Unit 1: Topic and History
I. What is multicriteria optimization and decision analysis?
II. How has this field developed? What were major historical steps?
III. Examples of multicriteria optimization problems. What are criteria, search space, and constraints?
Multicriteria Optimization and Decision Analysis

- **Definition:** *Multicriteria Decision Aiding (MCDA) (or: Multiattribute Decision Analysis)* is a scientific field that studies evaluation of a finite number of alternatives based on multiple criteria. It provides methods to compare, evaluate, and rank solutions.

- **Definition:** *Multicriteria Optimization (MCO) (or: Multicriteria Design, Multicriteria Mathematical Programming)* is a scientific field that studies search for optimal solutions given multiple criteria and constraints. Here, usually, the search space is very large and not all solutions can be inspected.

- **Definition:** *Multiobjective Decision Making (MCDM)* deals with MCDA and MCO or combinations of these.
Search Space, Objectives, and Constraints

• The search space is the space of alternative solutions considered in the MCDM problem
• The objectives are values associated with the solutions in the search space that are to be minimized or maximized
• The constraints are values associated with the solutions in the search space that have to stay in a prescribed domain
• Often the values of the objectives and constraints are obtained by functions that map from the search space to these values.
• These are called objective functions or, respectively, constraint functions.
Motivation: Some Multicriteria Problems

(A) Select best travel destination from a catalogue.
    Seach space: Catalogue
    Objectives: Sun $\rightarrow$ max, Comfort $\rightarrow$ max, and Travel Distance $\rightarrow$ min
    Constraints: Budget

(B) Find a optimal molecule for a drug:
    Search space: All drug-like molecules (chemical space)
    Objectives: Effectivity $\rightarrow$ max, SideEffects $\rightarrow$ min, Cost $\rightarrow$ min
    Constraints: Stability, Solubility in blood, non-toxic

(C) How to control industrial processes:
    Objectives: Set of possible settings of the control parameters
    Criteria: Profitability $\rightarrow$ max, Emissions $\rightarrow$ min
    Constraints: Stability, Safety, Physical feasibility

Other examples: SPAM classifiers, train schedules, reactor, computer hardware

What are criteria in these problems? What is the set of alternatives? Why is there a conflict?
Example: Optimization of Chemical Reactor

Steady State Chemical Reactor:

Consider a steady state chemical reactor. Steady state means that its state does not change over time (operated at equilibrium state).

• Search space:
  • $x_1$: Heating power at different point in time between (50 W – 70 W)
  • $x_2$: Stirring Power (1Watt – 5Watt, each)
  • $x_3$: Throughput in Liter/hour

• Objectives:
  • Profit/t = $\text{PriceProduct} \times \text{MassProduct}/t - \text{Cost}/t \rightarrow \text{max}$
  • Waste = $\text{WasteProduct}/t$

• Constraint
  • Temperature at outlet < 370 Kelvin
Application Example: SPAM Filtering (2013)

Def.: **False positive** result is given if a false hypothesis is accepted.  
Def.: **False negative** result is given if a true hypothesis is rejected.  
Def. **Binary classification of instances**: In **binary classification** in a given set of instances (e.g. images, emails) all instances need to be classified. The hypothesis is: A given instance belongs to a certain class.

**Applied to Spam filtering**: Hypothesis: Email (=instance) is a SPAM.  
- A false positive occurs when spam filtering spam blocking techniques wrongly classify a legitimate email message as spam.  
- A false negative occurs when a spam email is not detected as spam, but is classified as non-spam.

Multicriteria Problem: False negative rate $\rightarrow$ min, False positive rate$\rightarrow$min


Vitor Basto Fernandes,  
Portuguese Computer Scientist  
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Ukrainian Computer Scientist
False Positives minimal (no spam is accepted)

False Negatives minimal (no good email is rejected)
Early roots

- The earliest known reference relating to Multiple Criteria Decision Making can be traced to Benjamin Franklin (1706–1790).
- He allegedly had a simple paper system for deciding important issues.
  - Take a sheet of paper.
  - On one side, write the arguments in favor of a decision;
  - on the other side, write the arguments against.
  - Strike out arguments on each side of the paper that are relatively of equal importance.
  - When all the arguments on one side are struck out, the side which has the remaining arguments is the side of the argument that should be supported.

