Solutions of the exercises of Chapters 4 and 5 for

Theorie van Concurrency

4.1 See Fig.1.

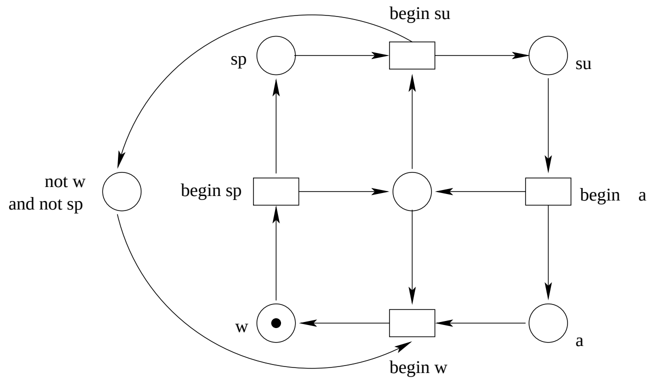

Figure 1: EN system for ex. 4.1

4.2 a) i) ${}^\bullet p_2 = \{t_1, t_2, t_5\}$

ii) $({}^\bullet p_2)^\bullet = (\{t_1, t_2, t_5\})^\bullet = \{p_1, p_2, p_3\}$

iii) $\{t_2, t_3\}^\bullet = \{p_1, p_2, p_3, p_4\} = P_N$

iv) $(\{t_2, t_3\}^\bullet)^\bullet = (\{p_1, p_2, p_3, p_4\})^\bullet = \{t_1, t_2, t_3, t_4, t_5\} = T_N$

v) $\text{nbh}(\{p_3, p_4\}) = \{p_3, p_4\}^\bullet \cup {}^\bullet\{p_3, p_4\} = \{t_5\} \cup \{t_1, t_2, t_3, t_4\} = T_N$

vi) $\text{nbh}(\{p_2, t_3\}) = \{p_2, t_3\}^\bullet \cup {}^\bullet\{p_2, t_3\} = \{t_3, t_4, p_4, p_1\} \cup \{t_1, t_2, t_5, p_2\} = X_N \setminus \{p_3\}$

vii) $F_N^+ \cap (T_N \times T_N)$ is the set of all pairs of transitions (t, t') such that there is a nonempty path from t to t' in the net. Since N, as a graph, is strongly connected, there is such a path for all (t, t'). Hence $F_N^+ \cap (T_N \times T_N) = T_N \times T_N$.

b) i) No.

ii) Yes, because there are no $p, q \in P \mid ({}^\bullet p = {}^\bullet q) \wedge (p^\bullet = q^\bullet)$

iii) No, ${}^\bullet t_1 = {}^\bullet t_2$ and $t_1{}^\bullet = t_2{}^\bullet$ but $t_1 \neq t_2$.

4.3 a) $P^\bullet \subseteq T$ and ${}^\bullet P \subseteq T$ by Definition 1(2): $F \subseteq (P \times T) \cup (T \times P)$. In the other direction, $T \subseteq P^\bullet$ and $T \subseteq {}^\bullet P$ by Definition 1(3): for every $t \in T$ there exists $p \in P$ such that $(p, t) \in F$, and for every $t \in T$ there exists $p \in P$ such that $(t, p) \in F$.

b) We first note that always $\text{nbh}(T) \subseteq P$, by Definition 1(2). Next we show that for all $p \in P$ and $t \in T$, $p \in \text{nbh}(t) \Leftrightarrow t \in \text{nbh}(p)$. Proof: $p \in {}^\bullet t \cup t^\bullet$ iff $p \in {}^\bullet t$ or $p \in t^\bullet$ iff $(p, t) \in F$ or $(t, p) \in F$ iff $t \in p^\bullet$ or $t \in {}^\bullet p$ iff $t \in {}^\bullet p \cup p^\bullet$. Now, $P \subseteq \text{nbh}(T)$ iff $\forall p \in P\ \exists t \in T : p \in \text{nbh}(t)$ iff $\forall p \in P\ \exists t \in T : t \in \text{nbh}(p)$ iff $\forall p \in P : \text{nbh}(p) \neq \emptyset$ iff N has no isolated places.

c) By symmetry it suffices to show that N' is P-simple if N is P-simple. Assume that $N \equiv_\beta^\alpha N'$ and that N is P-simple. Consider $p', q' \in P_{N'}$ with ${}^\bullet(p') = {}^\bullet(q')$ and $(p')^\bullet = (q')^\bullet$. We will show that $p' = q'$. Let $\alpha(p) = p'$ and $\alpha(q) = q'$ for $p, q \in P_N$. Then ${}^\bullet\alpha(p) = {}^\bullet\alpha(q)$ and $\alpha(p)^\bullet = \alpha(q)^\bullet$. Since (α, β) is an isomorphism from N to N': ${}^\bullet\alpha(p) = \beta({}^\bullet p)$, ${}^\bullet\alpha(q) = \beta({}^\bullet q)$, $\alpha(p)^\bullet = \beta(p^\bullet)$, and $\alpha(q)^\bullet = \beta(q^\bullet)$. Hence $\beta({}^\bullet p) = \beta({}^\bullet q)$ and $\beta(p^\bullet) = \beta(q^\bullet)$. Since β is a bijection, this implies that ${}^\bullet p = {}^\bullet q$ and $p^\bullet = q^\bullet$. Thus, because N is P-simple, $p = q$. And so $p' = \alpha(p) = \alpha(q) = q'$. Hence N' is P-simple.

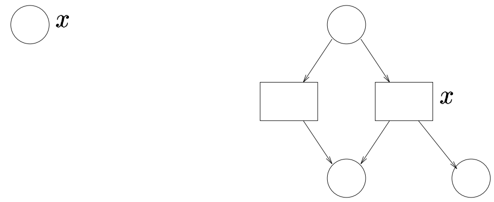

Figure 2: Two nets for ex. 4.4

4.4 See Fig.2.

In the leftmost net ${}^{\bullet}x = \emptyset$ and so $({}^{\bullet}x)^{\bullet} = \emptyset$ and $(({}^{\bullet}x)^{\bullet})^{\bullet} = \emptyset$. Also $x^{\bullet} = \emptyset$.

