Introduction to Set Notation

- Definition - Set

  A set is a unordered collection of zero or more distinct objects.
- The objects that make up a set are called

  elements or **members** of the set.

Specifying Sets

Two main ways:

1. Listing the members of the set.

   NOTATION: $A = \{a_1, a_2, \ldots, a_n\}$ or $A = \{a_1, a_2, \ldots\}$

   examples: $A = \{a, e, i, o, u\}$

   $N = \{1, 2, 3, 4, \ldots\}$

2. State the properties that characterize the members in the set.

   NOTATION: $B = \{x: x \text{ satisfies } \ldots\}$

   example: $B = \{x : x \text{ is an even integer}, x > 0\}$

   We read this as “B is the set of x such that x is an even integer and x is grater than zero”. Note that we can’t list all the members in the set B.

Note: Sets are often denoted by capitals, elements are usually lower-case.

Some Properties of Sets

- The order in which the elements are presented in a set is not important:

  A = {a, e, i, o, u} and B = {e, o, u, a, i} both define the same set

- The members of a set can be anything, even sets.
- In a set the same member does not appear more than once.

  F = {a, e, i, o, a, u} is incorrect since the element ‘a’ repeats.

“Element in/Member of” Notation

- Consider the set A = {a, e, i, o, u} then
- We write “‘a’ is a member of ‘A’” as: $a \in A$
- We write “‘b’ is not a member of ‘A’” as: $b \notin A$

Universal Set and Empty Set

- The members of all the investigated sets in a particular problem usually belongs to some fixed large set. In probability theory this is usually the sample space Ω.
- The set that has no elements is called the empty set and is denoted by ∅ or {}.

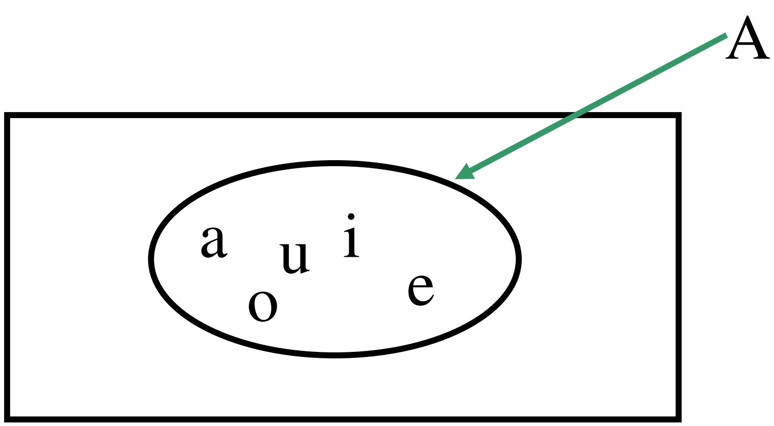
e.g. $\{x : x^2 = 4 \text{ and } x \text{ is an odd integer}\} = \emptyset$

Cardinality of a Set

- The number of elements in a set is called the cardinality of a set. Let ‘A’ be any set then its cardinality is denoted by |A|
- E.g. A = {a, e, i, o, u} then |A| = 5.

Venn Diagrams

- A pictorial way of representing sets.
- The universal set is represented by the interior of a rectangle and the other sets are represented by disks lying within the rectangle.
  - E.g. A = {a, e, i, o, u}

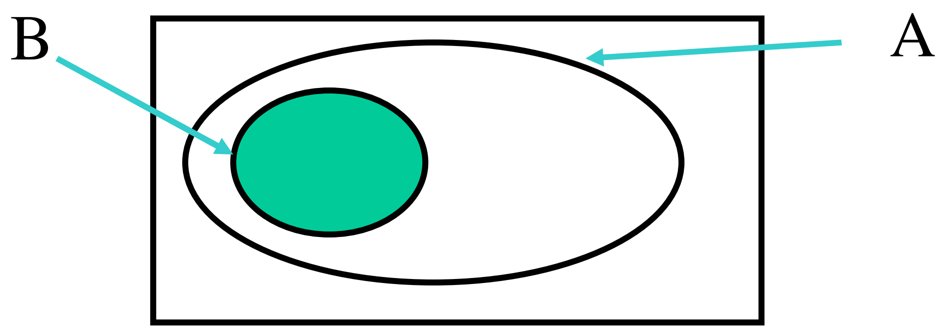


Subsets

- Set ‘A’ is called a subset of set ‘B’ if and only if every element of set ‘A’ is also an element of set ‘B’. We also say that ‘A’ *is contained in* ‘B’ or that ‘B’ *contains* ‘A’. It is denoted by $A \subseteq B$ or $B \supseteq A$.

