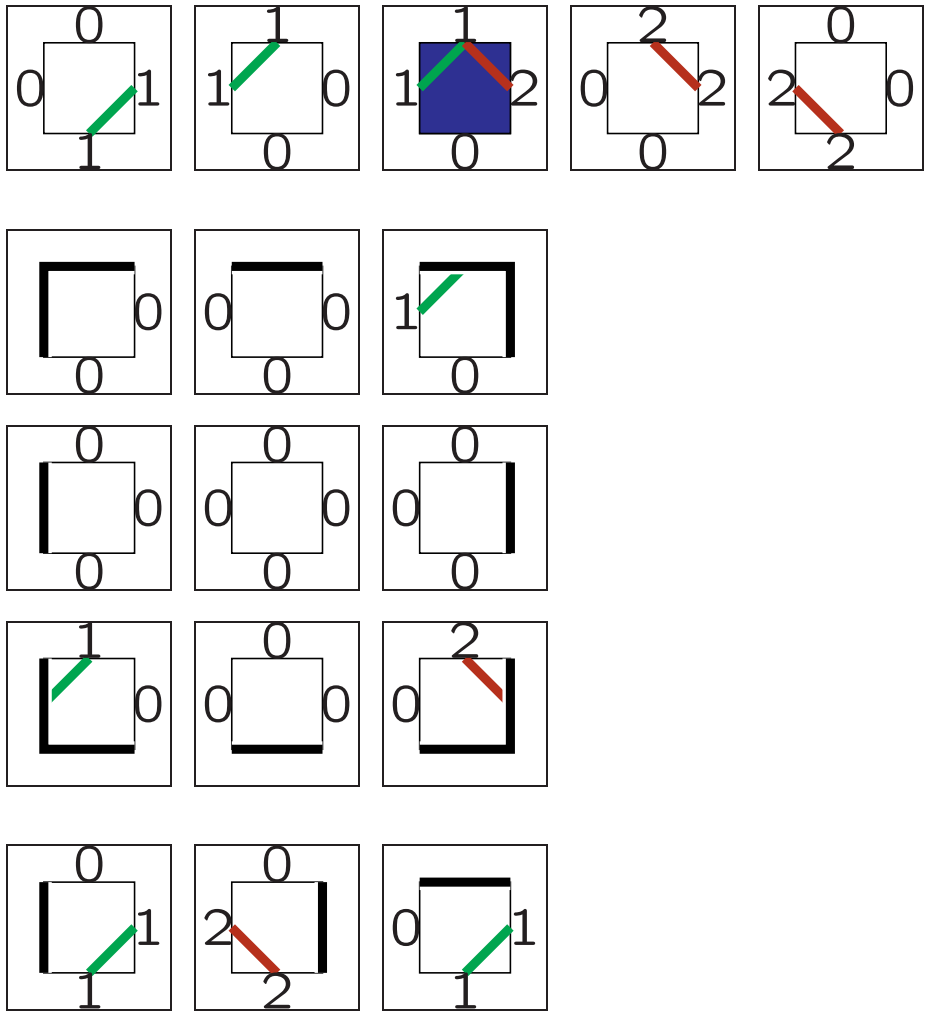






finite & rectangular




0 0 1 1
1 1 0 0
1 1 2 0
2 0 2 0
0 2 0 2
0 0 0
0 0 0 0
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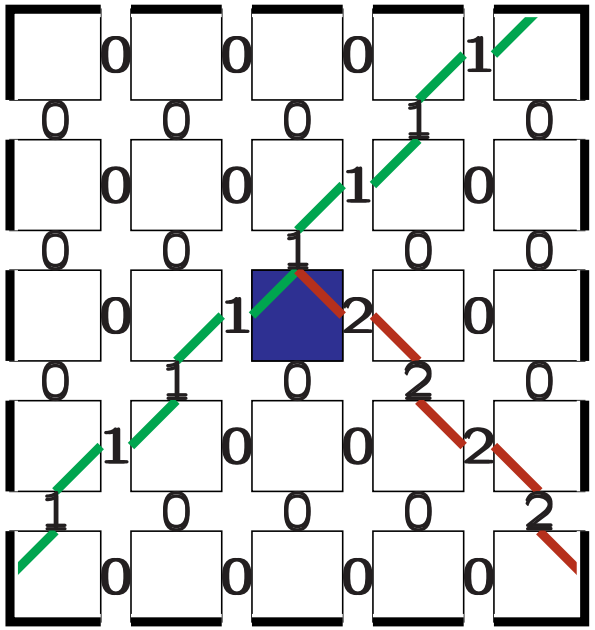
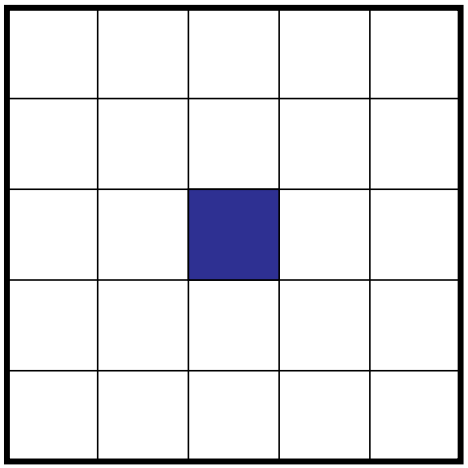
0 2 0 2
0 0 1 1





(Σ, T, c, φ)

T tiles with four sided markings
tiling: markings match

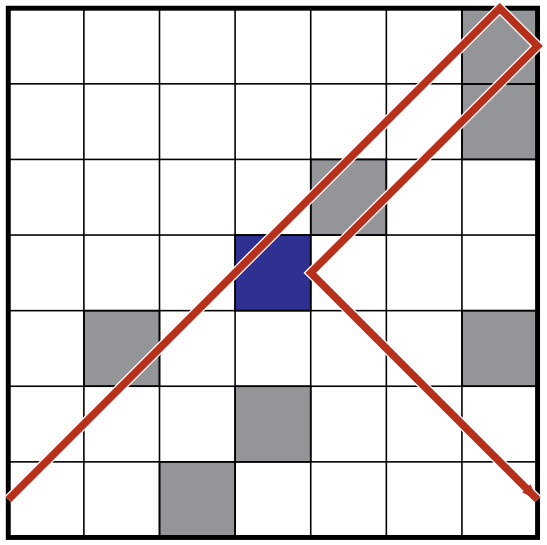
c border marking

$\varphi : T \to \Sigma$ tile $\mapsto$ 'colour'

