Logica (I&E)

najaar 2018

`http://liacs.leidenuniv.nl/~vlietrvan1/logica/`

Rudy van Vliet
kamer 140 Snellius, tel. 071-527 2876
rvvliet(at)liacs(dot)nl

college 1, maandag 3 september 2018

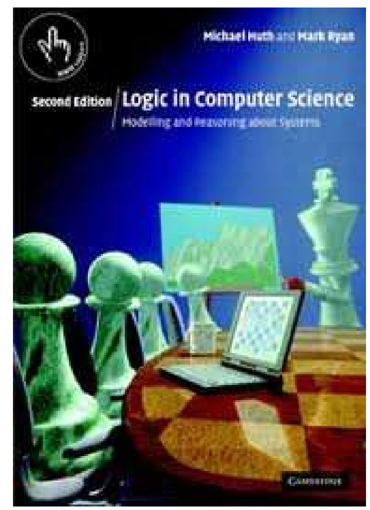
Praktische Informatie
PDF: A Brief History
1.1, 1.3: Propositions

Je moet de bal hebben om te schieten, en schieten om te scoren, maar dat is logisch.

Praktische Informatie

- hoorcollege: maandag, 11.00–12.45 (zaal 402)
(verplicht) werkcollege (Ruben Turkenburg): donderdag, 13.30–15.15 (zaal 408)
van maandag 3 september - donderdag 13 december 2018

- boek: Michael Huth & Mark Ryan:
Logic in Computer Science:
Modelling and Reasoning about Systems



- hoofdstuk 1: Propositional logic
- hoofdstuk 2: Predicate logic

- plus . . .

Praktische Informatie

- zelfde inhoud als Logica

- Engels vs. Nederlands

- 6 EC

- tentamens: donderdag 3 januari 2019, 10.00–13.00
donderdag 14 maart 2019, 10.00–13.00

Praktische Informatie

- Vijf huiswerkopgaven (individueel)

Niet verplicht, maar . . .
Algoritme om eindcijfer te berekenen:

```
cijferhuiswerkopgaven = gemiddelde van beste vier huiswerkopgaven

if (tentamencijfer >= 5.5)
  eindcijfer = max (6.0,
              70% * tentamencijfer + 30% * cijferhuiswerkopgaven)
else
  eindcijfer = tentamencijfer;
```

Cijfers van eerdere jaren niet meer geldig

Praktische Informatie

Website

`http://liacs.leidenuniv.nl/~vlietrvan1/logica/`

- slides
- overzicht van behandelde stof
- huiswerkopgaven

Logic

1. The ability to determine correct answers through a standardized process.

2. The study of formal inference.

3. A sequence of verified statements

4. Reasoning, as opposed to intuition.

5. The deduction of statements from a set of statements.

The First Age of Logic: Symbolic Logic

Sophists. . .

- All men are mortal.
- Socrates is a man.
- Therefore, Socrates is mortal.

‘All’ $\rightarrow$ ‘Some’. . .

Natural Language

Ambiguity

- Eric does not believe that Mary can pass any test.
- I only borrowed your car.

Paradoxes

- This sentence is a lie.
- The surprise paradox.

Therefore, logic in symbolic language

The Second Age of Logic: Algebraic Logic

- 1847, Boole: logic in terms of mathematical language
- Lewis Carol: Venn diagrams
- Fast algorithms

The Third Age of Logic: Mathematical Logic

- Paradox in mathematics

- Logic as language for mathematics

- Cantor: infinity

The Set $2^{\mathbb{N}}$ Is Uncountable

No list of subsets of $\mathbb{N}$ is complete,
i.e., every list $A_0, A_1, A_2, \ldots$ of subsets of $\mathbb{N}$ leaves out at least one.