Venn Diagram for a Proper Subset

- Note that if $B \subset A$ then the Venn diagram depicting those sets is as follows:



- If $B \subseteq A$ then the disc showing 'B' may overlap with the disc showing 'A' ($A = B$)

Power Set

- The set of all subsets of a set 'S' is called the power set of 'S'. It is denoted by P(S) or 2^S.
- So:

  $P(S) = \{x : x \subseteq S\}$
- E.g. $A = \{1, 2, 3\}$ then

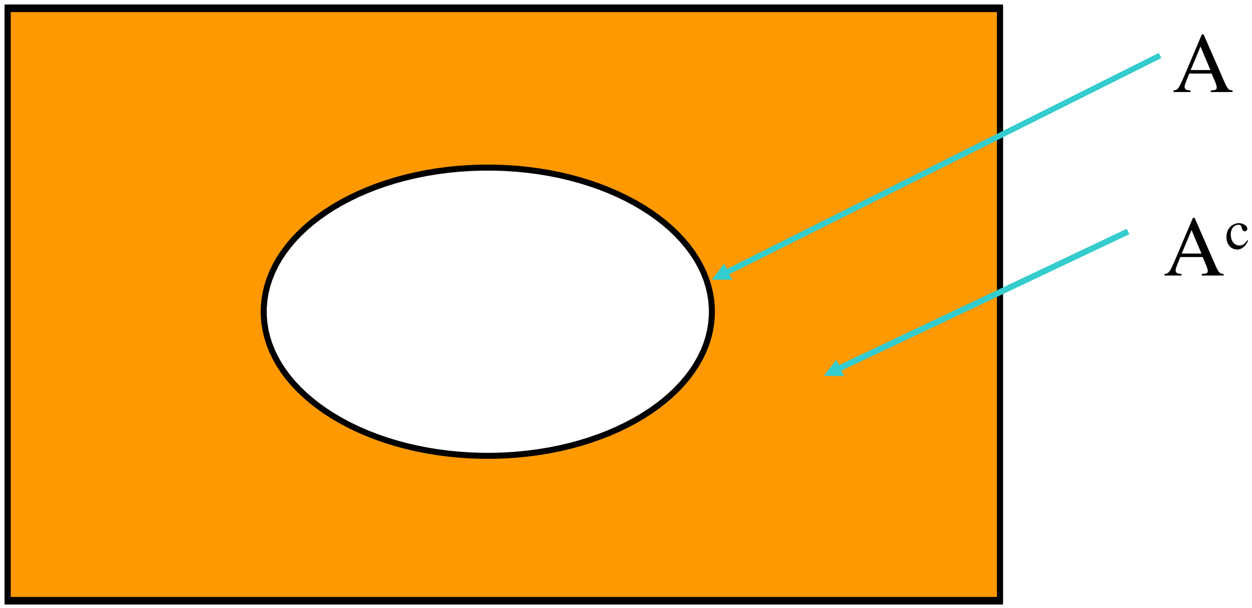
  $P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

- Note that $|P(S)| = 2^{|S|}$.
- E.g. $|P(A)| = 2^{|A|} = 2^3 = 8$.

Set Operations - Complement

- The (absolute) complement of a set 'A' is the set of elements which belong to Ω but which do not belong to A. This is denoted by A^c or $\bar{A}$ or $\tilde{A}$.
- In other words we can say:
- $A^c = \{x : x \in \Omega \wedge x \notin A\}$