Wang picture language

projection of local language

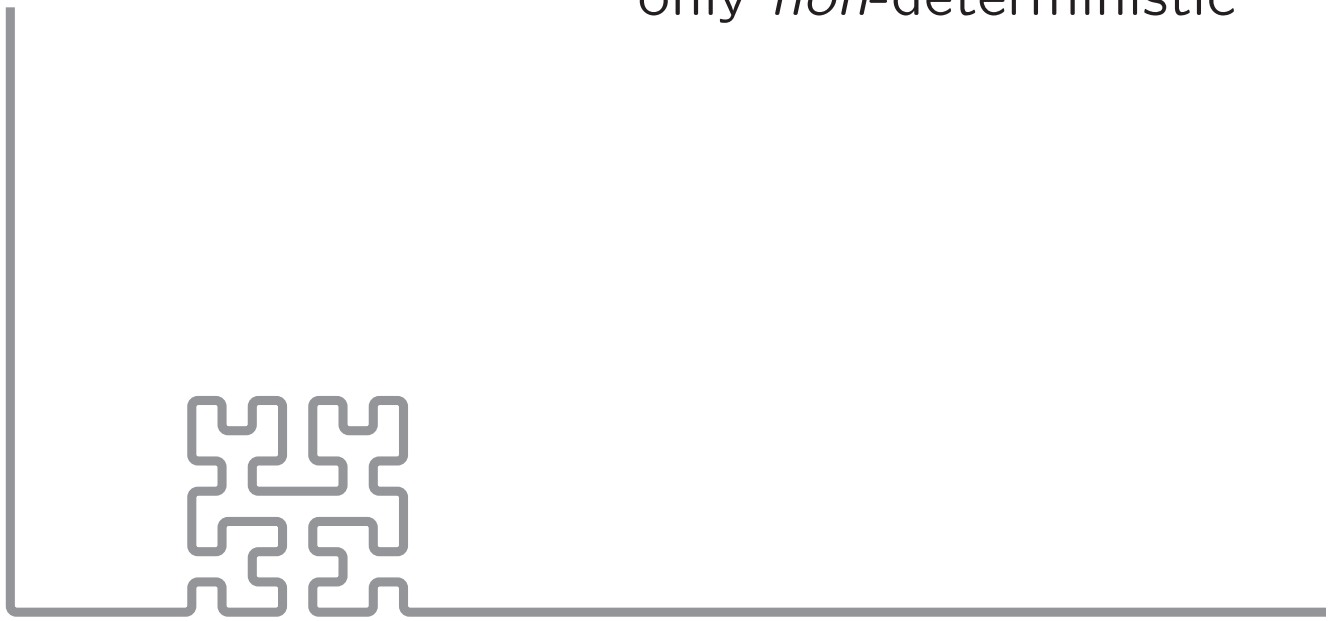
Giammarresi, Restivo: REC

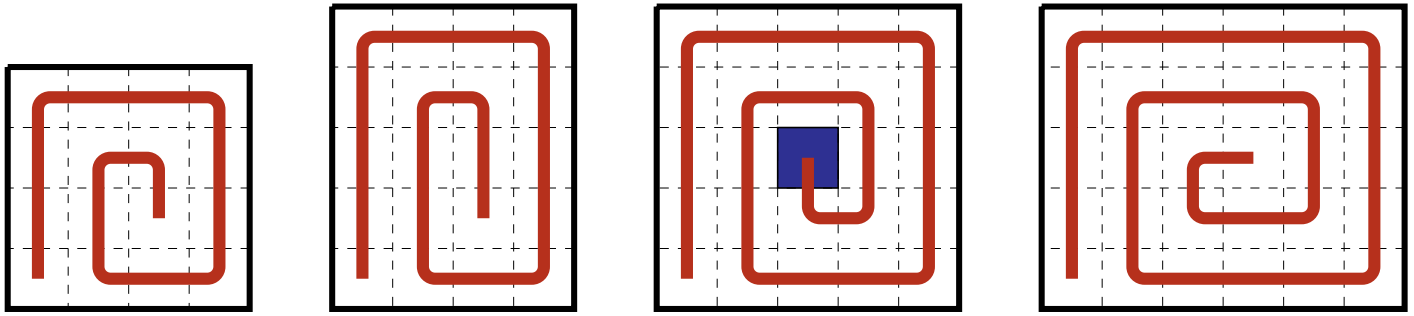


square

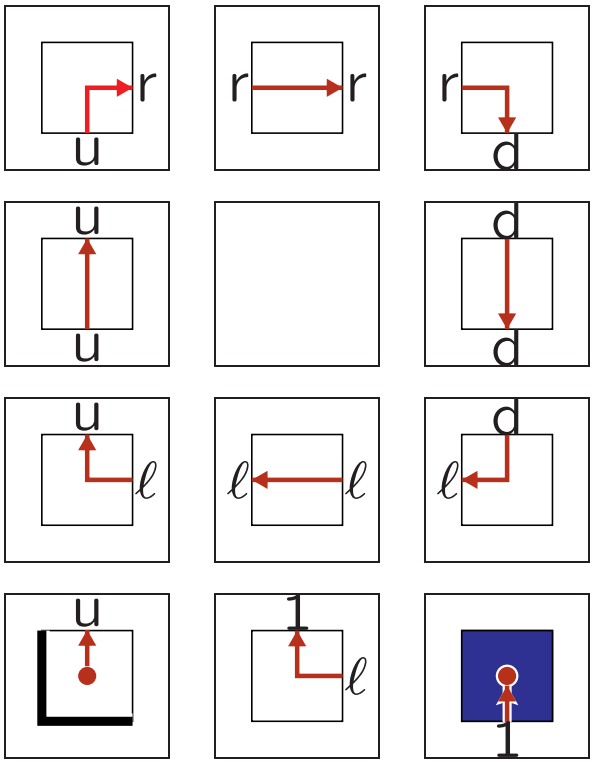
middle ok

only *non*-deterministic





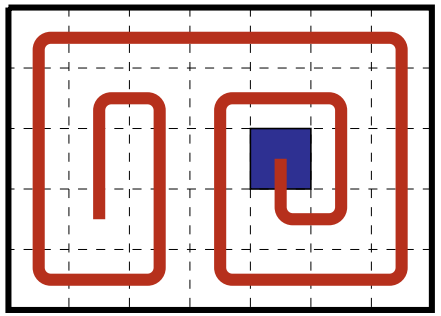
snake tiles



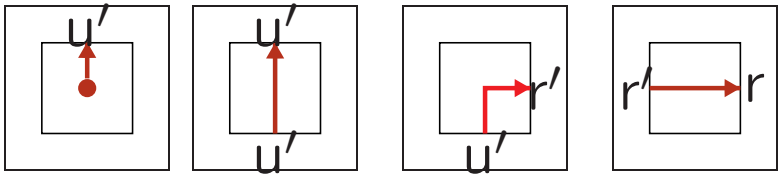
comment

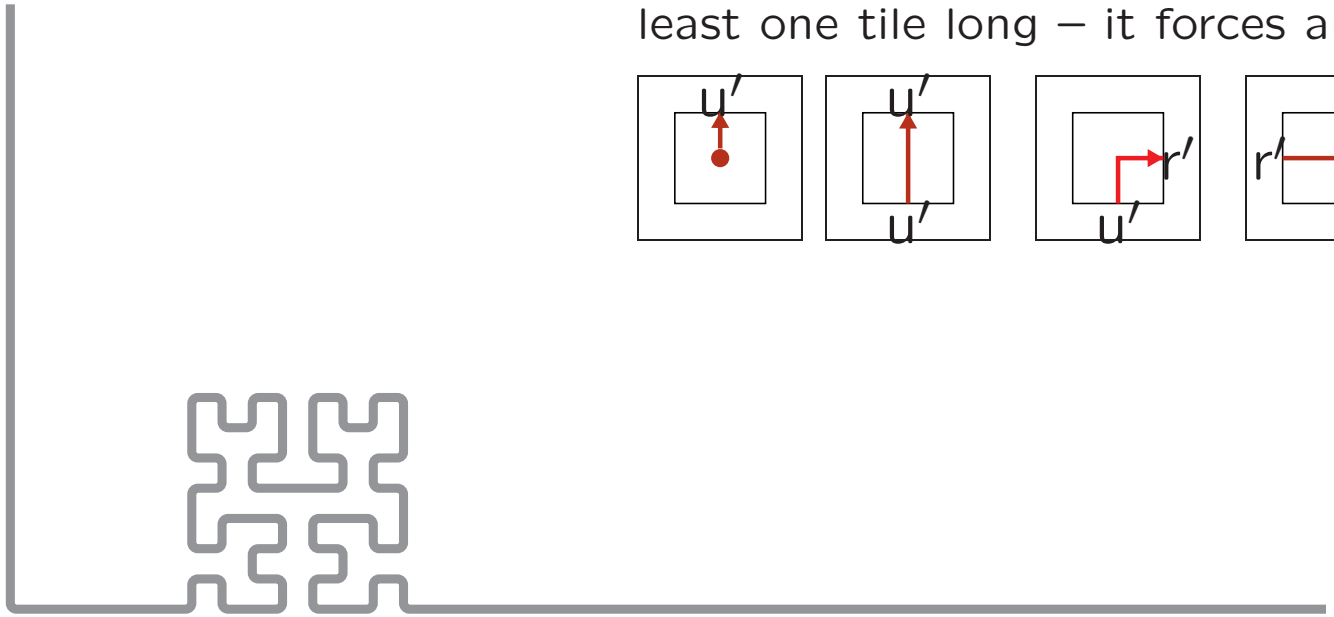
(added after lecture)

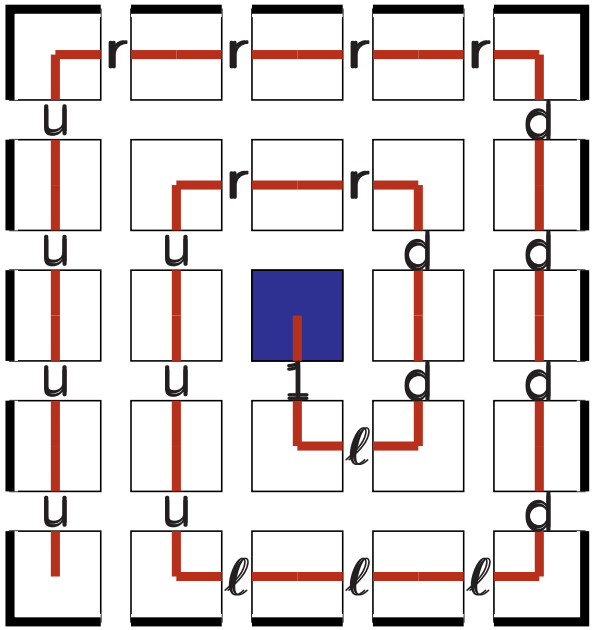
if the first tile does not recognize the border, the snake may spiral inward and/or outward!

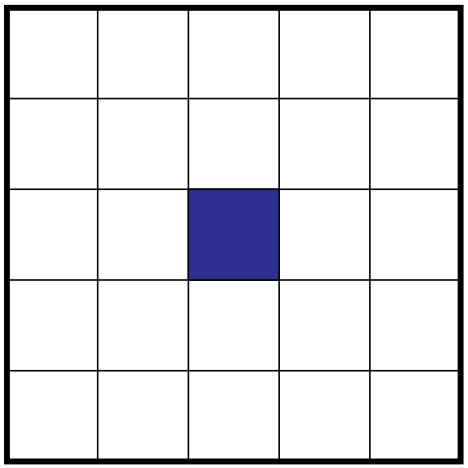


solution: make first horizontal segment to the right at least one tile long – it forces an inward spiral









rectangular snake tilings

$(\Sigma, T, i, f, \varphi)$

T tiles with two sided markings

tiling: markings match

& single visit

perfectly quilting

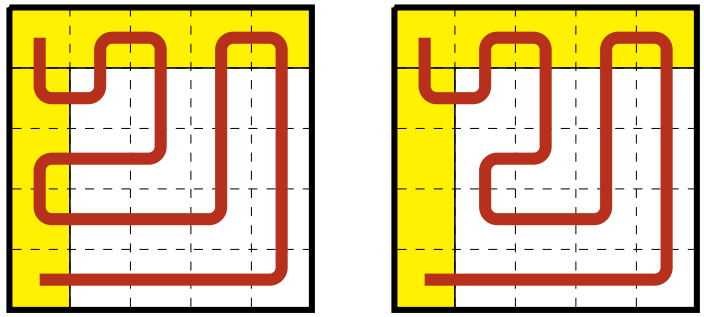
i, f initial, final tile

$\varphi : T \to \Sigma$ tile $\mapsto$ 'colour'

PQRS picture language

regular description

example from Marcella Anselmo

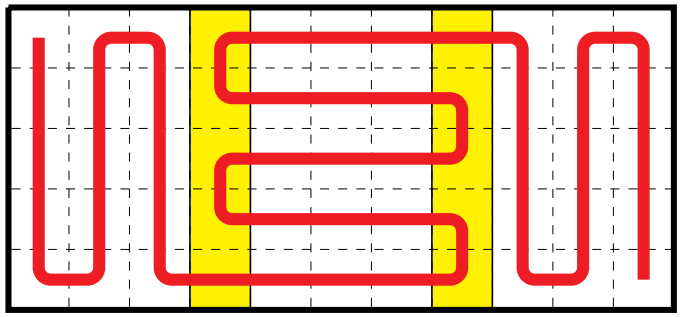


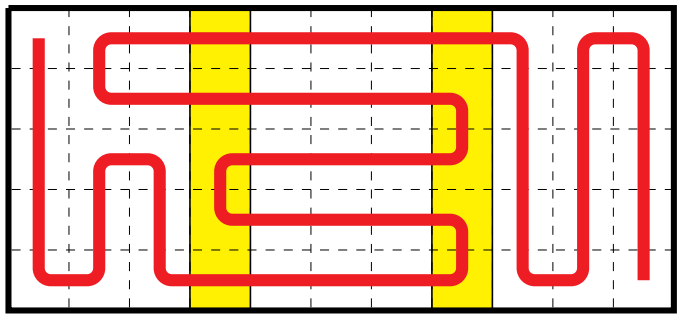
not PQRS

comment

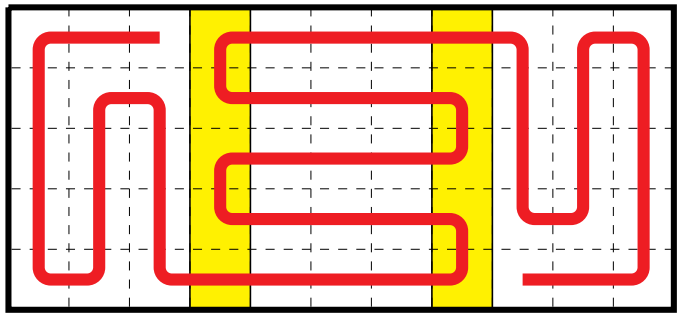
(added after lecture)

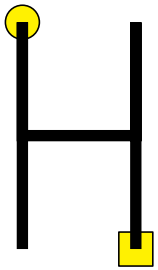
again the solution at the bottom depends on the fact whether the first snake tile recognizes the border; the technical results are independent of this,





oops!