In the rightmost net $({}^{\bullet}x)^{\bullet} = T$ and $T^{\bullet} = x^{\bullet}$.

4.5 a) t_3 **con** $C \Leftrightarrow p_2 \in C \wedge p_1, p_4 \notin C$, so $C = \{p_2\}[= \clubsuit]$ or $C = \{p_2, p_3\}[= \spadesuit]$.
From $\clubsuit$ we have $p_2[t_3\rangle\{p_1, p_4\}$, and from $\spadesuit$ we have $\{p_2, p_3\}[t_3\rangle\{p_1, p_3, p_4\}$.

b) $\{p_1\}[uvwx\rangle\{p_1, p_4\}$
$\{p_1\}[t_1\rangle\{p_2, p_3\}[t_4\rangle\{p_3, p_4\}[t_5\rangle\{p_1, p_2\}[t_4\rangle\{p_1, p_4\}$

4.6 See Fig.3.

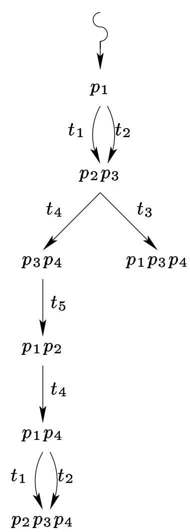

Figure 3: $SCG(M)$ for ex. 4.6

From Fig.3 it is easy to see that

$$
\begin{aligned}
FS(M) \;=\;& \{\lambda, t_1, t_2, t_1t_3, t_2t_3, t_1t_4, t_2t_4, \\
& t_1t_4t_5, t_2t_4t_5, t_1t_4t_5t_4, t_2t_4t_5t_4, \\
& t_1t_4t_5t_4t_1, t_2t_4t_5t_4t_1, t_1t_4t_5t_4t_2, t_2t_4t_5t_4t_2\}
\end{aligned}
$$

$\mathbb{C}_M = \{\{p_1\}, \{p_2, p_3\}, \{p_1, p_3, p_4\}, \{p_3, p_4\}, \{p_1, p_2\}, \{p_1, p_4\}, \{p_2, p_3, p_4\}\}$,
i.e., all nodes of $SCG(M)$

use$(T_M) = T_M$ as all transitions can be fired starting from the initial configuration (all transitions are labels of edges in $SCG(M)$)

Considering $SCG(M)$, M does not have any live transition (in, e.g., configuration $\{p_1, p_3, p_4\} \in \mathbb{C}_M$ no transition can fire, and hence no nonempty transition sequence can fire).

4.7 [Lemma 7: $C[t\rangle D \Leftrightarrow C\backslash D = {}^\bullet t \wedge D\backslash C = t^\bullet$]

What we want to prove is:

$s, u \in T_M, C \subseteq P_M, sus$ **con** $C \Rightarrow s^\bullet \subseteq {}^\bullet u \wedge {}^\bullet s \subseteq u^\bullet$.

Let sus **con** C. Then $\exists\, D, E, F \subseteq P \mid C[s\rangle D[u\rangle E[s\rangle F$.

According to Lemma 7

${}^\bullet s = C\backslash D = E\backslash F$, ${}^\bullet u = D\backslash E$ and $s^\bullet = D\backslash C = F\backslash E$, $u^\bullet = E\backslash D$.

Consider now an arbitrary $p \in s^\bullet$.

If $p \in s^\bullet$ then $p \in D \wedge p \notin C \wedge p \in F \wedge p \notin E$ and so $p \in D\backslash E = {}^\bullet u$. Hence $s^\bullet \subseteq {}^\bullet u$.

Similarly for ${}^\bullet s \subseteq u^\bullet$.

4.8 [$t \in T$ is a live transition of M iff]

$$\forall\, C \in \mathbb{C}_M \;\exists\, y \in T_M^* \mid yt \textbf{ con } C$$

$$\Updownarrow$$

$$\forall\, x \in FS(M) \;\exists\, y \in T_M^* \mid xyt \in FS(M)$$

$\Downarrow$) If $x \in FS(M)$ then x **con** $(C_{in})_M$ and so $\exists\, C \in \mathbb{C}_M \mid (C_{in})_M[x\rangle C$. As t is live, $\exists\, y \in T_M^* \mid yt$ **con** C. Hence xyt **con** $(C_{in})_M$ so $xyt \in FS(M)$.

$\Uparrow$) If $C \in \mathbb{C}_M$ then $\exists\, x \in FS(M) \mid (C_{in})_M[x\rangle C$. According to the hypothesis $\exists\, y \in T_M^* \mid xyt \in FS(M)$ and so xyt **con** $(C_{in})_M$. Hence yt **con** C (because the C with $(C_{in})_M[x\rangle C$ is unique). So t is live.

4.9 See Fig.4.

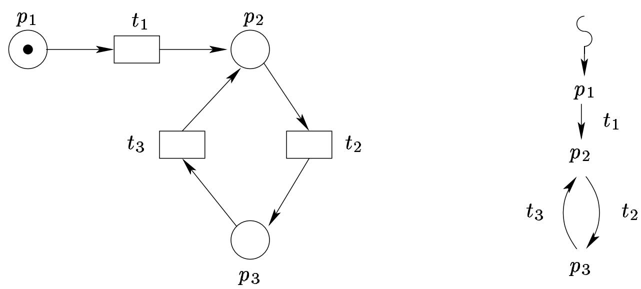

Figure 4: EN system and SCG for ex. 4.9

t_2 and t_3 are live, t_1 is not (but t_1 is useful).

$FS(M) = \lambda + t_1(t_2t_3)^*(\lambda + t_2)$

4.10
a) See Fig.5, without the dotted line.
b) No transition of M is live because in configuration $\{p_5\}$ no transition has concession.
c) $\{t_2, t_5\}$
d) It is in Fig.5, considering also the dotted line. A CG is a SCG with the concurrent steps added. By c), **disj**($\{t_2, t_5\}$). The only configuration $C \in \mathbb{C}_M$ with $\{t_2, t_5\}$ **con** C is $\{p_2, p_4\}$, and $\{p_2, p_4\}[\{t_2, t_5\}\rangle\{p_1, p_3\}$. Note the diamond in Fig.5.

