1) a. Choose $p \dashv \vdash \text{‘shines today’}$, and $q \dashv \vdash \text{‘shines tomorrow’}$. We get $p \rightarrow (\neg q)$.

   c. Choose $p \dashv \vdash \text{‘shines today’}$, and $r \dashv \vdash \text{‘rains today’}$. We explicity have to express exclusive or, $(p \lor r) \land \neg(p \land r)$.

   d. Both options, or only one of them?

2) ‘In full’ suggests to add also the outer parentheses we usually omit.

   \[
   ( (\neg p) \rightarrow (p \land r) ) \\
   ( (p \rightarrow q) \rightarrow ((s \lor t) \rightarrow (r \rightarrow q))) \\
   ( (p \lor q) \rightarrow ((\neg p) \lor r) )
   \]

   $p \lor q \land r$ has no fixed meaning if we strictly follow the rules of the exercise. The binding order of $\lor$ and $\land$ is not specified, associativity seems to hold only for $\rightarrow$.

3) b. Sorry, I mean natural language ‘and’, to have two tables, to compare the two related formulas.

   c. We see from the table that $(q \land (r \rightarrow q))$ is actually equivalent to $q$.

   \[
   \begin{array}{ccc|c}
   p & q & r & p \lor (\neg(q \land (r \rightarrow q))) \\
   \hline
   T & T & T & T \\
   T & T & F & T \\
   T & F & T & T \\
   T & F & F & F \\
   T & F & F & F \\
   F & T & T & T \\
   F & T & F & T \\
   F & F & T & F \\
   F & F & F & F \\
   \end{array}
   \]

4) a. $(\forall x)A(m, P(x))$ is not correct as ‘$P(x)$’ means the logical statement ‘$x$ is a professor’ (true or false) and can not be written in place of a person ‘professor $x$’.

   $(\forall x)(P(x) \rightarrow A(m, x))$ For every $x$, when professor, then Mary admires $x$

   This is not the same as $(\forall x)(P(x) \rightarrow A(m, x))$ which also states that all $x$ are professors. We assume there are also students.

   b. $(\exists x)(P(x) \land A(x, m))$ There is a person who is professor and admires Mary.

   c. No student attended every lecture. Tricky. There is no single student that attended all lectures. $\neg(\exists x)(\forall y)(S(x) \land L(y) \land B(x, y))$

      Every student missed at least one lecture: $(\forall x)(S(x) \rightarrow (\exists y)(L(y) \land \neg B(x, y)))$

5) a. For every person there is a person that is the mother of the first. $(\forall x)(\exists y)M(y, x)$

   b. $(\forall x)(\exists y)M(y, x) \land (\forall x)(\exists y)F(y, x)$

      or $(\forall x)(\exists y)(\exists z)(M(y, x) \land M(z, x))$

   d. $(\exists x)(\exists y)(F(Ed, x) \land F(x, y))$

   e. Probably: $(\forall x)(\exists y)F(x, y) \rightarrow (\exists y)(F(x, y) \lor M(x, y))$
6) In the first case $\phi$ is not true. If we choose $x = b$ and $y = a$ then $R(x, y)$ is true, as $(b, a)$ is in the relation given. However we can not find $z \in A$ that makes $R(y, z)$ true, as there is no pair $(b, z)$ in the relation. This makes $R(x, y) \rightarrow R(y, z)$ false (for this $x$, this $y$ and all $z$). Consequently $\phi$ is false.

Second case. Assume $x$ and $y$ are chosen that make $R(x, y)$ true. We consider cases to find $z$.
- $x = b$, $y = c$ then we choose $z = b$,
- $x = a$, $y = b$ then we choose $z = c$,
- $x = c$, $y = b$ then we choose $z = c$.

Hence we can always find $z$, which makes the statement true.