‘regular description’

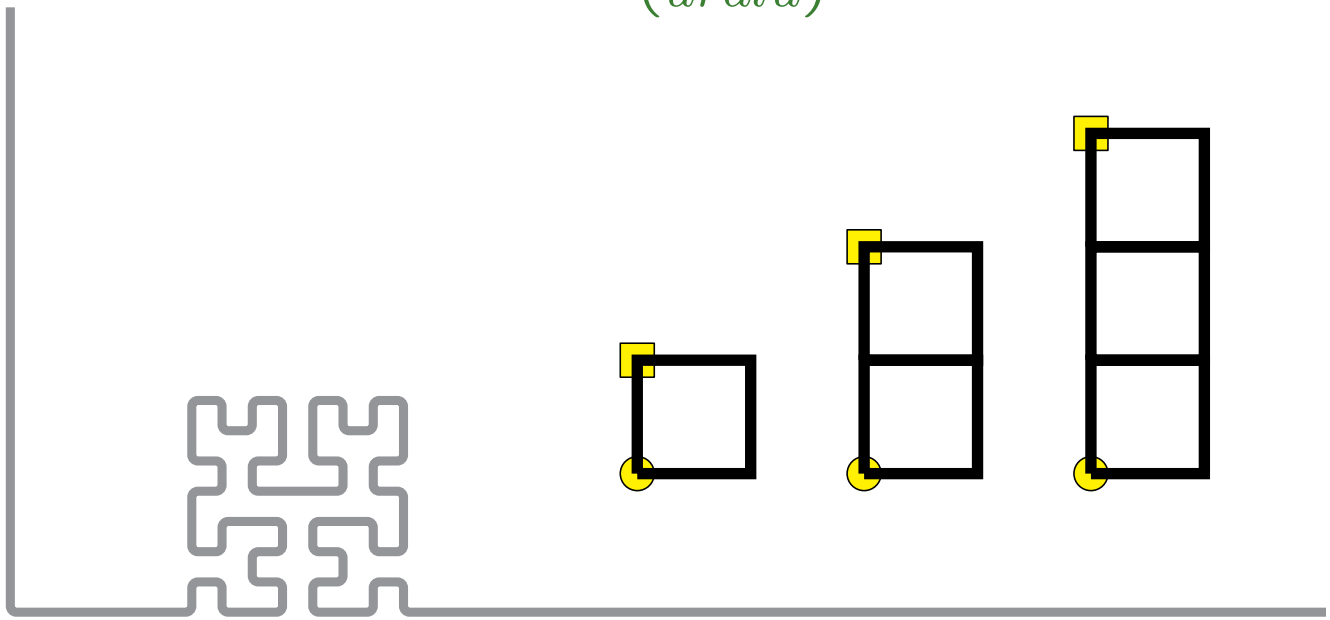
chain code picture language

$D = \{r, d, l, u\}$

d^2urud^2

overlap possible

$(urdlu)^*$

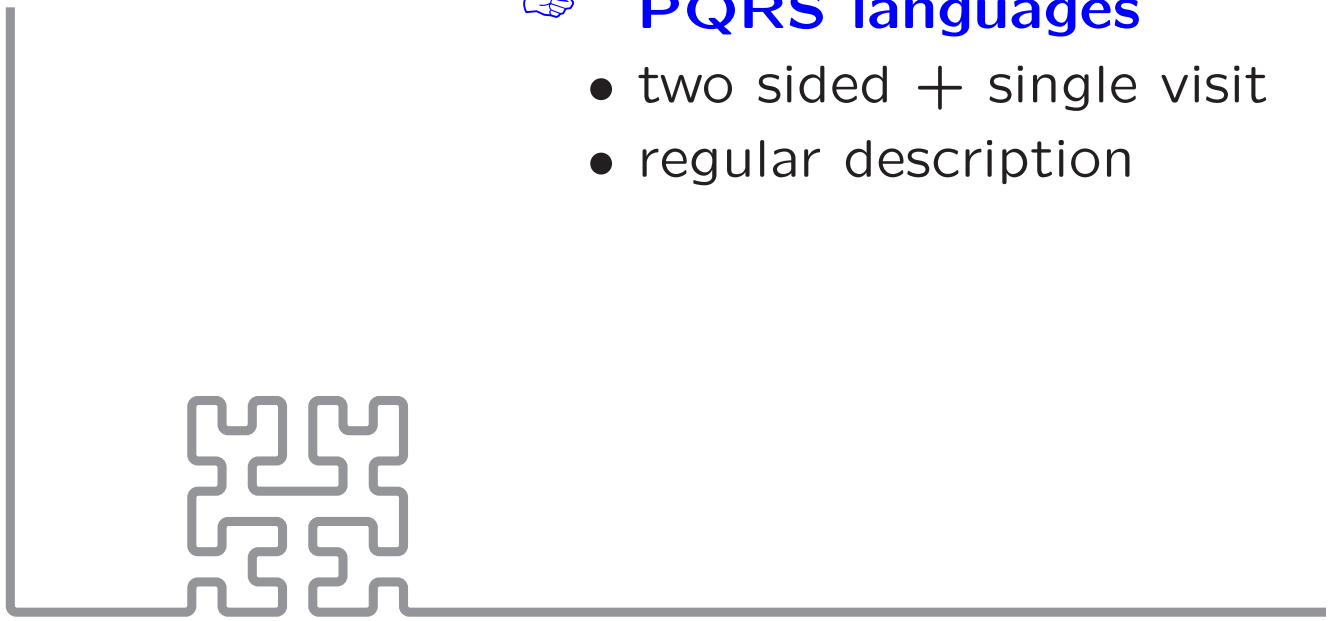


☞ **Wang languages**

- four sided
- pojection local language

☞ **PQRS languages**

- two sided + single visit
- regular description



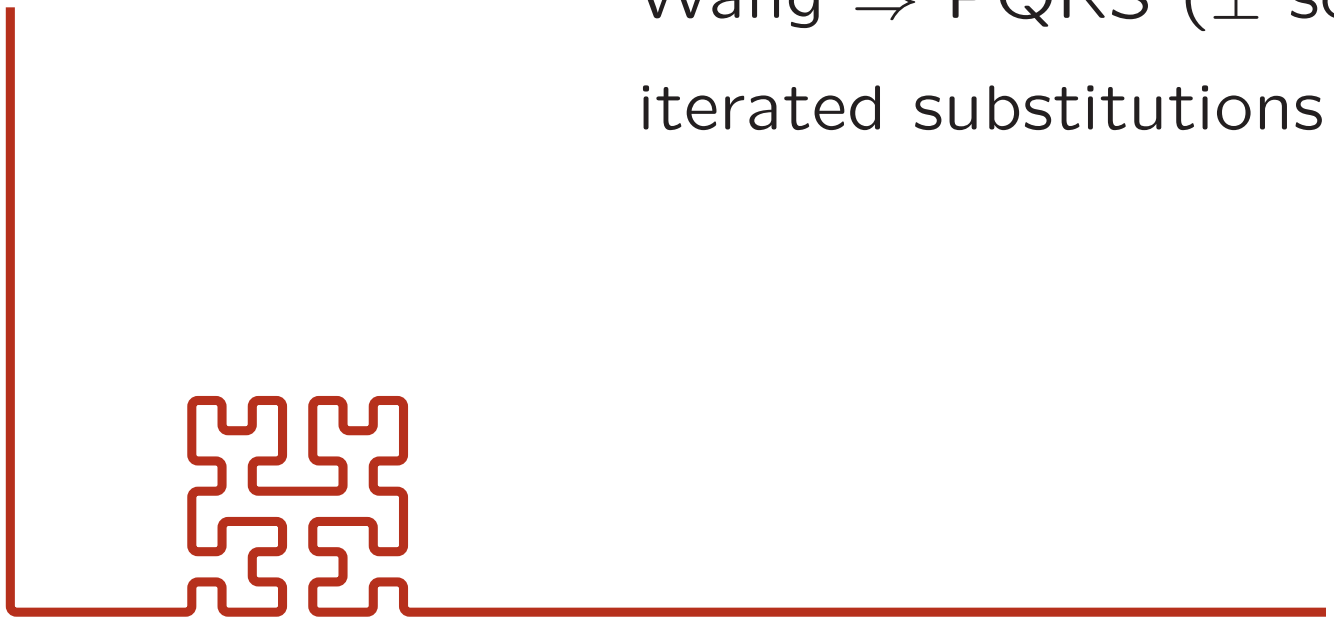
two families

PQRS $\Rightarrow$ Wang

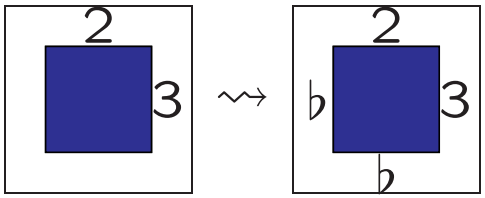
undecidability

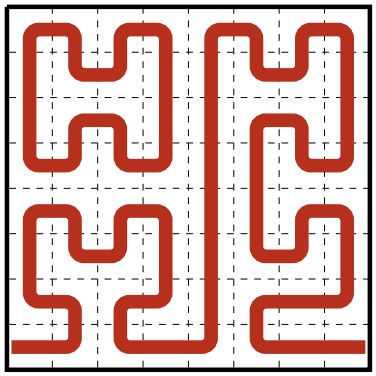
Wang $\Rightarrow$ PQRS ($\pm$ scaling)

iterated substitutions



Lemma. Every PQRS language is a Wang language.

