

# Assignment 2: Multicriteria Optimization and Decision Making

LIACS, Algorithms Group

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Assignments can be done in pairs or alone. Submit assignments until the 11th of December. You may hand them in before the lecture or submit online to [emmerich@liacs.nl](mailto:emmerich@liacs.nl).

## 1 Graphical and Analytical Solution of Pareto Optimization

A heliomite is a cone shaped bio-reactor for hydrogen production (Figure 1). In order to build a large scale hydrogen farm the following trade-off between the cost of the heliomite, determined by its volume, and its visible surface should be maximized for a radius  $r$  (in meters) in  $[1, 5]$  and a height  $h$  (in meters) in  $[1, 10]$ . The area of the visible surface to be maximized is given by  $f_A = \pi r \sqrt{r^2 + h^2}$ , and the volume to be minimized is given by  $f_V = \frac{1}{3} \pi r^2 h$ .

- Q1.1** Determine the efficient set graphically and indicate it in the diagram of Fig. 3!
- Q1.2** Suppose the 'aesthetical' constraint  $\frac{3}{2} > h/r > 2$  is added to the diagram. How does the shape of the efficient set change?
- Q1.3** Suppose we would want to reverse the problem and maximize the volume and minimize the area of the cones (e.g., to build a cone shaped hydrogen tank with minimal area exposed to the sun). What would be the efficient set in this case?
- Q1.5** Derive a functional expression of the Pareto-front for Q1.1 and Q1.3 analytically! Do NOT use constraint Q1.2! How would you classify the Pareto fronts (convex, concave, (dis)connected)? What happens, if the objective functions are scaled by a constant factor?

## 2 Theory of (Pareto) Landscapes

Figure 2 describes a linearly ordered landscape. The values in the circles identify the index of solutions, e.g.  $x_1$  has height  $f(x_1) = 1$  and  $x_4$  has height  $f(x_4) = 4$ . The lines connecting the nodes define the neighborhood-graph, e.g.  $N(x_8) = \{x_{20}, x_{14}, x_{13}\}$ .

- Q2.1** Identify all local optima and saddle points between them!
- Q2.2** Draw the barrier tree of the landscape using the flooding algorithm!
- Q2.3** Introduce a second objective function (to be minimized) by enumerating columnwise, i.e.  $f_2(x_1) = 1, f_2(x_{16}) = 2, f_2(x_6) = 3, f_2(x_{15}) = 4, f_2(x_{11}) = 5, f_2(x_7) = 6, f_2(x_{12}) = 7, f_2(x_2) = 8, f_2(x_4) = 9, f_2(x_9) = 10, f_2(x_{17}) = 11, f_2(x_{10}) = 12, f_2(x_{20}) = 13, f_2(x_{14}) = 14, f_2(x_5) = 15, f_2(x_{18}) = 16, f_2(x_8) = 17, f_2(x_{13}) = 18, f_2(x_{19}) = 19, f_2(x_3) = 20$ . Identify (1) all locally efficient points, (2) all globally efficient points, (3) the valley below  $x_{14}$ !

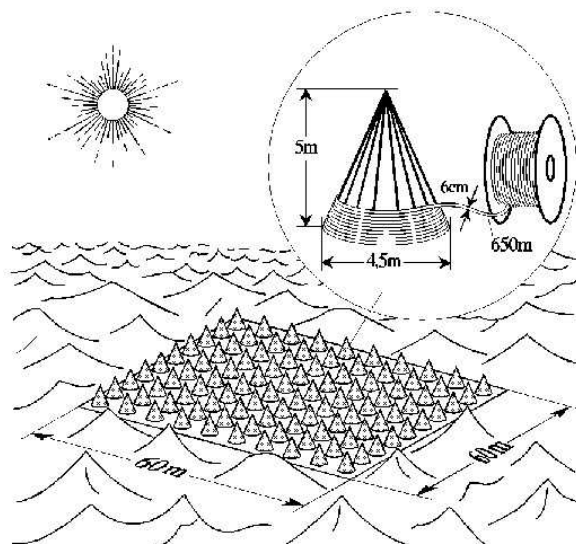


Figure 1: Hydrogen farm in the Sahara consisting of cone-shaped heliostats (c.f. <http://www.bionik.tu-berlin.de/institut/xs2arbas/xs2arbas.html>)

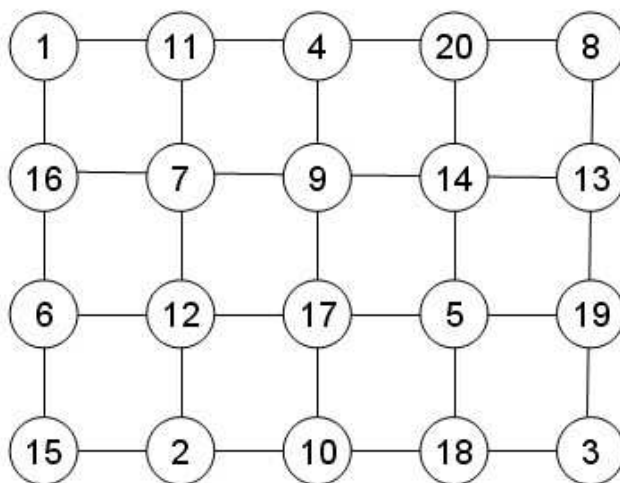


Figure 2: A discrete landscape. All solutions (circles) have a unique fitness value and solutions connected by a single line are direct neighbors.