Venn Diagram for the Complement



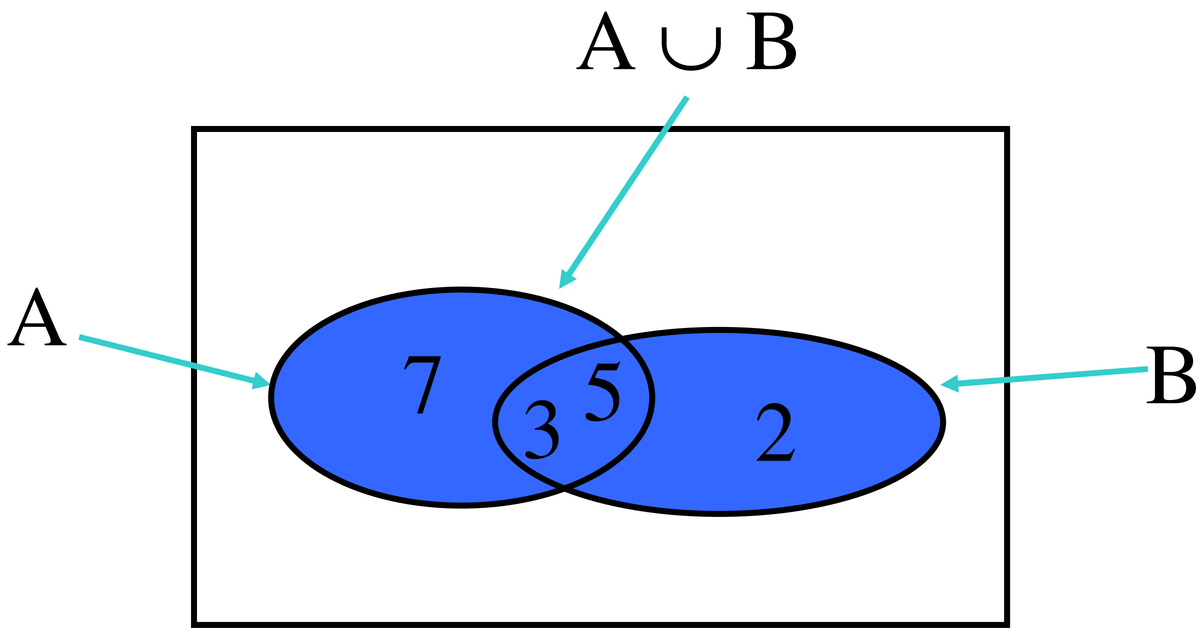
Set Operations - ∪nion

- Union of two sets 'A' and 'B' is the set of all elements which belong to either 'A' or 'B' or both. This is denoted by $A \cup B$.
- In other words we can say:

  $A \cup B = \{x : x \in A \vee x \in B\}$
- E.g. $A = \{3, 5, 7\}$, $B = \{2, 3, 5\}$