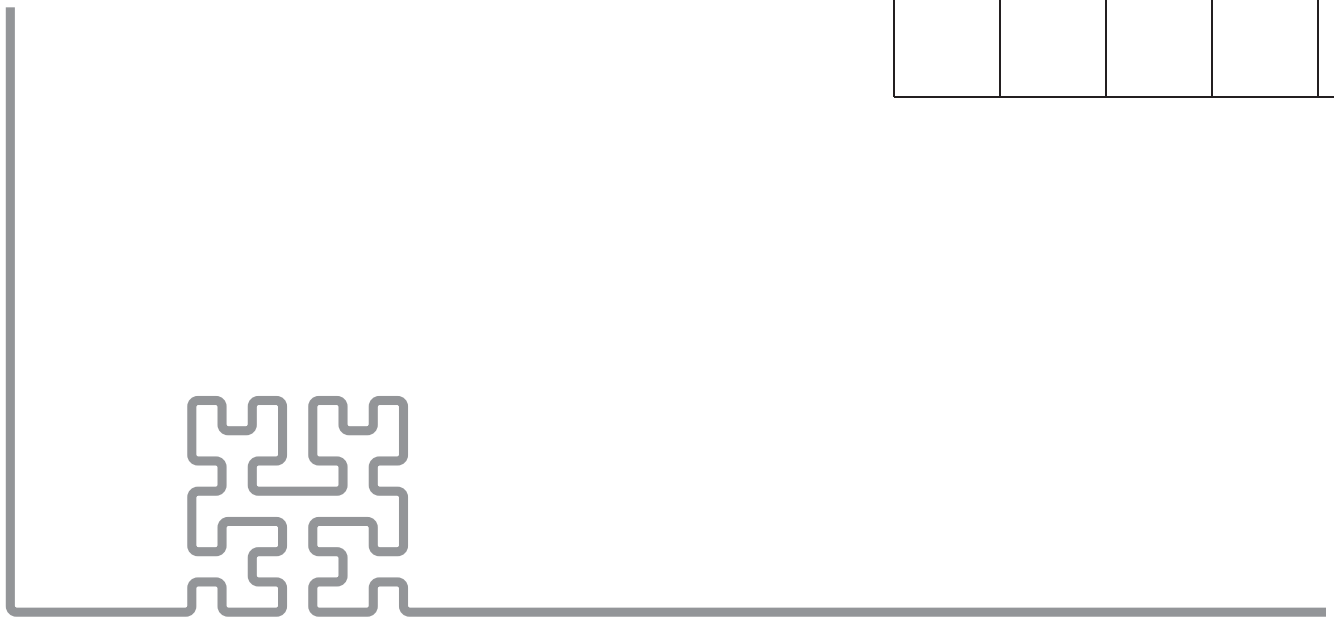


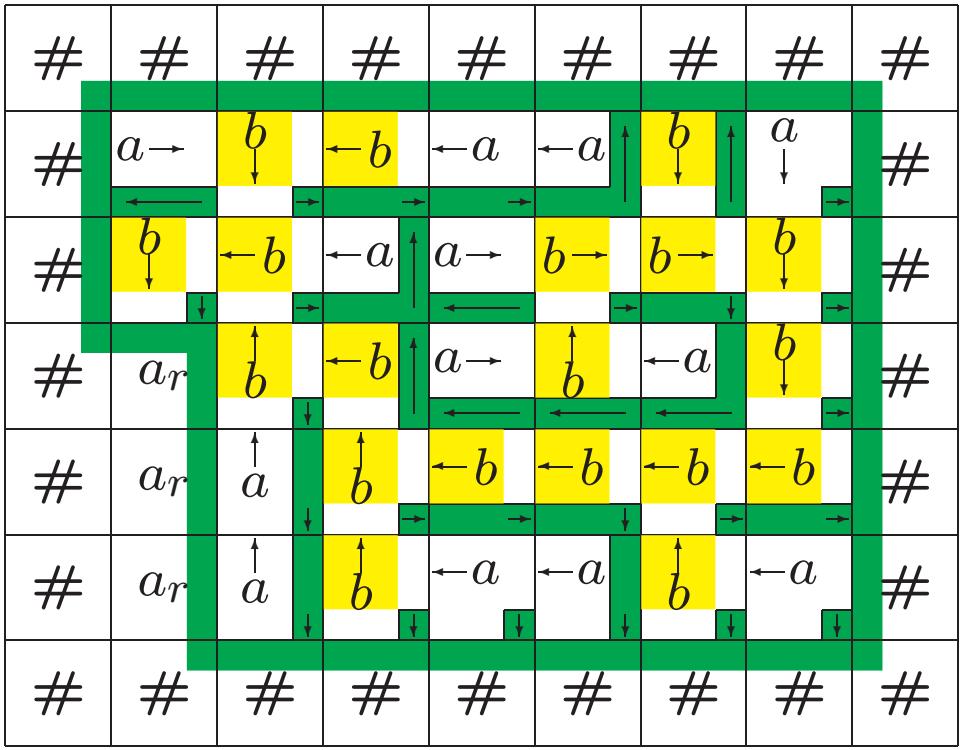


single connected snake?

	a	b	b	a	a	b	a	
	b	b	a	a	b	b	b	
	a	b	b	a	b	a	b	
	a	a	b	b	b	b	b	
	a	a	b	a	a	b	a	

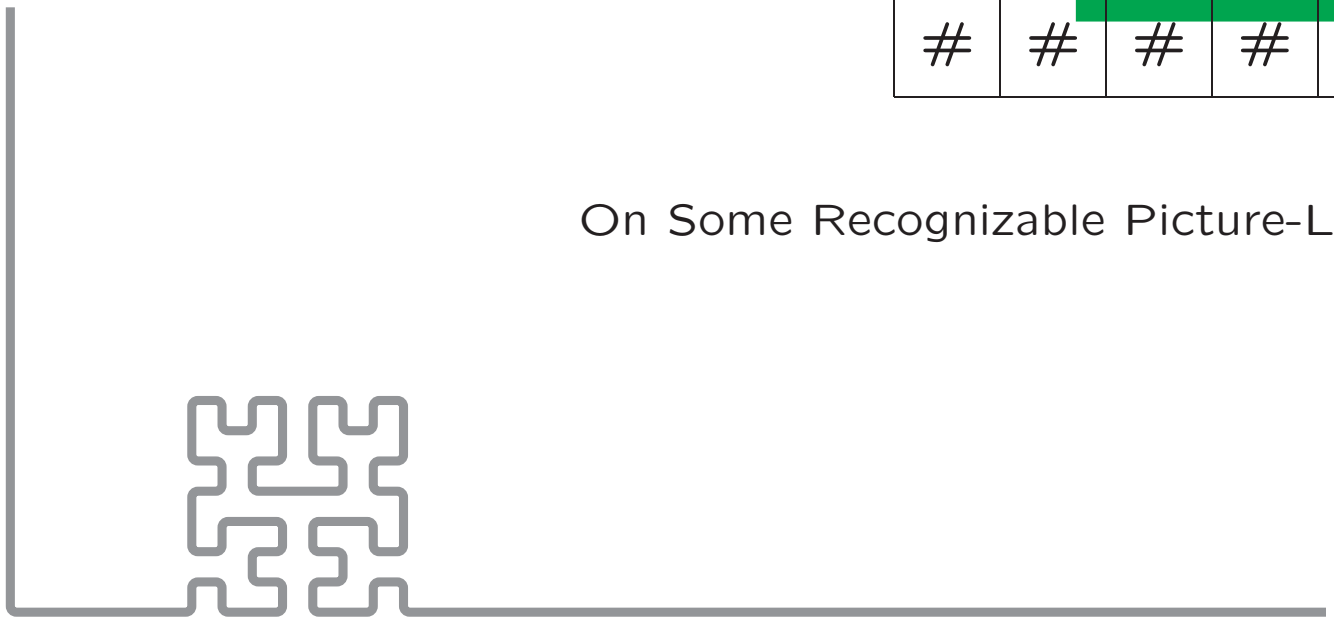
using Wang tiles?

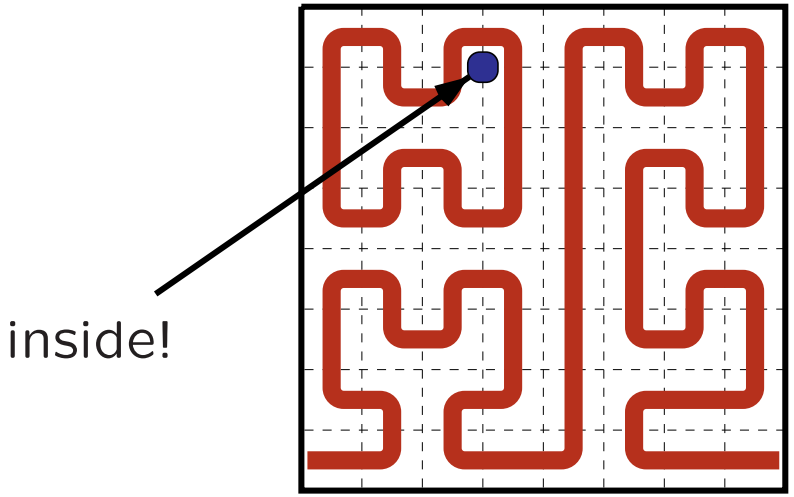


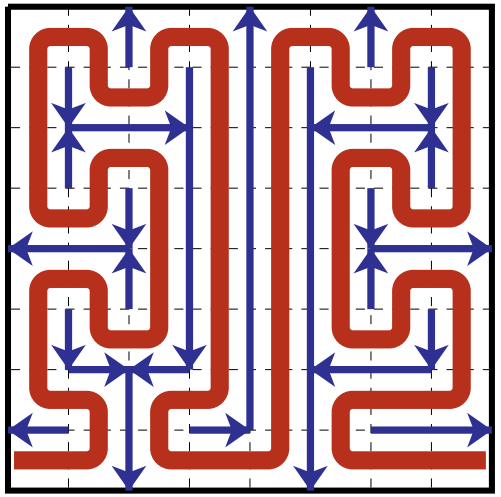


Klaus Reinhardt

On Some Recognizable Picture-Languages, MFCS 1998




inside!



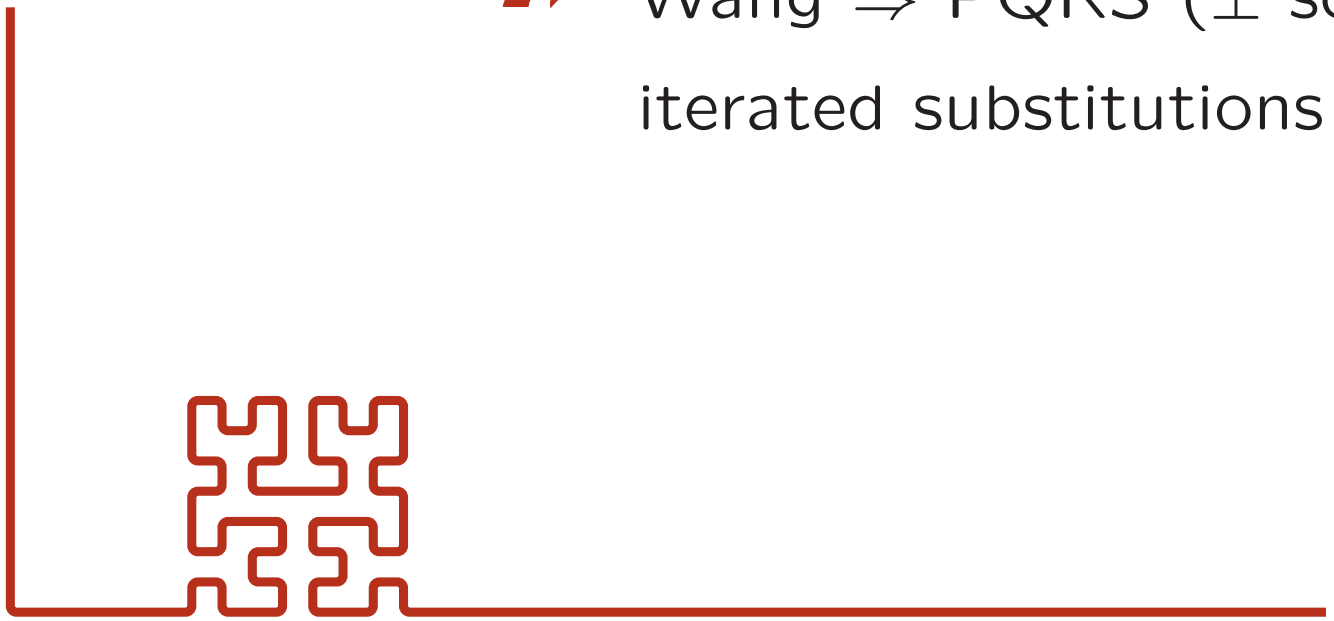
two families

PQRS $\Rightarrow$ Wang

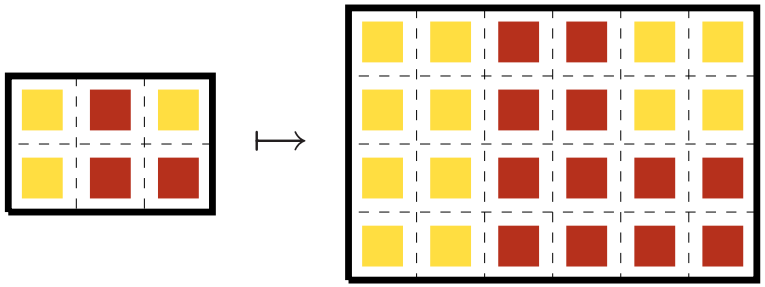
undecidability

Wang $\Rightarrow$ PQRS ($\pm$ scaling)

iterated substitutions



multiplication



Lemma. Every Wang language is a PQRS language (after multiplication).