### 3 Weighted Scalarization and Lagrange Multipliers

Supposed a weighted scalarization technique is used in order to solve an multiobjective problem. The aggregate function is  $w_1y_1 + w_2y_2, w_1 + w_2 = 1, w_1 \in [0, 1], w_2 \in [0, 1]$ . The Pareto front of the problem is  $(\sqrt{y_1} + \sqrt{y_2})^2 = 1, (y_1, y_2) \in [0, 1]^2$ . Use Lagrange multiplier rule to find the points on the Pareto front that minimize  $w_1y_1 + w_2y_2$ !

### 4 Heuristic Optimization - Part I

Implement the following heuristic for continuous bi-objective optimization in a programming language of your choice:

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#### Algorithm 1 Steady state EMOA

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t ← 0
initialize P0 ← x1, ..., xμ randomly within the variable bounds.
for j ∈ {1, ..., T} do
    Draw uniformly randomly a solution xparent out of Pt
    for i ∈ {1, ..., N} do
        xinew ← xiparent + σ · normal(0, 1)
    end for
    Q = Pt ∪ {xnew}
    xworst ← solution with lowest rank in Q
    Pt+1 ← Q \ {xworst}
    t ← t + 1
end for

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Implement the following two ranking criteria:

1. sort by means of  $w_1f_1(\mathbf{x}) + w_2f_2(\mathbf{x})$ . Choose randomly new weights between 0 and 1 in each generation (VEGA)
2. the contribution to the hypervolume for ranking solutions (SMS-EMOA) - if the contribution is zero, decide randomly!

Moreover, implement the following parameterized test problem (EBN Problem):

$$f_1(\mathbf{x}) = \frac{1}{n^\gamma} \left( \sum_{i=1}^N (x_i)^2 \right)^\gamma \rightarrow \min, f_2(\mathbf{x}) = \frac{1}{n^\gamma} \left( \sum_{i=1}^N (1 - x_i)^2 \right)^\gamma \rightarrow \min, \mathbf{x} \in [0, 1]^{10}$$

**Q4.1** Visualize the obtained sets (populations) for  $\sigma = 0.01, T = 20000, N = 10, \mu = 10$  with some plot program<sup>1</sup> for  $\gamma = 1$  and  $\gamma = 0.25$  and  $\gamma = 0.5$ . Explain the different distributions obtained with the algorithms!

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<sup>1</sup>e.g. gnuplot (freeware), Octave (freeware) or MS Excel

## 5 Heuristic Optimization - Part II

Suppose, you would like to go by car from city A (e.g. Amsterdam) to city B (e.g. Oslo). Your two objectives are to minimize the cost of the journey and at the same time to minimize the time of the journey. An example connectivity matrix in table 1 is given for the time it takes to travel from city  $A$  to city  $B$  and the cost of it. If there is no direct connection the time and cost values are set to infinity.

	Amsterdam	Hamburg	Kiel	Copenhagen	Frederikshavn	Göteborg	Malmö	Oslo
Amsterdam	0	40	1000	1000	1000	1000	1000	218
Hamburg	5	0	8	1000	1000	1000	1000	150
Kiel	1000	1	0	45	32	100	1000	250
Copenhagen	1000	1000	3.5	0	1000	1000	30	1000
Frederikshavn	1000	1000	5	1000	0	60	80	75
Göteborg	1000	1000	12	1000	2	0	35	41
Malmö	1000	1000	1000	1	10	3	0	1000
Oslo	2.5	2	25	1000	8	4	1000	0

Table 1: Costs[Euro] (upper diagonal) and times[in hours] (lower diagonal) for directly connected cities. A value of 1000 is a penalty value for not directly connected cities. The table can be downloaded in ASCII format under <https://www.liacs.nl/emmerich/table.txt>

**Q5.1** Based on the SMS-EMOA introduced in the previous section, develop an heuristic for approximating the Pareto sets for the multi-objective shortest path problems:

- Represent the solutions as permutations, (i.e. 0 1 2 7 3 1 4 5 6 represents path Amsterdam Hamburg Kiel Oslo).
- Use swapping of a random element with its neighbor as minimal move. In one mutation  $N_{swap}$  minimal moves are conducted!
- Experiment with different orders of magnitude for values of  $N_{swap}$  and report/compare your results! Compare to complete enumeration!