4.11
a) See Fig.6. Note that when disregarding the initial configuration and transitions t_3 and t_6, the picture is a cube.

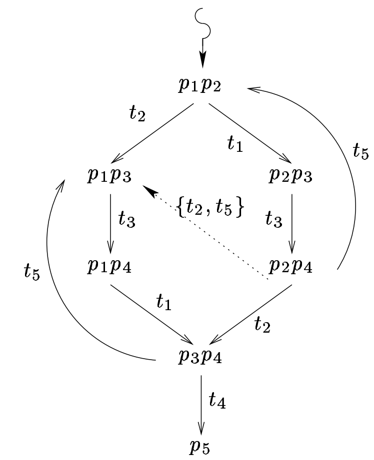

Figure 5: $SCG(M)$ for ex. 4.10

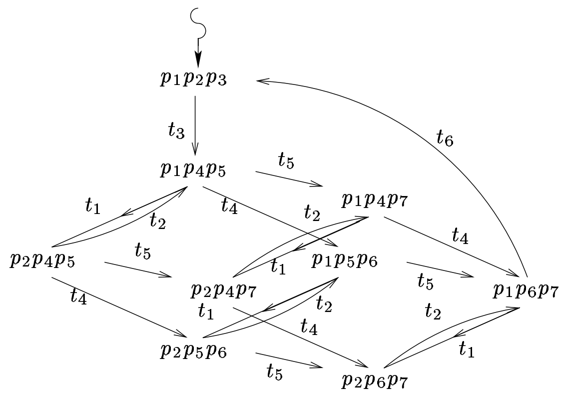

Figure 6: $SCG(M)$ for ex. 4.11

b) All transitions are live (because all transitions are useful, and the graph $SCG(M)$ is strongly connected).

c) All diamonds and cubes in $SCG(M)$:
$\{t_1,t_4\},\{t_1,t_5\},\{t_2,t_4\},\{t_2,t_5\},\{t_4,t_5\},\{t_1,t_4,t_5\},\{t_2,t_4,t_5\}$.
$CG(M)$ has 12 more edges than $SCG(M)$, two for each doubleton and one for each tripleton. The tripletons have concession in $\{p_1,p_4,p_5\}$ and $\{p_2,p_4,p_5\}$, respectively.

4.12 (a) We prove the Hint (in the literature this property is called *confluence*). We first note that the statement holds for $|u|=0$, i.e., $u=\lambda$ and $F=C$ (take $E=D$, $y=\lambda$, and $x=v$). For symmetric reasons it als holds for $|v|=0$. Thus, in what follows we may always assume that $|u|>0$ and $|v|>0$.

The proof of the Hint is by induction on $|u|+|v|$. The basis of the induction, i.e., the case that $|u|+|v|=0$, is handled by the remark above. For the induction step, assume that the statement holds whenever $|u|+|v|\leq n$ (the induction hypothesis). We now show the statement for $|u|+|v|=n+1$, see Fig.7. Consider $C,D,F\in\mathbb{C}_M$ such that $F[u\rangle C$ and $F[v\rangle D$. Since, by the remark above, we may assume that $|u|>0$ and $|v|>0$, let $u=su'$ and $v=tv'$, with $s,t\in T$ and $u',v'\in T^*$. Moreover, let $F[s\rangle F_1$ and $F[t\rangle F_2$. Since s **con** F and t **con** F and M is conflict-free, $\{s,t\}$ **con** F. Hence, by Lemma 17, there exists $E_0\in\mathbb{C}_M$ such that $F_1[t\rangle E_0$ and $F_2[s\rangle E_0$. We now apply the induction hypothesis to $F_1[u'\rangle C$ and

$F_1[t\rangle E_0$; note that $|u'| + |t| < |su'| + |tv'| = |u| + |v|$. Thus, there exist $E_1 \in \mathbb{C}_M$ and $x_1, y_1 \in T^*$ such that $C[x_1\rangle E_1$, $E_0[y_1\rangle E_1$, $|x_1| \leq |t|$, and $|y_1| \leq |u'|$. Similarly, applying the induction hypothesis to $F_2[s\rangle E_0$ and $F_2[v'\rangle D$, there exist $E_2 \in \mathbb{C}_M$ and $x_2, y_2 \in T^*$ such that $E_0[x_2\rangle E_2$, $D[y_2\rangle E_2$, $|x_2| \leq |v'|$, and $|y_2| \leq |s|$. Finally, we apply the induction hypothesis to $E_0[y_1\rangle E_1$ and $E_0[x_2\rangle E_2$; note that $|y_1| + |x_2| \leq |u'| + |v'| < |u| + |v|$. Hence there exist $E \in \mathbb{C}_M$ and $x_1', y_2' \in T^*$ such that $E_1[x_1'\rangle E$, $E_2[y_2'\rangle E$, $|x_1'| \leq |x_2|$, and $|y_2'| \leq |y_1|$. Now take $x = x_1 x_1'$ and $y = y_2 y_2'$. Then $C[x\rangle E$ because $C[x_1\rangle E_1[x_1'\rangle E$, $D[y\rangle E$ because $D[y_2\rangle E_2[y_2'\rangle E$, $|x| = |x_1 x_1'| \leq |tv'| = |v|$ because $|x_1| \leq |t|$ and $|x_1'| \leq |x_2| \leq |v'|$, and $|y| = |y_2 y_2'| \leq |su'| = |u|$ because $|y_2| \leq |s|$ and $|y_2'| \leq |y_1| \leq |u'|$. Thus, we have found the required E, x, and y.