Supposedly Franklin used this in making important decisions.

Source: http://www.mcdmsociety.org/facts.html
Development

- Vilfredo Pareto (1848–1923), an Italian economist who used the concept of Pareto efficiency in his studies of economic efficiency and income distribution.
- At the same time Francis Edgeworth defined ‘indifference curves’, the ‘core’ of an exchange economy, and the so-called ‘Edgeworth box’ based on a concept of local Pareto optimality for two criteria.
- When Kuhn and Tucker formulated optimality conditions for nonlinear optimization with constraints in 1951, they also considered problems with multiple objectives.

Vilfredo Pareto, Italian economist, 1848-1923  
Francis Edgeworth, British Economist,  
Albert William Tucker, Canadian Mathematician, 1905-1995

Around 1900  
1951
Development

• Ralph Keeney and Howard Raiffa published an important work in 1976. This book was instrumental in establishing the theory of multiattribute value theory (including utility theory) as a discipline. It became a standard reference and text for many generations of study of decision analysis and MCDM.

• Ralph Steuer's professor, John Evans, suggested the topic of developing a multiple criteria simplex method to compute all efficient extreme points. Inspiration was drawn from earlier works of Karlin, Koopmans, and Geoffrion. Steuers ADBASE computer code for generating efficient points became important. (1986)
Kahnemann and Tversky studied the psychological aspects of decision making and pointed out (seemingly) irrational components in human decision making.

In the closely related field of game theory, John von Neumann and later John Nash studied decisions in games with conflicting parties.
Development

• Kaiza Miettinen published a book on Nonlinear Multiobjective Optimization which became a standard reference on deterministic methods for solving mathematical programming with multiple criteria.

• Kalyanmoy Deb published a seminal book on Evolutionary Multicriteria Optimization, including NSGA-II algorithm. The work on NSGA-II became the most cited computer science paper 2000-2010.

• Since then EMO is a very active field of research, not only in economics but also in (computer) science and engineering.
Development of MCDM field

# MCDM Journals

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<thead>
<tr>
<th>Rank</th>
<th>Top-20 Sources (includes both journals and PROCEEDINGS)</th>
<th>#</th>
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<tr>
<td>1.</td>
<td>European Journal of Operational Research</td>
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<td>Journal of the Operational Research Society</td>
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<td>Fuzzy Sets and Systems</td>
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<td>Journal of Mathematical Analysis and Applications</td>
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<td>International Journal of Production Economics</td>
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<td>Lecture Notes in Economics and Mathematical Systems</td>
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<td>16.</td>
<td>Evolutionary Multi-criterion Optimization, Proceedings</td>
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<td>17.</td>
<td>IEEE Transactions on Power Systems</td>
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<td>Engineering Optimization</td>
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<td>Water Resources Research</td>
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Articles in Evolutionary Multicriterion Optimization

Recorded by Carlos Coello Coello
He maintains a page with all publications in EMO: http://www.lania.mx/~ccoello/EMOO/
The fields of multicriteria decision analysis and multicriteria optimization are distinguished by whether a small finite set is considered or search in a large search space. The fields evolved in parallel, first in economics and later for other disciplines, especially engineering. Recently there is an increasing focus on algorithms and integration in machine learning. In general, multicriteria optimization problems can be defined by the following components: search space, objectives, constraints. Examples of multicriteria problems are selection from a catalogue (MCDA), drug discovery, and industrial process optimization. Also (SPAM) classification can be viewed as a multicriteria optimization problem. Here the false positive rate ($\rightarrow \min$) and the false negative rate ($\rightarrow \min$) are possible objectives.