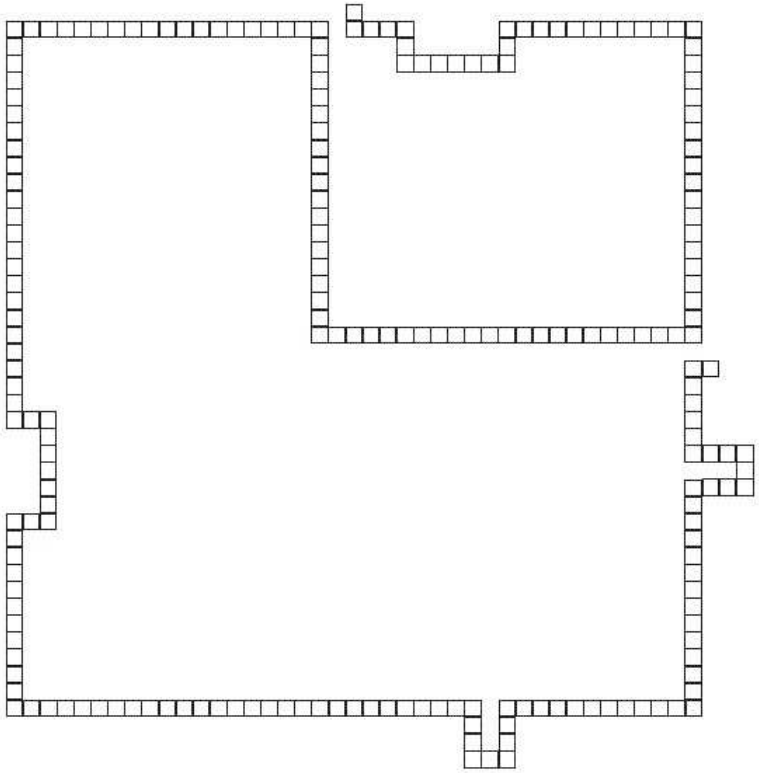
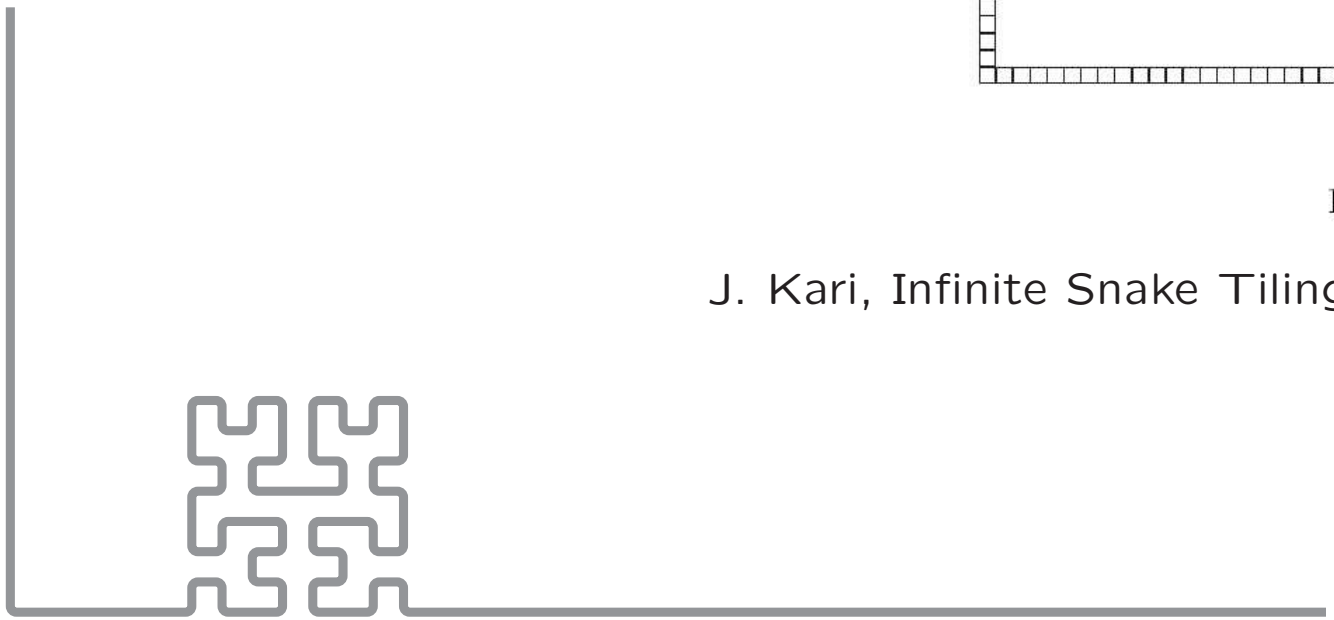
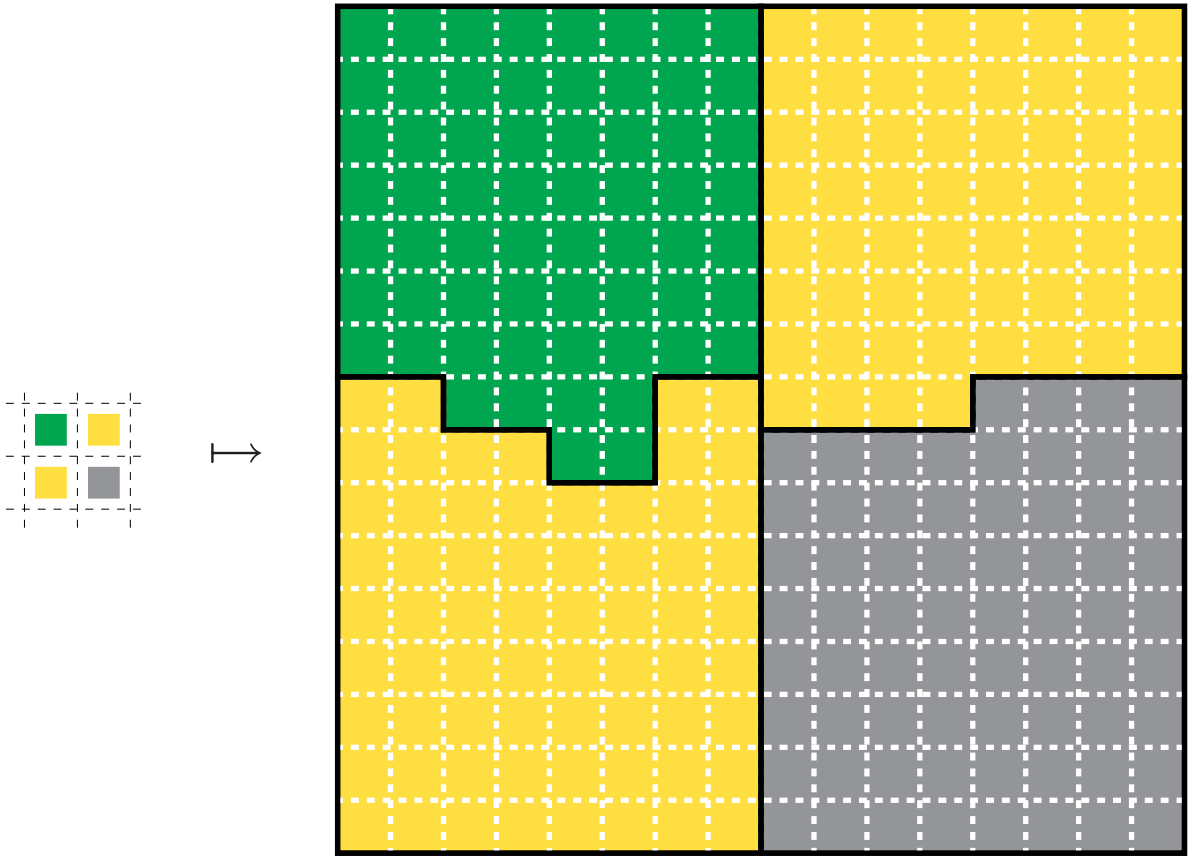


Fig. 2. A motif.