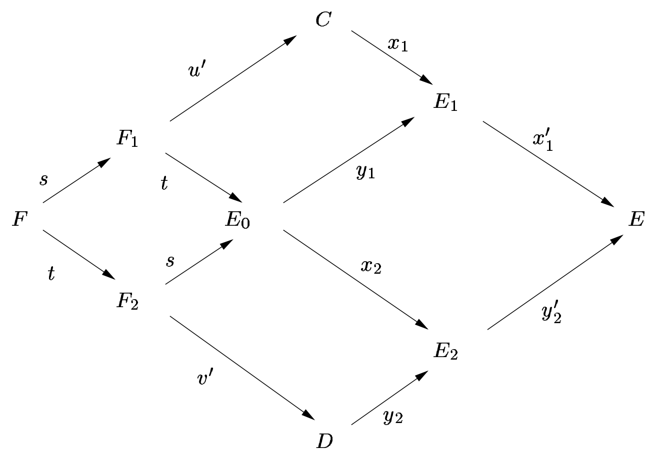

Figure 7: Proof of confluence for ex. 1.11(a)

(b) See also Fig.8. It should be clear that M has no live transitions if there is a reachable configuration in which no transition has concession. It remains to show the other direction. Let $T = \{t_1, \ldots, t_n\}$ and assume that every transition $t_i \in T$ is not live. Let, for all $1 \leq i \leq n$, C_i be a configuration such that $\nexists x \in T^*$: $x t_i$ **con** C_i. By at most $n-1$ applications of item (a) we obtain that there exist $E \in \mathbb{C}_M$ en $x_i \in T^*$ such that $C_i[x_i\rangle E$, for all $1 \leq i \leq n$. Thus, in configuration E no transition has concession.

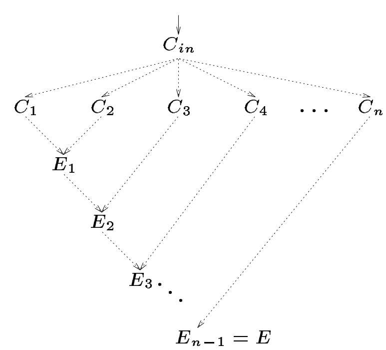

Figure 8: $SCG(M)$ for ex. 1.11(b)

4.13 a) $\{t_2\}$

i) No because $\{t_1, t_4\}$ has no concession in $(C_{in})_M$

ii) $\{t_1, t_3\}$ **con** $(C_{in})_M$
$\mathbf{cfl}(t_1, \{p_1, p_2, p_3, p_4\}) = \{t_2\}[= \clubsuit]$
$\{p_1, p_2, p_3, p_4\}[t_3\rangle\{p_1, p_2, p_7\}$
$\mathbf{cfl}(t_1, \{p_1, p_2, p_7\}) = \{t_4\}[= \spadesuit]$ as not $\mathbf{disj}(\{t_1, t_4\})$
$\clubsuit \neq \spadesuit$ so $((C_{in})_M, t_1, t_3)$ is a confusion

b) (C_{in}, t_1, t_3) is neither *c.i.* or *c.d.* because neither $\clubsuit \subset \spadesuit$ nor $\spadesuit \subset \clubsuit$.
Let us study $((C_{in})_M, t_3, t_1)$ to see if it is symmetrical.
$\{t_3, t_1\}$ **con** $(C_{in})_M$ obviously
$\mathbf{cfl}(t_3, \{p_1, p_2, p_3, p_4\}) = \{t_2\}[= \clubsuit]$
$\{p_1, p_2, p_3, p_4\}[t_1\rangle\{p_3, p_4, p_5\}$
$\mathbf{cfl}(t_3, \{p_3, p_4, p_5\}) = \emptyset[= \spadesuit]$
$\clubsuit \supset \spadesuit$ so $((C_{in})_M, t_3, t_1)$ is a *c.d.* confusion, and it is symmetric [as $((C_{in})_M, t_1, t_3)$ is a confusion]

4.14 In ex. 4.10 the only concurrent step was $\{t_2, t_5\}$ so let us study it.
$(\{p_2, p_4\}, t_5, t_2)$:
$\mathbf{cfl}(t_5, \{p_2, p_4\}) = \emptyset[= \clubsuit]$
$\{p_2, p_4\}[t_2\rangle\{p_3, p_4\}$
$\mathbf{cfl}(t_5, \{p_3, p_4\}) = \{t_4\}[= \spadesuit]$
$\clubsuit \subset \spadesuit$ and $(\{p_2, p_4\}, t_5, t_2)$ is a *c.i.* confusion

$(\{p_2, p_4\}, t_2, t_5)$:
$\mathbf{cfl}(t_2, \{p_2, p_4\}) = \emptyset[= \clubsuit]$
$\{p_2, p_4\}[t_5\rangle\{p_1, p_2\}$
$\mathbf{cfl}(t_2, \{p_1, p_2\}) = \{t_1\}[= \spadesuit]$
$\clubsuit \subset \spadesuit$ and $(\{p_2, p_4\}, t_2, t_5)$ is a *c.i.* confusion
so it is a symmetric confusion.

In ex.4.11, all conflict sets are empty except $\mathbf{cfl}(t_1, \{p_1, p_6, p_7\}) = \{t_6\}$. Hence $(\{p_1, p_4, p_7\}, t_1, t_4)$ and $(\{p_1, p_5, p_6\}, t_1, t_5)$ are c.i. confusions. They are not symmetric.

5.1 a) See Fig.9.
$SCG(M) \equiv SCG(M_1)$ with $\alpha(p_1) = p_1, \alpha(p_2) = p_2, \alpha(p_3) = p_3, \alpha(p_4) = p_4p_6, \alpha(p_5) = p_5p_6, \alpha(p_2p_5) = p_2p_5, \alpha(p_3p_5) = p_3p_5, \alpha(p_4p_5) = p_4p_5p_6$
$\beta(t_i) = t_i$ for all i
$M \approx M_1$, and so also $M \approx_w M_1$ and $M \approx_{fs} M_1$

b) See Fig.10.
$M \not\approx M_2$ because the two SCG are not isomorphic.
$\alpha = \{(C, C), (C, C \cup \{p_6\}) \mid C \in \mathbb{C}_M\} \backslash \{(\{p_1\}, \{p_1\})\}$
β is the identity
$M \approx_w M_2$, and so $M \approx_{fs} M_2$

c) See Fig.11.
$M \not\approx M_3$ because the two SCG are not isomorphic.
$M \not\approx_{fs} M_3$ because M has 8 firing sequences of length 5, whereas M_3 has only 6 firing sequences of length 5. Since $M \not\approx_{fs} M_3$, also $M \not\approx_w M_3$.