The Set $2^{\mathbb{N}}$ Is Uncountable (continued)

$$A = \{i \in \mathbb{N} \mid i \notin A_i\}$$

$$\begin{aligned}
A_0 &= \{0, 2, 5, 9, \ldots\} \\
A_1 &= \{1, 2, 3, 8, 12, \ldots\} \\
A_2 &= \{0, 3, 6\} \\
A_3 &= \emptyset \\
A_4 &= \{4\} \\
A_5 &= \{2, 3, 5, 7, 11, \ldots\} \\
A_6 &= \{8, 16, 24, \ldots\} \\
A_7 &= \mathbb{N} \\
A_8 &= \{1, 3, 5, 7, 9, \ldots\} \\
A_9 &= \{n \in \mathbb{N} \mid n > 12\} \\
&\ldots
\end{aligned}$$

	0	1	2	3	4	5	6	7	8	9	...
$A_0 = \{0, 2, 5, 9, \ldots\}$	1	0	1	0	0	1	0	0	0	1	...
$A_1 = \{1, 2, 3, 8, 12, \ldots\}$	0	1	1	1	0	0	0	0	1	0	...
$A_2 = \{0, 3, 6\}$	1	0	0	1	0	0	1	0	0	0	...
$A_3 = \emptyset$	0	0	0	0	0	0	0	0	0	0	...
$A_4 = \{4\}$	0	0	0	0	1	0	0	0	0	0	...
$A_5 = \{2, 3, 5, 7, 11, \ldots\}$	0	0	1	1	0	1	0	1	0	0	...
$A_6 = \{8, 16, 24, \ldots\}$	0	0	0	0	0	0	0	0	1	0	...
$A_7 = \mathbb{N}$	1	1	1	1	1	1	1	1	1	1	...
$A_8 = \{1, 3, 5, 7, 9, \ldots\}$	0	1	0	1	0	1	0	1	0	1	...
$A_9 = \{n \in \mathbb{N} \mid n > 12\}$	0	0	0	0	0	0	0	0	0	0	...
...						...					

	0	1	2	3	4	5	6	7	8	9	...
$A_0 = \{0, 2, 5, 9, \ldots\}$	1	0	1	0	0	1	0	0	0	1	...
$A_1 = \{1, 2, 3, 8, 12, \ldots\}$	0	1	1	1	0	0	0	0	1	0	...
$A_2 = \{0, 3, 6\}$	1	0	0	1	0	0	1	0	0	0	...
$A_3 = \emptyset$	0	0	0	0	0	0	0	0	0	0	...
$A_4 = \{4\}$	0	0	0	0	1	0	0	0	0	0	...
$A_5 = \{2, 3, 5, 7, 11, \ldots\}$	0	0	1	1	0	1	0	1	0	0	...
$A_6 = \{8, 16, 24, \ldots\}$	0	0	0	0	0	0	0	0	1	0	...
$A_7 = \mathbb{N}$	1	1	1	1	1	1	1	1	1	1	...
$A_8 = \{1, 3, 5, 7, 9, \ldots\}$	0	1	0	1	0	1	0	1	0	1	...
$A_9 = \{n \in \mathbb{N} \mid n > 12\}$	0	0	0	0	0	0	0	0	0	0	...
...						...					
$A = \{2, 3, 6, 8, 9, \ldots\}$	0	0	1	1	0	0	1	0	1	1	...

Hence, there are uncountably many subsets of $\mathbb{N}$.