J. Kari, Infinite Snake Tiling Problems, DLT2002

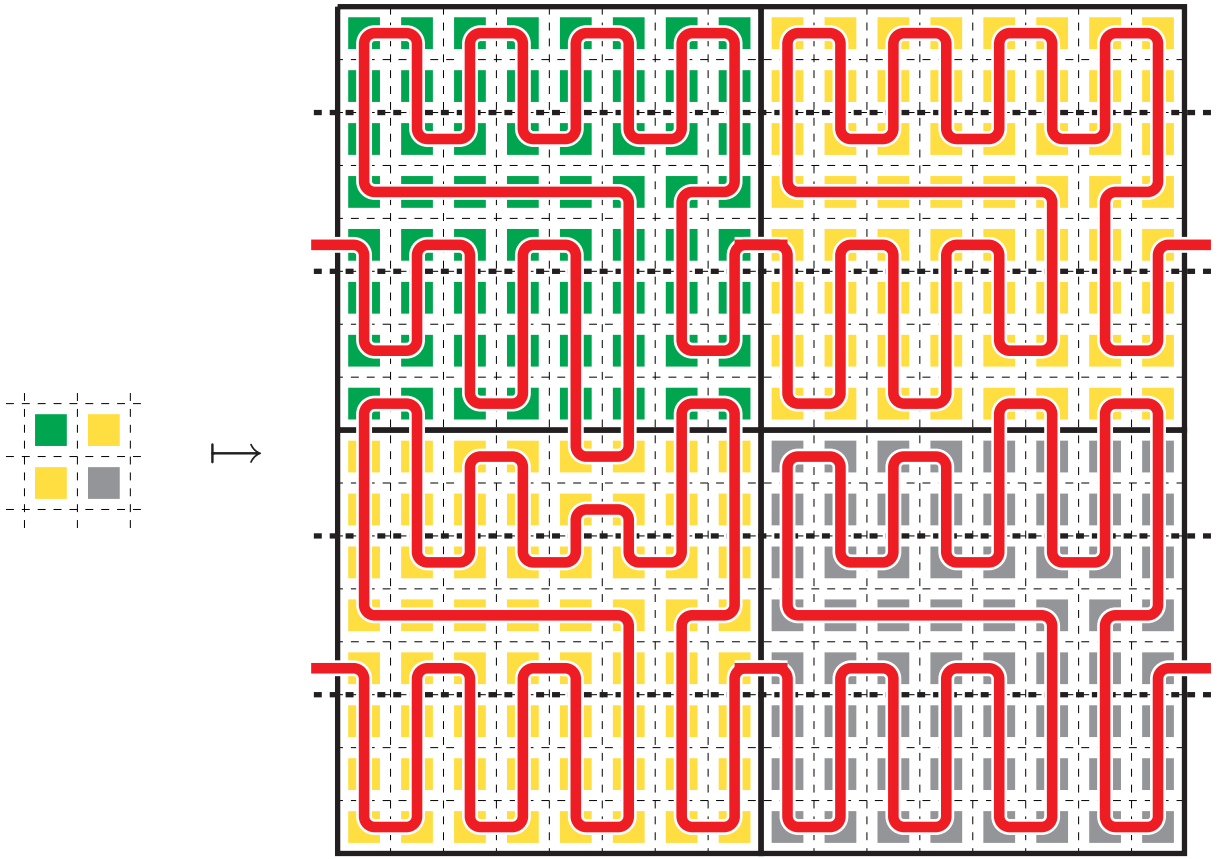


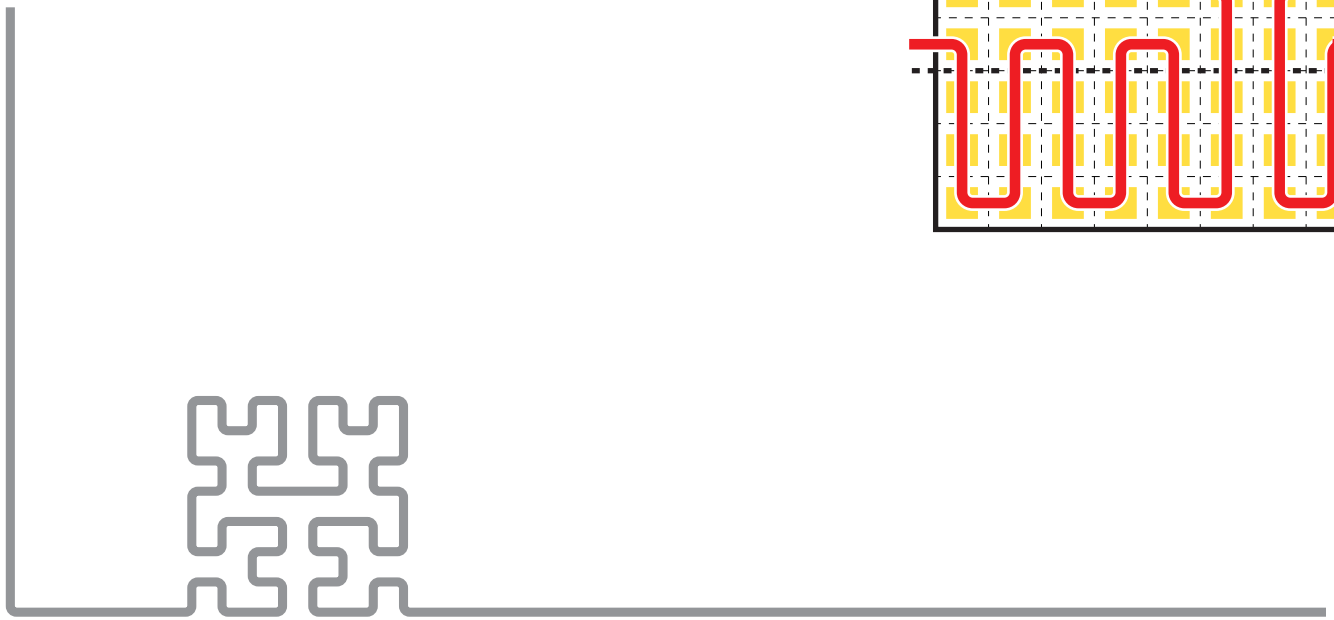


four sides $\mapsto$ two sides

horizontal: markings

vertical: form





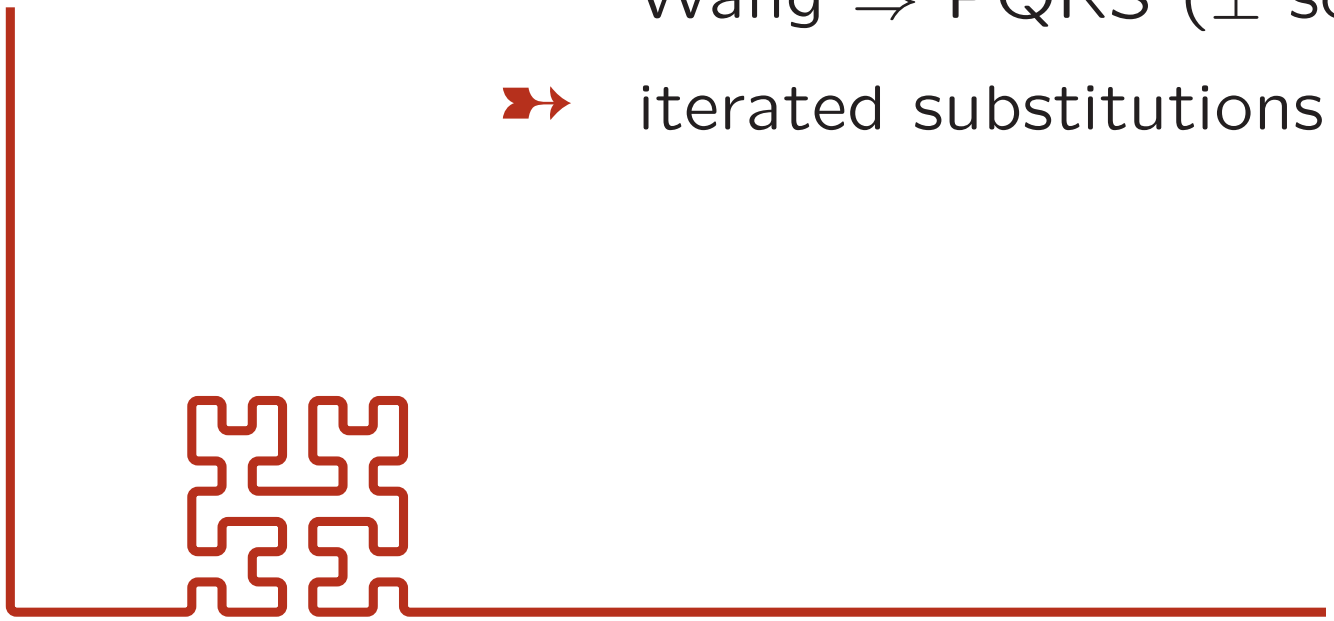
two families

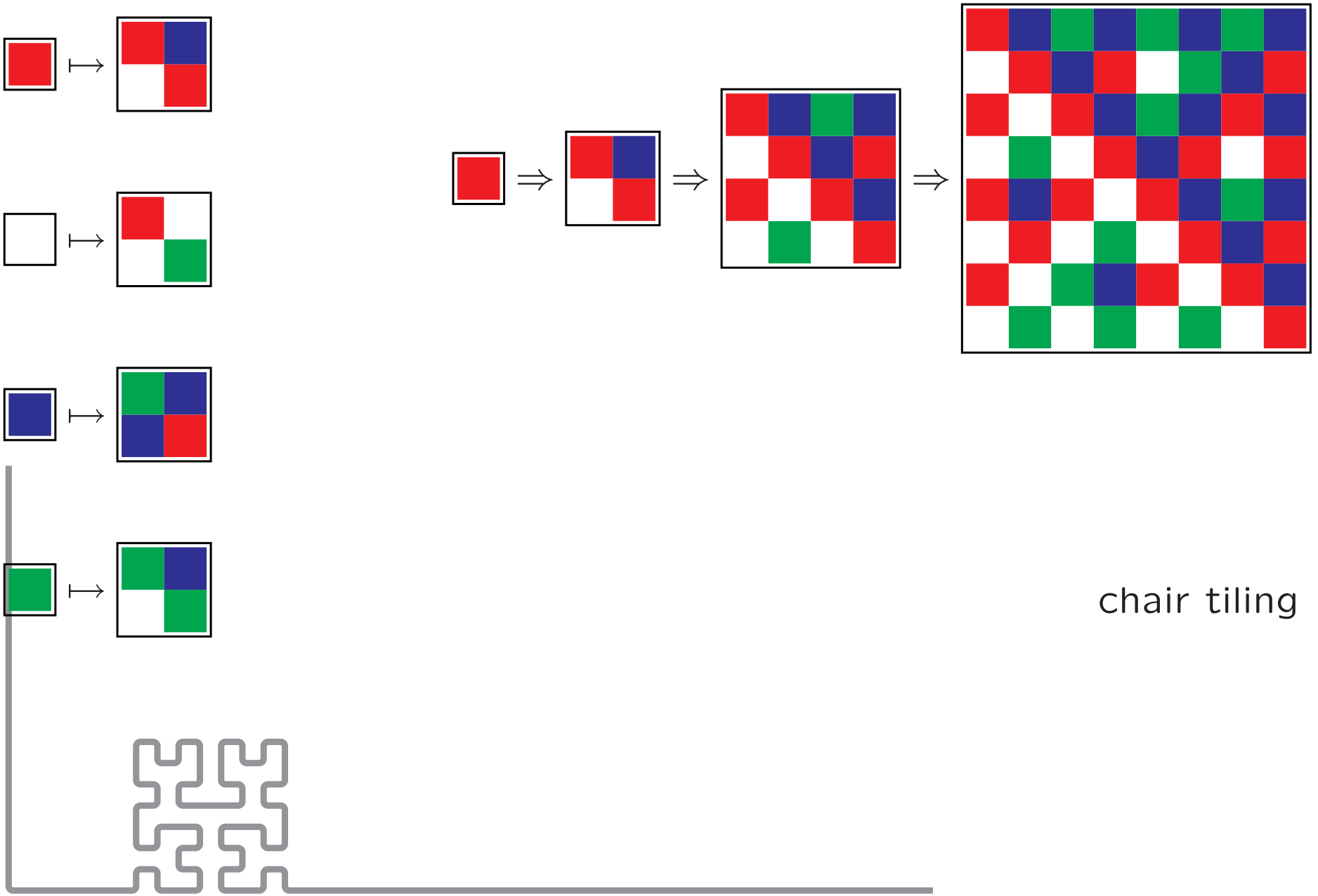
PQRS $\Rightarrow$ Wang

undecidability

Wang $\Rightarrow$ PQRS ($\pm$ scaling)

iterated substitutions

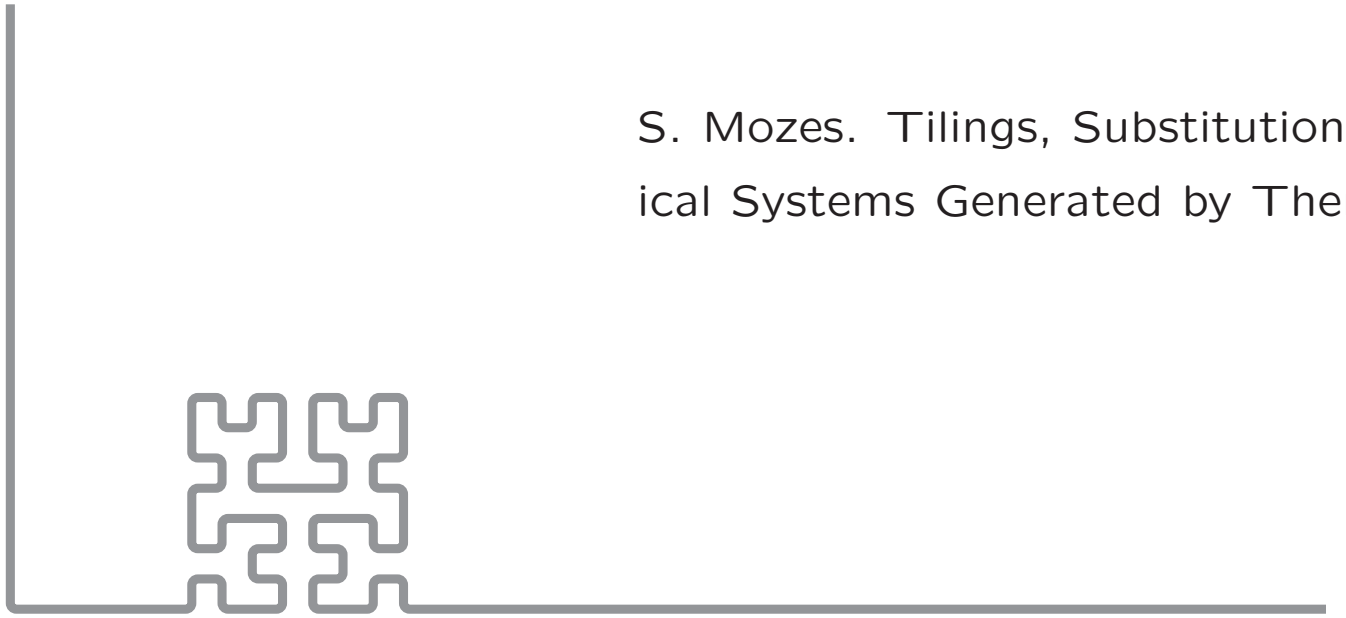




chair tiling

Lemma. Every iterated substitution defines a Wang language.