5.2 We show that α and β satisfy the conditions of Definition 31.

(1) (C_{in}, C'_{in}) is in α because $x = \lambda$ satisfies the requirements: $C_{in}[\lambda\rangle_M C_{in}$ and $C'_{in}[\lambda\rangle_{M'} C'_{in}$ by Definition 8(1).

(2) Assume that $(C, C') \in \alpha$ and $C[t\rangle_M D$. By definition of α, there exists $y \in T^*$ such that $C_{in}[y\rangle_M C$ and $C'_{in}[\beta(y)\rangle_{M'} C'$. Hence $yt \in FS(M)$ and so, since M and M' are firing sequence equivalent, with bijection β, $\beta(yt) \in FS(M')$. Note that $\beta(yt) = \beta(y)\beta(t)$. Thus, $\beta(y)\beta(t)$ has concession in C'_{in}. Since firing $\beta(y)$ leads from C'_{in} to the unique configuration

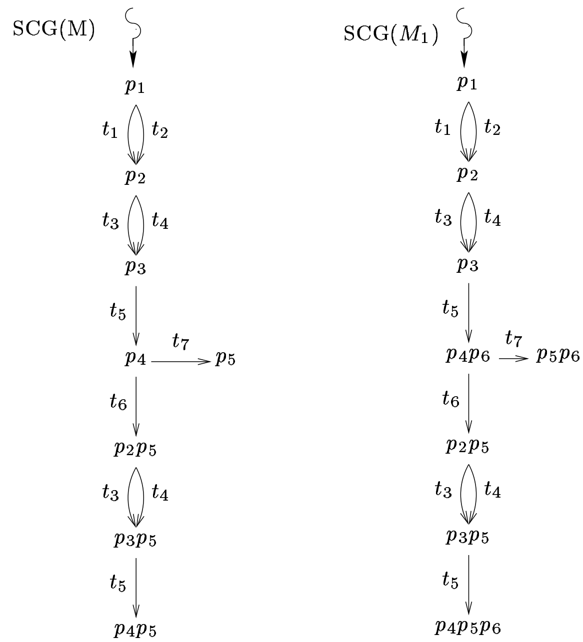

Figure 9: $SCG(M)$ and $SCG(M_1)$ for ex. 5.1(a)

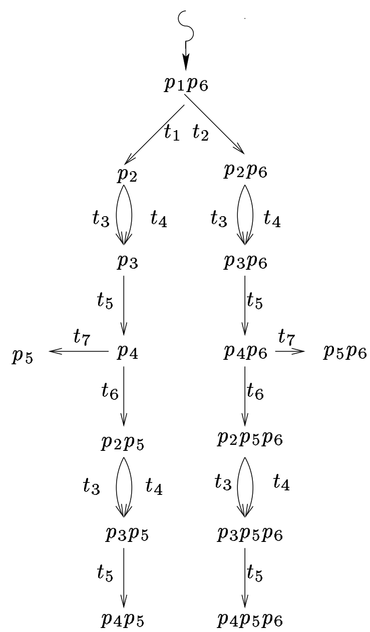

Figure 10: $SCG(M_2)$ for ex. 5.1(b)

C', this means that $\beta(t)$ has concession in C'. Let $C'[\beta(t)\rangle_{M'} D'$. Then $(D, D') \in \alpha$ because $x = yt$ satisfies the requirements: $C_{in}[y\rangle_M C[t\rangle_M D$ and $C'_{in}[\beta(y)\rangle_{M'} C'[\beta(t)\rangle_{M'} D'$.

(3) The proof is entirely analogous to the one of (2).

5.3 See Fig.12.

Considering the $SCG(M)$ represented in Fig.12 it is possible to remove t_2, t_5 and t_7 and

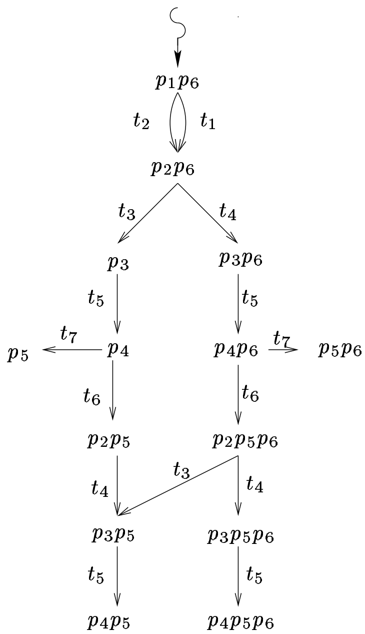

Figure 11: $SCG(M_3)$ for ex. 5.1(c)

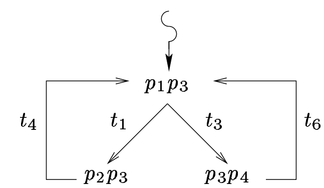

Figure 12: $SCG(M)$ for ex. 5.3

subsequently p_3 and p_5. This gives the EN system depicted in Fig.13. The SCG of this system is obtained from Fig.12 by removing all occurrences of p_3.

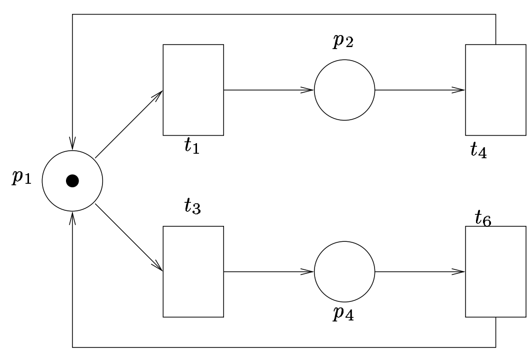

Figure 13: Strongly reduced EN system for ex. 5.3

5.4 Using Lemma 42, it is straightforward to show that for all places p and q, there is a path from p to q in und(M) iff there is an $x \in T^*$ such that $\{p\}[x\rangle\{q\}$.