  $A \cup B = \{3, 5, 7, 2, 3, 5\} = \{2, 3, 5, 7\}$

Venn Diagram Representation for Union



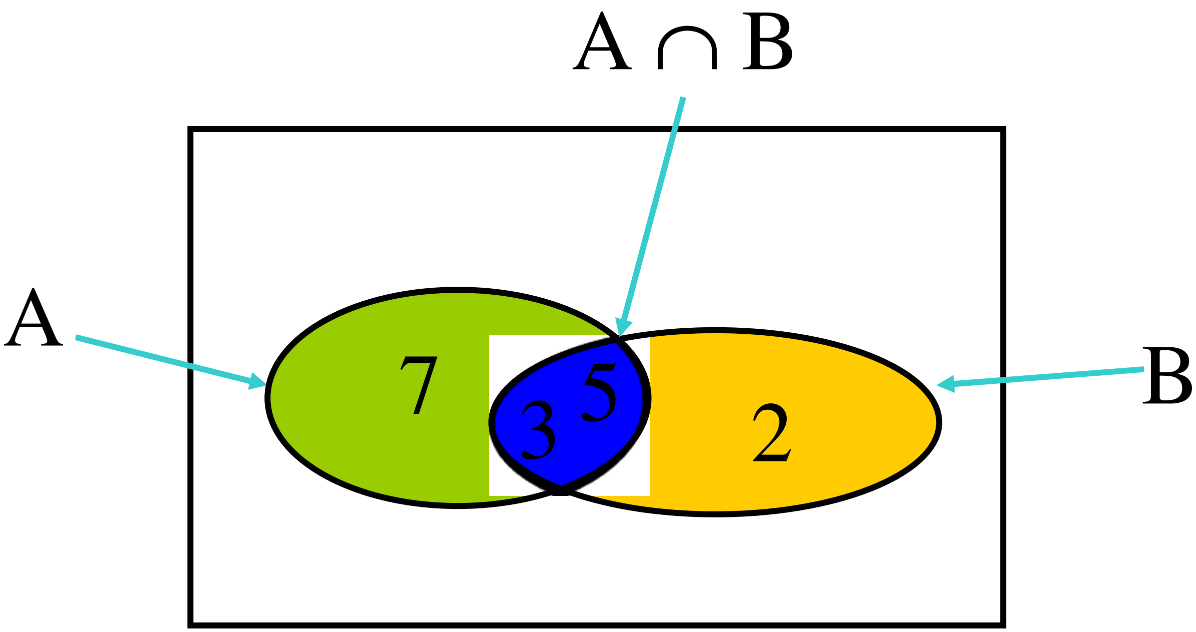
Set Operations - I$\cap$tersection

- Intersection of two sets 'A' and 'B' is the set of all elements which belong to both 'A' and 'B'. This is denoted by $A \cap B$.
- In other words we can say:

  $A \cap B = \{x : x \in A \wedge x \in B\}$
- E.g. $A = \{3, 5, 7\}$, $B = \{2, 3, 5\}$

  $A \cap B = \{3, 5\}$

Venn Diagram Representation for Intersection



Set Operations - Difference

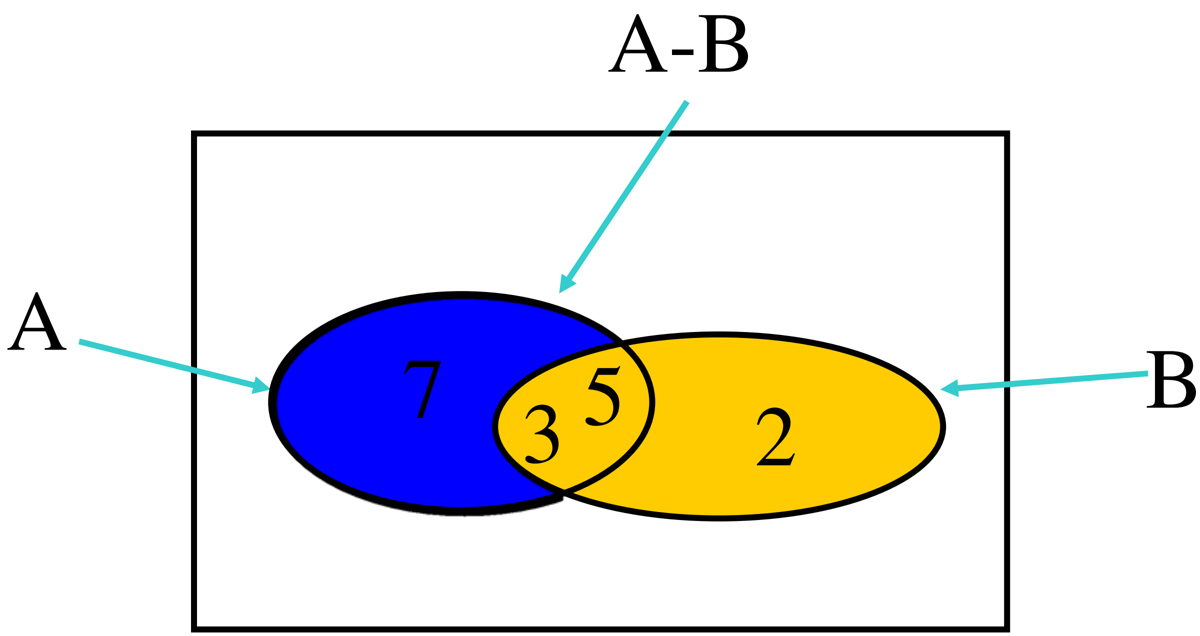
- The difference or the relative complement of a set ‘B’ with respect to a set ‘A’ is the set of elements which belong to ‘A’ but which do not belong to ‘B’. This is denoted by A – B or A \ B.
- In other words we can say:

  $A - B = \{x : x \in A \wedge x \notin B\}$

- E.g. A = {3, 5, 7}, B = {2, 3, 5}

  $A - B = \{3, 5, 7\} - \{2, 3, 5\} = \{7\}$

Venn Diagram Representation for Difference



Thanks

- Next time:
  - Relation between sets and probability
  - See study guide