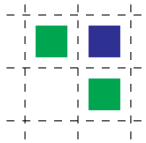
S. Mozes. Tilings, Substitution Systems and Dynamical Systems Generated by Them. (1989)



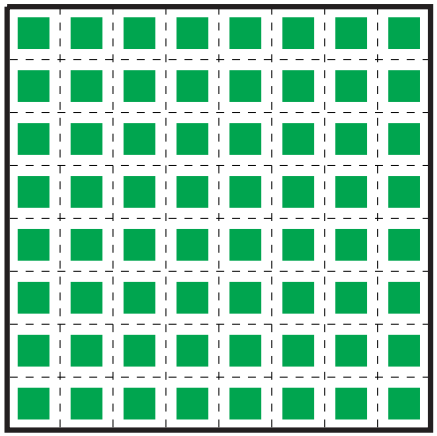
iterated substitution
⇒ Wang
⇒ PQRS + multiplication

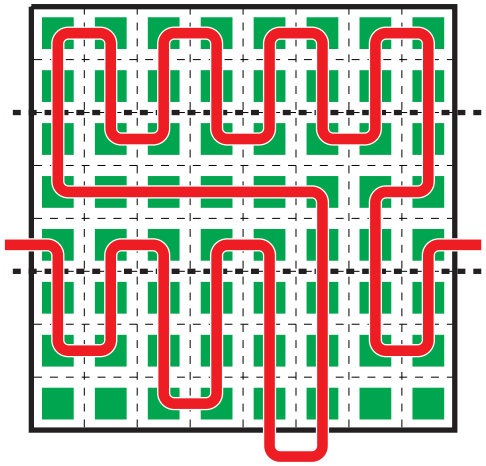
Lemma. Every iterated substitution defines a PQRS language. (no multiplication needed)

$T(n)$ nth iteration
2^k multiplication



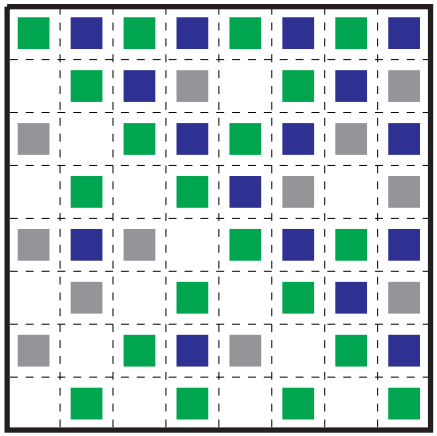
$\mapsto$

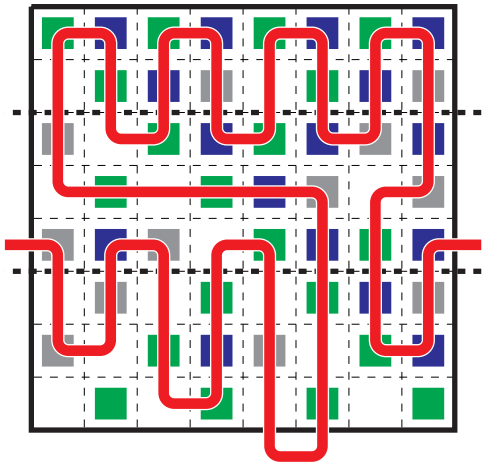




$T(n) \mapsto 2^k \cdot T(n)$

$T(n) \mapsto T(n+k)$





two families

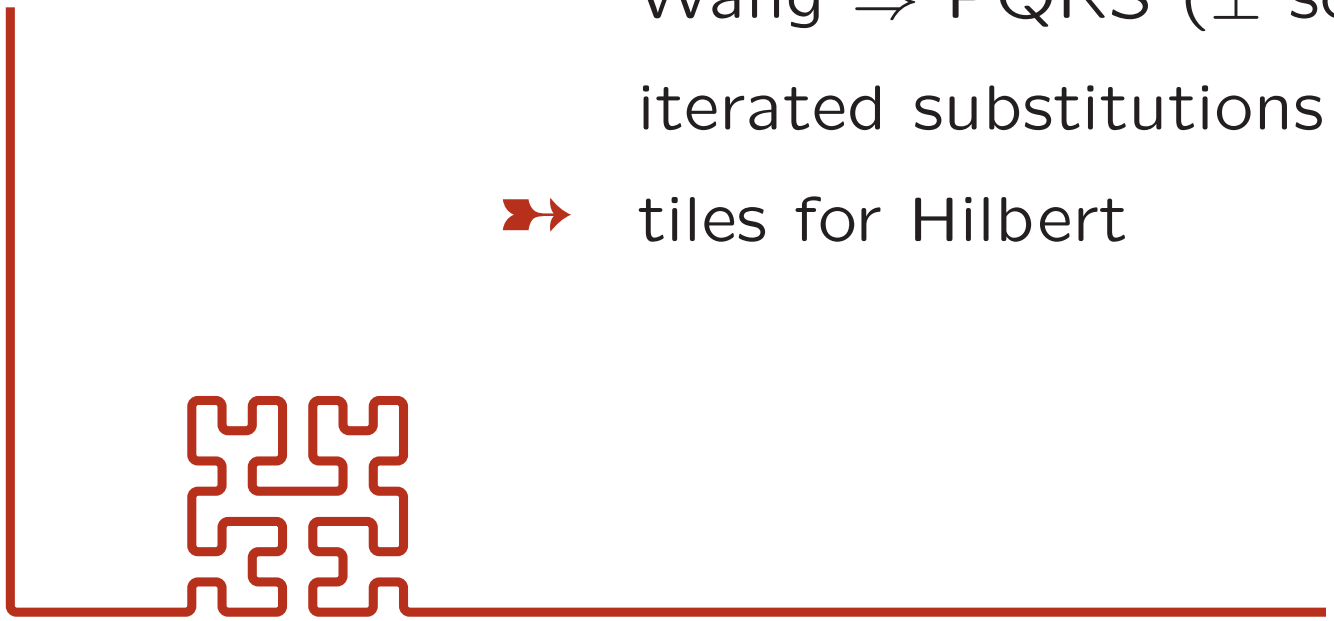
PQRS $\Rightarrow$ Wang

undecidability

Wang $\Rightarrow$ PQRS ($\pm$ scaling)

iterated substitutions

tiles for Hilbert



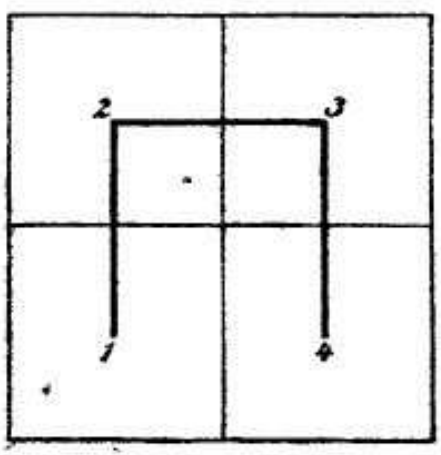


Fig. 1.

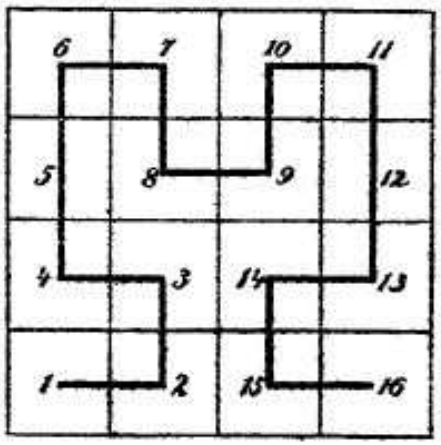


Fig. 2.

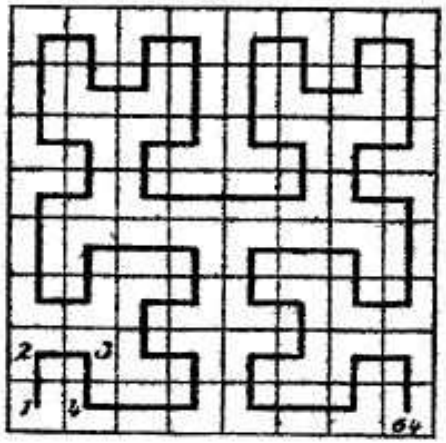
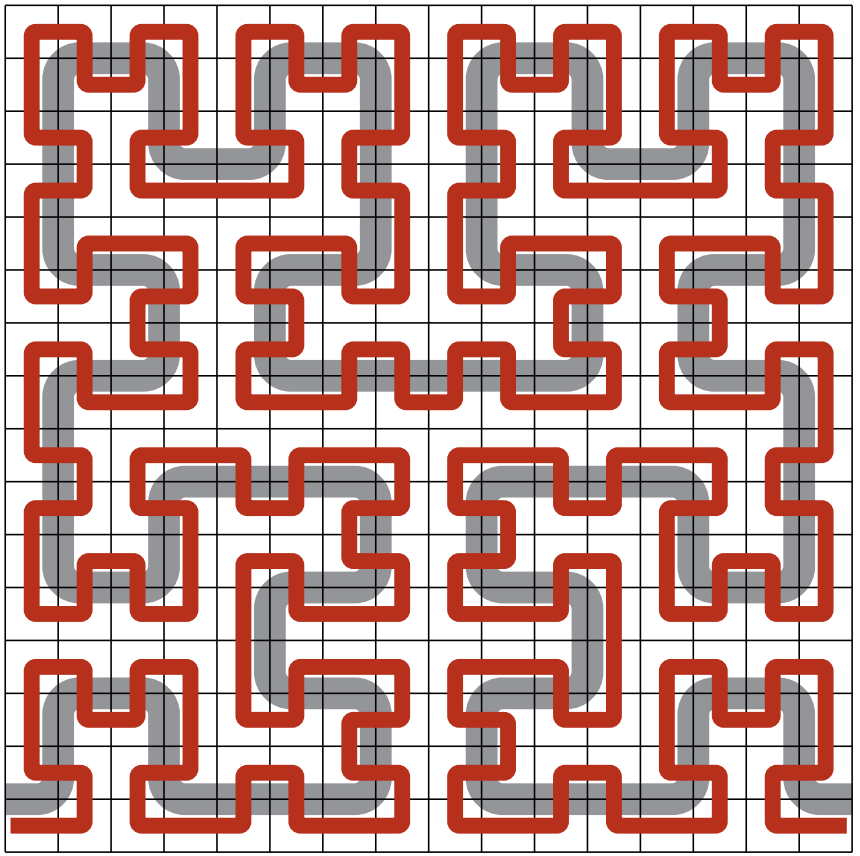
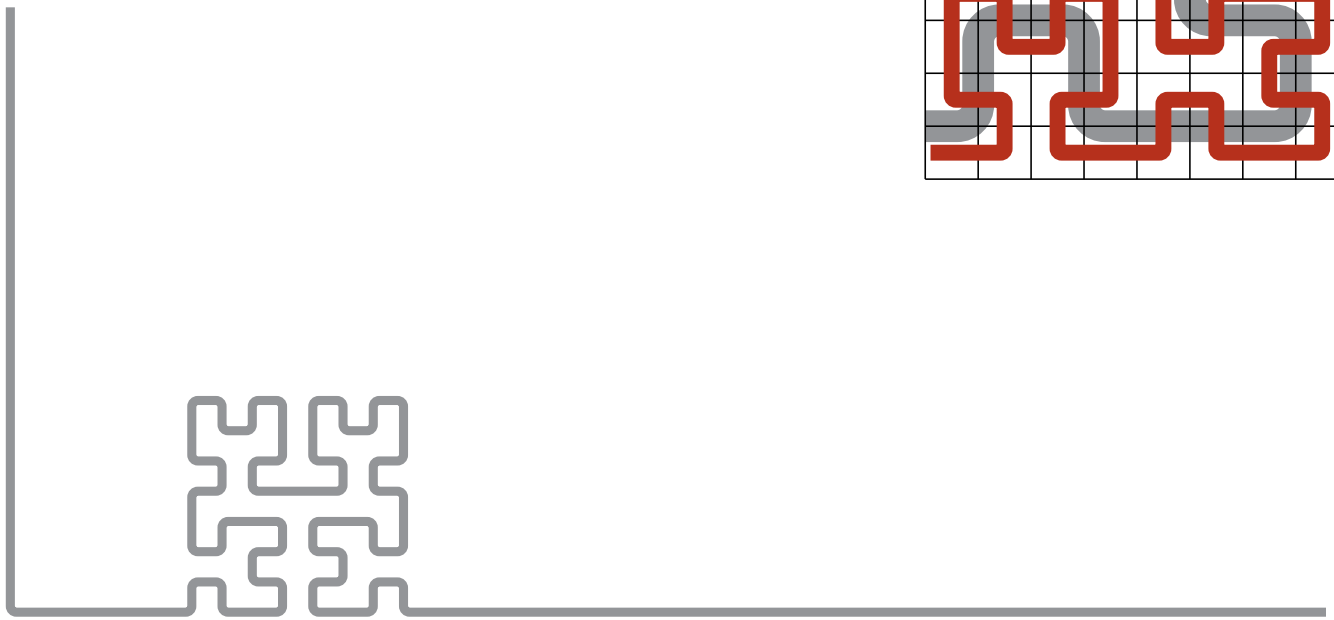