($\Leftarrow$) Assume that und(M) is strongly connected, and let $t \in T$. To show that t is live, let $C \in \mathbb{C}_M$. Then $C = \{p\}$ and ${}^\bullet t = \{q\}$ for some $p, q \in P$. Since und(M) is strongly connected, there is a path from p to q, and hence there is an $x \in T^*$ such that $\{p\}[x\rangle\{q\}$. Hence xt **con** C.

($\Rightarrow$) Assume now that all transitions are live. We first show that there is a path from every place p to every transition t. By Lemma 42, $\{p\} \in \mathbb{C}_M$. Since t is live, there is an $x \in T^*$ such that $\{p\}[x\rangle\{q\}$, where $\{q\} = {}^\bullet t$. Hence there is a path from p to q, and so also one from p to t. Since every transition has a nonempty output-set, there is also a path from every transition to every other transition. To prove the remaining two cases, it suffices to show that every place p has a nonempty input set. Suppose ${}^\bullet p = \emptyset$. Since there are no isolated places, $p^\bullet$ contains a transition t. Clearly, t is not live, a contradiction.

5.5 $\Rightarrow$) Let $M \approx^{\alpha}_{\beta} M'$ with M' sequential. Consider $C, D \in \mathbb{C}_M$ and $t \in T_M$ such that t **con** C and t **con** D. Then $\beta(t)$ **con** $\alpha(C)$ and $\beta(t)$ **con** $\alpha(D)$. As M' is sequential $\#\alpha(C) = 1$. Since ${}^\bullet\beta(t) \subseteq \alpha(C)$ and ${}^\bullet\beta(t) \neq \emptyset$, we get that $\alpha(C) = {}^\bullet\beta(t)$. Similarly $\alpha(D) = {}^\bullet\beta(t)$ and hence $\alpha(C) = \alpha(D)$. Since α is injective, $C = D$.

$\Leftarrow$) Let M be an EN system so that $\forall C, D \in \mathbb{C}_M, t \in T_M$, if t **con** C and t **con** D then $C = D$. For the sequential configuration graph $SCG(M)$ this means that every useful transition occurs exactly once as the label of an edge. Thus, $SCG(M)$ can be turned into a sequential EN system, with the configurations of M as places. We define $M' = (P', T', F', C'_{in})$ with $P' = \mathbb{C}_M$, $T' = \mathbf{use}(T_M)$, $F' = \{(C,t),(t,D) \mid C[t\rangle_M D,\ C, D \in \mathbb{C}_M, t \in T_M\}$, and $C'_{in} = \{C_{in}\}$.
Clearly, for every $t \in T'$, $({}^\bullet t)_{M'} = \{C\}$ and $(t^\bullet)_{M'} = \{D\}$, where C and D are the unique configurations in $\mathbb{C}_M$ such that $C[t\rangle_M D$. Thus $\#({}^\bullet t) = \#(t^\bullet) = 1$ for all $t \in T'$. Since also $\#C'_{in} = 1$, M' is sequential by Lemma 41. We now use Lemma 30 to show that $M \approx^{\alpha}_{\beta} M'$. Let $\alpha : \mathbb{C}_M \to \mathcal{P}(P')$ be defined by $\alpha(C) = \{C\}$, and let β be the identity on $\mathbf{use}(T_M)$. Then α is injective and β is a bijection, and moreover:

(1) $\alpha(C_{in}) = \{C_{in}\} = C'_{in}$, and
(2) if $C[t\rangle_M D$, then $\{C\}[t\rangle_{M'}\{D\}$ (because $({}^\bullet t)_{M'} = \{C\}$ and $(t^\bullet)_{M'} = \{D\}$ as observed above); moreover, if t $\mathbf{con}_{M'}$ $\{C\}$ then $(C, t) \in F'$ and so t $\mathbf{con}_M$ C.

5.6 a) We first note that it is immediate from the previous exercise that every sequential EN system has the property. Next we note that it is straightforward to show that the given property is equivalent with the following property:

For all $y, z, x \in T^*_M$ and $t \in T_M$:
if $yt \in FS(M)$ and $zt \in FS(M)$, then $(yx \in FS(M) \Leftrightarrow zx \in FS(M))$.

This shows that the property depends on $FS(M)$ only. Since firing sequence equivalent systems have the same $FS(M)$, modulo a bijection β, the property is preserved by $\approx_{fs}$. Hence every system that is firing sequence equivalent with a sequential system has the property.

b) We first show that if $C[t\rangle D$, and $C'[t\rangle D'$, then $(C, C') \in \alpha$ and $(D, D') \in \alpha$. Since t **con** C and t **con** C', it follows from the property (and the definition of α) that $(C, C') \in \alpha$. To prove that $(D, D') \in \alpha$, consider an arbitrary $x \in T^*_M$. If x **con** D, then tx **con** C and hence, because $(C, C') \in \alpha$, also tx **con** C'. This implies that x **con** D'. The other direction is proved in exactly the same way.

For $C \in \mathbb{C}_M$, we denote by $[C]$ the equivalence class of C with respect to α, i.e., $[C] = \{C' \in \mathbb{C}_M \mid (C, C') \in \alpha\}$. Intuitively, M' is constructed by taking $SCG(M)$, identifying α-equivalent configurations, and then applying the construction of Exercise 5.5. Formally we define $M' = (P', T', F', C'_{in})$ with $P' = \{[C] \mid C \in \mathbb{C}_M\}$, $T' = \mathbf{use}(T_M)$, $F' = \{([C], t), (t, [D]) \mid C[t\rangle_M D,\ C, D \in \mathbb{C}_M, t \in T_M\}$, and $C'_{in} = \{[C_{in}]\}$.
By the statement shown above, the input and output sets of each transition in M' are singletons. Since also C'_{in} is a singleton, M' is sequential by Lemma 41. To show that