The Third Age of Logic: Mathematical Logic

- Hilbert: devise single logical formalism to derive all mathematical truth

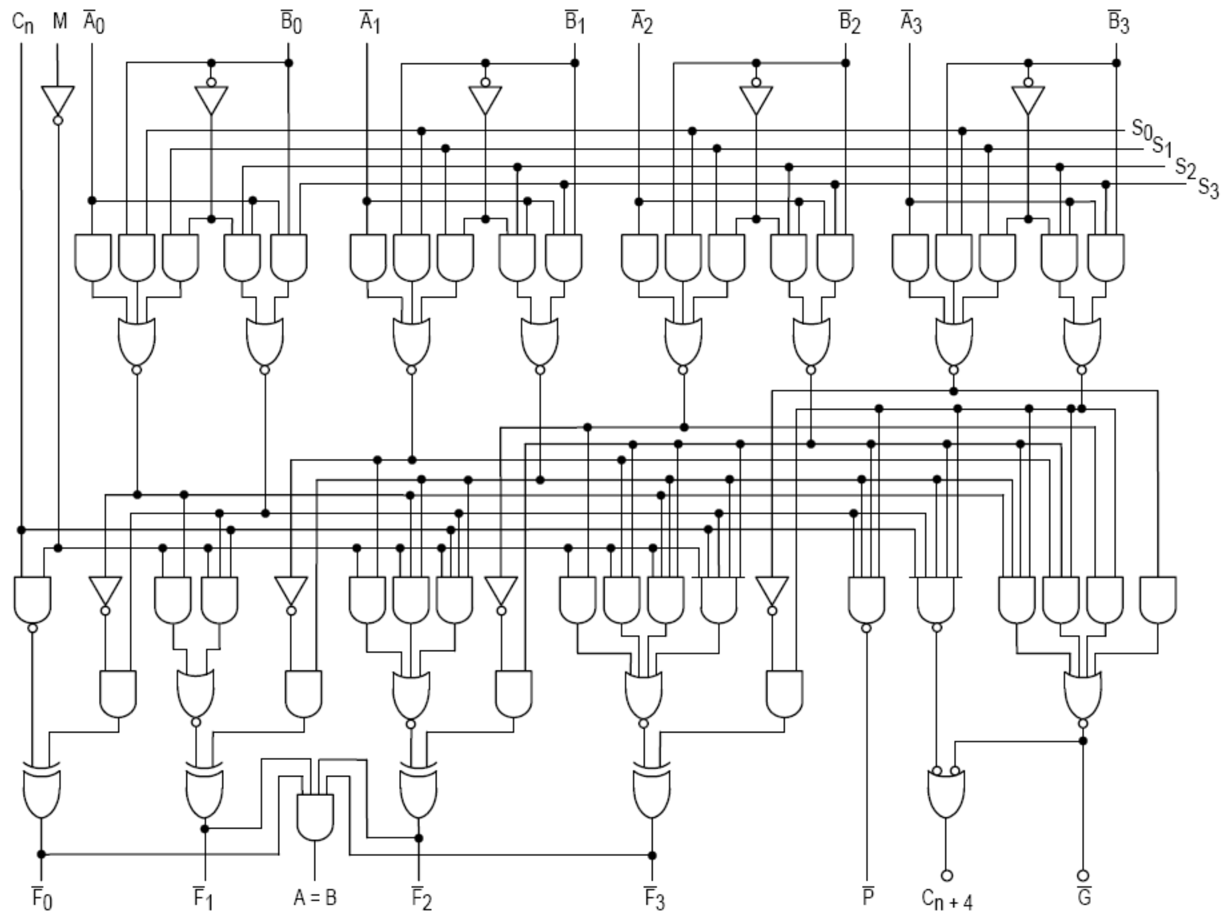
- Russell: paradox in set theory

- Gödel: incompleteness theorems

- Church and Turing: unsolvable problems

The Fourth Age of Logic: Logic in Computer Science

- Boolean circuits



The Fourth Age of Logic: Logic in Computer Science

- NP-completeness
- SQL $\equiv$ first-order logic
- Formal semantics of programming languages
- Design validation and verification: temporal logic
- Expert systems in AI
- Security

1. Propositional logic

Example 1.1. If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late.
Therefore, . . .

Propositional logic

Example 1.1. If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late.
Therefore, there were taxis at the station.

Propositional logic

Example 1.2. If it is raining and Jane does not have her umbrella with her, then she will get wet. Jane is not wet. It is raining. *Therefore,*

Propositional logic

Example 1.2. If it is raining and Jane does not have her umbrella with her, then she will get wet. Jane is not wet. It is raining. *Therefore,* Jane has her umbrella with her.

Propositional logic

Example 1.1. If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late.
Therefore, there were taxis at the station.

Example 1.2. If it is raining and Jane does not have her umbrella with her, then she will get wet. Jane is not wet. It is raining.
Therefore, Jane has her umbrella with her.

General structure: . . .

Propositional logic

Example 1.1. If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late.
Therefore, there were taxis at the station.

Example 1.2. If it is raining and Jane does not have her umbrella with her, then she will get wet. Jane is not wet. It is raining.
Therefore, Jane has her umbrella with her.

General structure:
If p and not q, then r. Not r. p. *Therefore,* q.

1.1. Declarative sentences

Proposition = declarative sentence

$\{$ true, false $\}$

1.1. Declarative sentences

(2) Jane reacted violently to Jack's accusations.

(3) Every even natural number > 2 is the sum of two prime numbers.

(4) All Martians like pepperoni on their pizza.

(5) Albert Camus etait un écrivain francais.

(6) Within five years, Feyenoord will be champion of the Eredivisie again.

1.1. Declarative sentences

Non-declarative:

- Could you please pass me the salt?
- May fortune come your way.

Reasoning about computer programs

Building up sentences

Atomic = indecomposable sentences

- p: I won the lottery last week.
- q: I purchased a lottery ticket.
- r: I won last week's sweepstakes.