**Q5.2** Find improvements of the heuristic, e.g. by changing the variation operator, the selection scheme, and/or the representation of solutions and report the results!

**Q5.3** Implement an branch-and-bound algorithm that solves the problem. Provide the source code or pseudo-code of your implementation! How many distance-measurements are needed to obtain the Pareto front?

**Q5.4** Discuss the dis/advantage heuristic, exhaustive and deterministic search for this example! (Comparison criteria: speed/efficiency, difficulty of implementation and transparency, flexibility (i.e. to changes of the objective function), scalability).

Suxes!

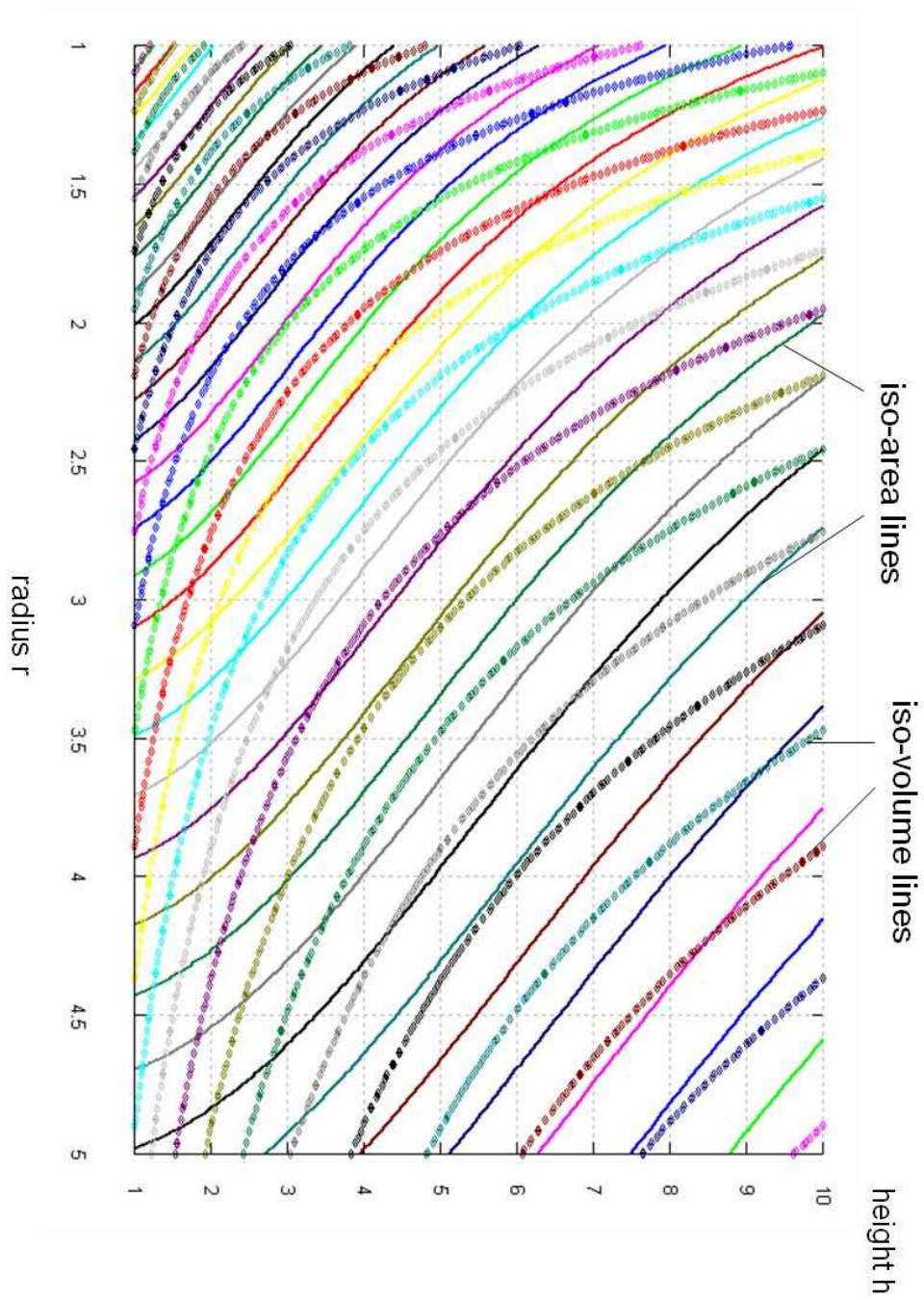


Figure 3: Iso-heightlines for the heliomite areas and volume.