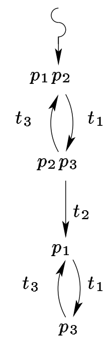

Figure 14: $SCG(M)$ for ex. 5.6(d) part 1

$M' \approx_{fs} M$, we show instead that $M' \approx_w M$, see Theorem 33. Define the relation $\alpha' \subseteq \mathbb{C}_M \times \mathcal{P}(P')$ as follows: $\alpha' = \{(C, \{[C]\}) \mid C \in \mathbb{C}_M\}$. Let us show that this α', with β the identity on $\mathbf{use}(T_M)$, satisfies the requirements of Definition 31. The first requirement is obviously satisfied. To show the second, note that if $C[t\rangle_M D$, then, by definition of M' and its sequentiality, $\{[C]\}[t\rangle_{M'}\{[D]\}$. To show the third, assume that $\{[C']\}[t\rangle_{M'}\{[D']\}$ and that $(C, \{[C']\}) \in \alpha'$. This means that $C'[t\rangle_M D'$, and that $[C] = [C']$, i.e., $(C, C') \in \alpha$. Since t **con** C' and $(C, C') \in \alpha$, also t **con** C. Let $C[t\rangle_M D$. Then, as above, $\{[C]\}[t\rangle_{M'}\{[D]\}$, and since $\{[C]\} = \{[C']\}$, also $\{[D]\} = \{[D']\}$, i.e., $(D, \{[D']\}) \in \alpha'$. This proves the three requirements of Definition 31. It easily follows from them that α' and β are of the correct type, i.e., $\alpha' \subseteq \mathbb{C}_M \times \mathbb{C}_{M'}$ and $T' = \mathbf{use}(T_{M'})$ (cf. the proof of Lemma 30).

c) For $C \in \mathbb{C}_M$, we denote by M_C the EN system that is obtained from M by changing the initial configuration into C. With this notation, the property can be reformulated as follows:

For all $C, D \in \mathbb{C}_M$ and $t \in T_M$:
if t **con** C and t **con** D, then $FS(M_C) = FS(M_D)$.

The algorithm to decide the property is now as follows. Construct $SCG(M)$. For every triple (C, D, t) (with $C, D \in \mathbb{C}_M$ and $t \in T_M$) such that t **con** C and t **con** D (which can be seen in $SCG(M)$), decide whether or not $FS(M_C) = FS(M_D)$. The latter can be decided because, as is known from Formal Language Theory, it is decidable for two finite automata whether or not they accept the same language (see the proof of Theorem 12).

d) Part 1. The SCG of the first EN system M is given in Fig.14.

Since t_1 **con** p_1p_2 and t_1 **con** p_1, M does not satisfy the property of Exercise 5.5, and so is not configuration equivalent with a sequential system. Moreover, it does not even satisfy the property of Exercise 5.6, because t_1t_2 **con** p_1p_2 but *not* t_1t_2 **con** p_1. Hence M is not firing sequence equivalent with any sequential system.

Part 2. The SCG of the second EN system M is given in Fig.15

Since t_4 **con** p_2 and t_4 **con** p_2p_4, M does not satisfy the property of Exercise 5.5, and so is not configuration equivalent with a sequential system. It does, however, satisfy the property of Exercise 5.6. In fact, with the terminology from c), $FS(M_{p_2}) = FS(M_{p_2p_4}) = (t_4t_3)^*(\lambda + t_4)$. And also for t_3 **con** p_3 and t_3 **con** p_3p_4 we see that $FS(M_{p_3}) = FS(M_{p_3p_4}) = (t_3t_4)^*(\lambda + t_3)$. Hence M is firing sequence equivalent with a sequential system M'. By the construction in b), M' is obtained from $SCG(M)$ by identifying configurations p_2 and p_2p_4, and identifying configurations p_3 and p_3p_4, and turning the resulting graph into a net. Note that M' can be obtained from M by removing p_4.

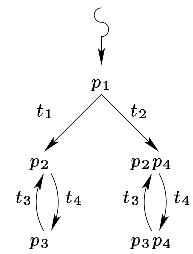

Figure 15: $SCG(M)$ for ex. 5.6(d) part 2

5.7 Using Theorem 49 we find that M has sequential components $\{p_1, p_2, p_3, p_4, p_5\}$, $\{p_1, p_6, p_7, p_8, p_9\}$, $\{p_{10}, p_{11}, p_{12}, p_{13}, p_{14}\}$, and $\{p_{15}, p_{16}, p_{17}, p_{18}\}$, which cover M. So, according to Theorem 59, M is contact-free. Note that $\{p_1, p_4, p_5, p_6, p_7, p_{10}, p_{11}, p_{17}, p_{18}\}$ is also a sequential component of M.

5.8 a) See Fig.16 (without the dotted lines).

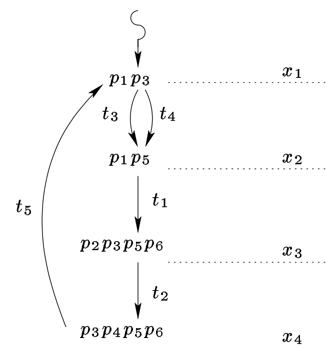

Figure 16: $SCG(M)$ for ex. 5.8(a)

b) According to Lemma 47 a subsystem is determined by a set of places $S \subseteq P$ such that $^\bullet S = S^\bullet$. The trivial subsystems are $\emptyset$ and P. We find the others by Lemma 47.

Note first (after considering t_2) that $p_2 \in S$ iff $p_4 \in S$. Similarly (considering t_3 or t_4), $p_3 \in S$ iff $p_5 \in S$. Furthermore (considering t_5), if $p_4 \in S$ then $p_1 \in S$. Similarly, if $p_5 \in S$ or $p_6 \in S$ then $p_1 \in S$. Consequently always $p_1 \in S$. Now, considering t_1, we conclude that $p_2 \in S$ or $p_3 \in S$ or $p_6 \in S$, where the 'or' is inclusive, giving six possibilities. Thus, apart from the trivial subsystems $\emptyset$ and P, we obtain the following subsystems: $\{p_1, p_2, p_4\}$, $\{p_1, p_6\}$, $\{p_1, p_3, p_5\}$, $\{p_1, p_2, p_4, p_6\}$, $\{p_1, p_2, p_3, p_4, p_5\}$, and $\{p_1, p_3, p_5, p_6\}$. Note that the last three are unions of the first three.