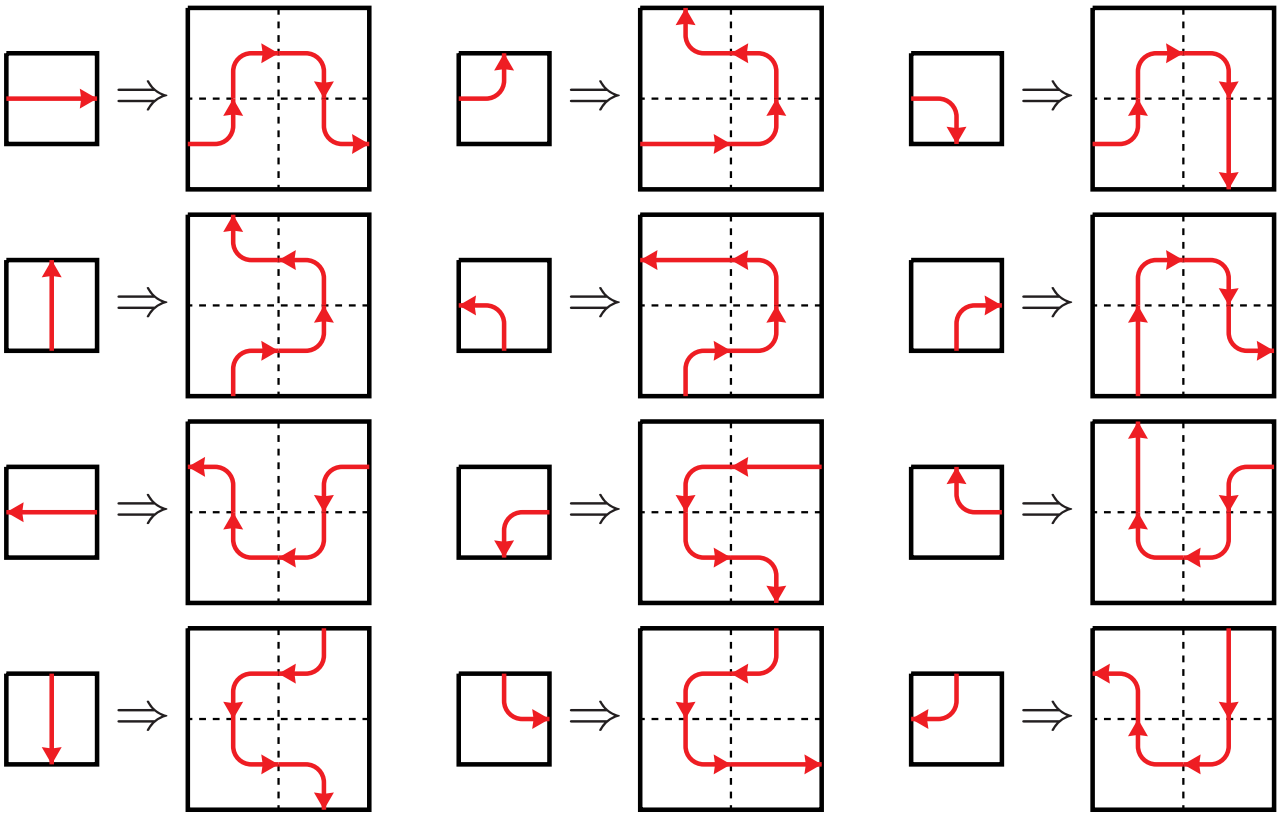
Fig. 3.

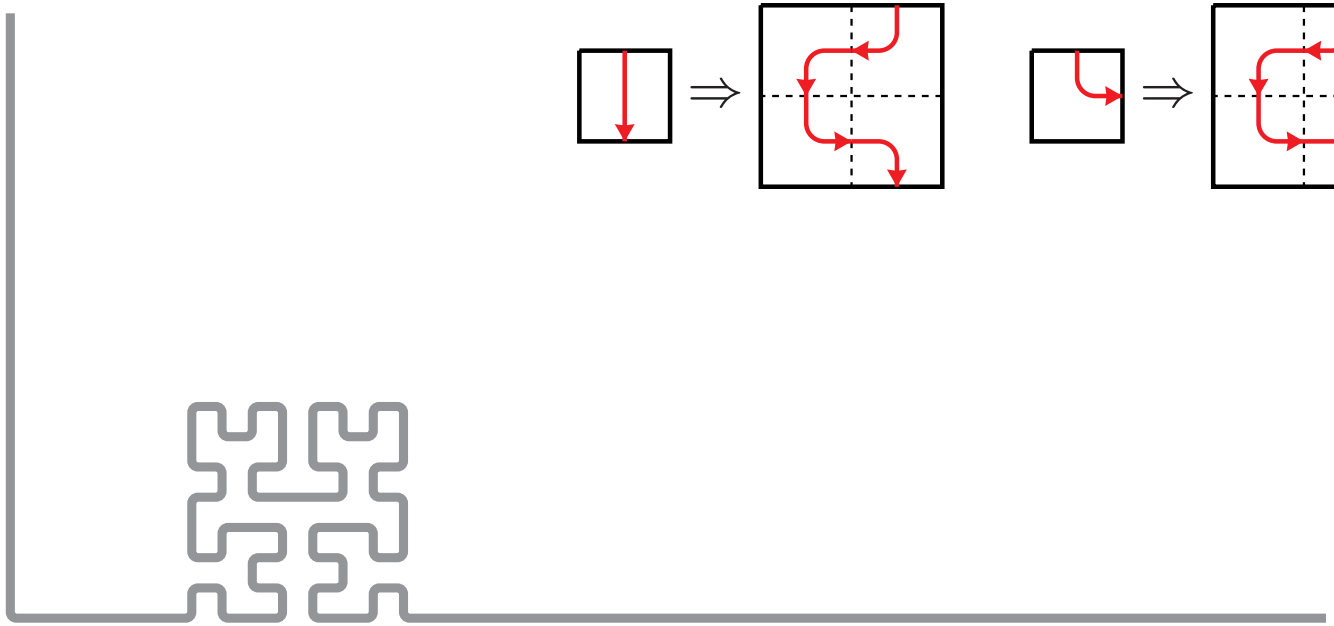
Ueber die stetige Abbildung einer Linie auf ein Flächenstück (1891)

David Hilbert in Königsberg i. Pr.









Observation. Hilbert is an iterated substitution.

Corollary. Hilbert is a PQRS language.

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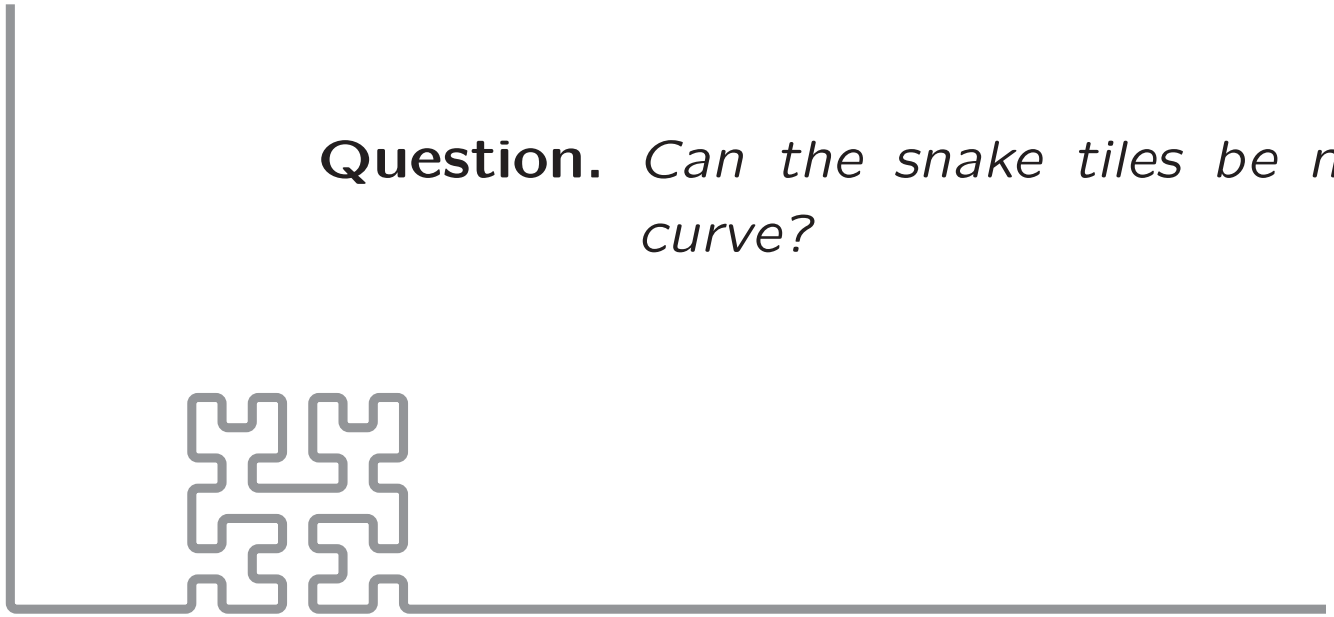
Question. *Can the snake tiles be made to follow the curve?*



kiitos!