By Theorem 49 the sequential subsystems are $\{p_1, p_2, p_4\}$ and $\{p_1, p_6\}$.

c) See Fig.17. The places that do not belong to a sequential component are p_3 and p_5. The dashed lines are the complement.

d) The goal of this part of the exercise is not completely clear. It is not possible to add one place to M and obtain a contact-free M'' that is configuration equivalent with M. In fact, let the place be q. Since t_1 has contact in $\{p_1, p_3\}$, (q, t_1) must be in F'' and q has no token in C''_{in}. Since, after firing t_3, t_1 must be enabled, there must be an edge (t_3, q) in F''. Thus, in M'', $\{p_1, p_3\}[t_3\rangle\{p_1, p_5, q\}[t_1\rangle\{p_2, p_3, p_5, p_6\}$. Since in the last configuration t_3 has contact, (q, t_3) must be in F'', but F'' cannot contain both (t_3, q) and (q, t_3) by Definition 1(4). Thus, such an M'' cannot be obtained in this way.

See fig. 11 ExBook

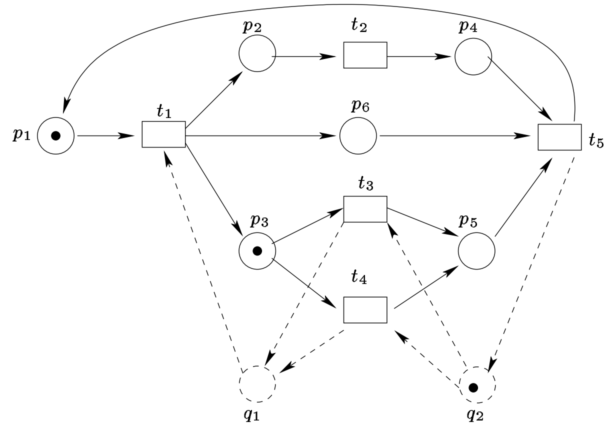

Figure 17: Contact-free EN system M' for ex. 5.8(c)

However, since M satisfies the condition of Exercise 5.5 (see Fig.16), it is configuration equivalent with a sequential (and hence contact-free) system M''. Using the construction in the proof of Exercise 5.5, M'' is obtained as depicted in Fig.18 (see also the dotted lines in Fig.16). Obviously M'' contains much less places than M'.

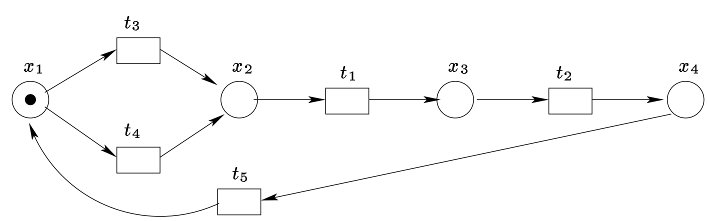

Figure 18: Contact-free EN system M'' for ex. 5.8(d)

e) Recall the subsystems of M from b). These are also the subsystems of M_1, because (by Lemma 47) a set S of places determines a subsystem iff ${}^\bullet S = S^\bullet$, and that does not depend on the initial configuration. But whether a subsystem is a sequential component *does* depend on the initial configuration. Now also the subsystem $\{p_1, p_3, p_5\}$ is sequential, by Theorem 49. So $\{p_1, p_2, p_4\}$, $\{p_1, p_6\}$, and $\{p_1, p_3, p_5\}$ are sequential components covering M_1.

5.9 a) See Fig.19.

b) See Fig.20 (without the **q**'s).

c) No because in configuration $\{p_5\}$ no transition has concession.

d) It is easy to see that all nonempty subsystems must contain p_1, and hence also p_5 and p_2. Thus, the subsystems are: $\emptyset$, $\{p_1, p_2, p_5\}$, and P.

e) p_3 and p_4 do not belong to any sequential component so we can complement them. See the detail in Fig.21.

f) See Fig.20 with the **q**'s.

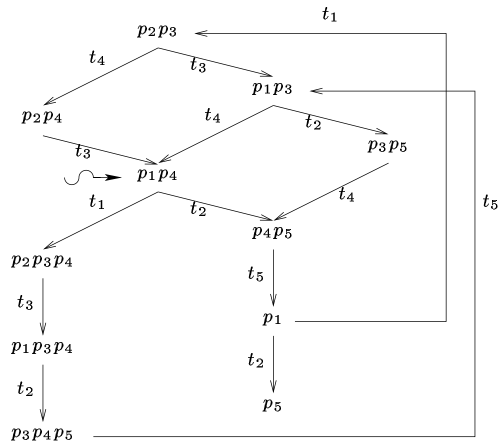

Figure 19: $SCG(M)$ for ex. 5.9(a)

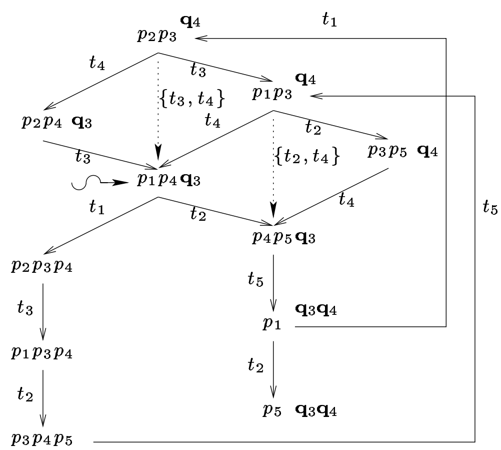

Figure 20: $CG(M)$ for ex. 5.9(b) and 5.9(f)

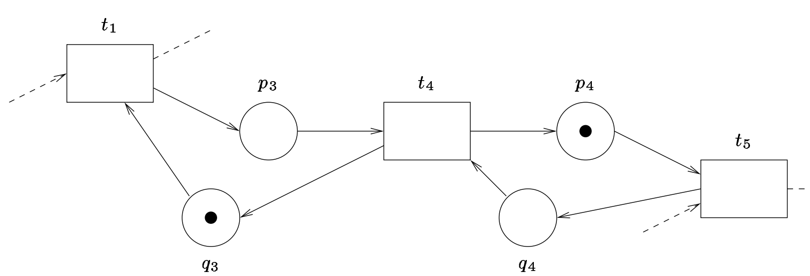

Figure 21: Contact-free EN system for ex. 5.9(e)