Rules:

- $\neg p$, *negation*
- $p \vee r$, *disjunction* (is not XOR)
- $p \wedge r$, *conjunction*
- $p \rightarrow q$ *implication*, *assumption* and *conclusion*

Binding priorities

$p \wedge q \to \neg r \vee q$ means . . .

Binding priorities

$p \wedge q \to \neg r \vee q$ means $(p \wedge q) \to ((\neg r) \vee q)$

Convention 1.3. $\neg$ binds more tightly than $\vee$ and $\wedge$, and the latter two bind more tightly than $\to$.
Implication $\to$ is *right associative*: . . .

Binding priorities

$p \wedge q \rightarrow \neg r \vee q$ means $(p \wedge q) \rightarrow ((\neg r) \vee q)$

Convention 1.3. $\neg$ binds more tightly than $\vee$ and $\wedge$, and the latter two bind more tightly than $\rightarrow$.
Implication $\rightarrow$ is *right associative*: expressions of the form $p \rightarrow q \rightarrow r$ denote $p \rightarrow (q \rightarrow r)$.

1.3. Propositional logic as a formal language

Well-formed formula built up of

$\{p, q, r, \ldots\} \cup \{p_1, p_2, p_3, \ldots\} \cup \{\neg, \wedge, \vee, \rightarrow, (,)\}$

$(\neg)() \vee pq \rightarrow$

Definition 1.27. The well-formed formulas of propositional logic are those which we obtain by using the construction rules below, and only those, finitely many times:

atom: Every propositional atom $p, q, r, \ldots$ and $p_1, p_2, p_3, \ldots$ is a well-formed formula.

$\neg$: if ϕ is a well-formed formula, then so is $(\neg\phi)$.

$\wedge$: if ϕ and ψ are well-formed formulas, then so is $(\phi \wedge \psi)$.

$\vee$: if ϕ and ψ are well-formed formulas, then so is $(\phi \vee \psi)$.

$\rightarrow$: if ϕ and ψ are well-formed formulas, then so is $(\phi \rightarrow \psi)$.

Backus Naur Form

(notation context free grammar)

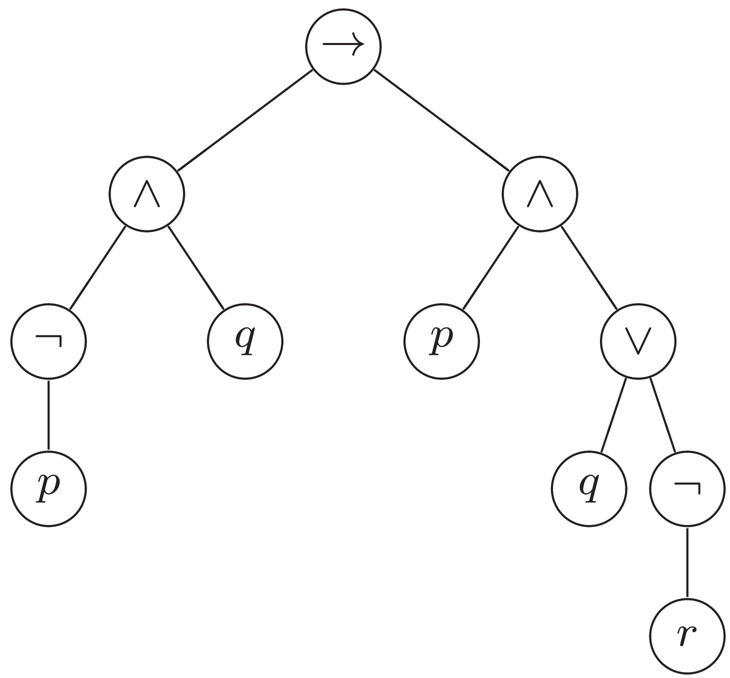
$$\phi ::= p \;\mid\; (\neg\phi) \;\mid\; (\phi \wedge \phi) \;\mid\; (\phi \vee \phi) \;\mid\; (\phi \rightarrow \phi)$$

ϕ, ϕ

$$(((\neg p) \wedge q) \rightarrow (p \wedge (q \vee (\neg r))))$$

Parse tree

$(((\neg p) \wedge q) \rightarrow (p \wedge (q \vee (\neg r))))$

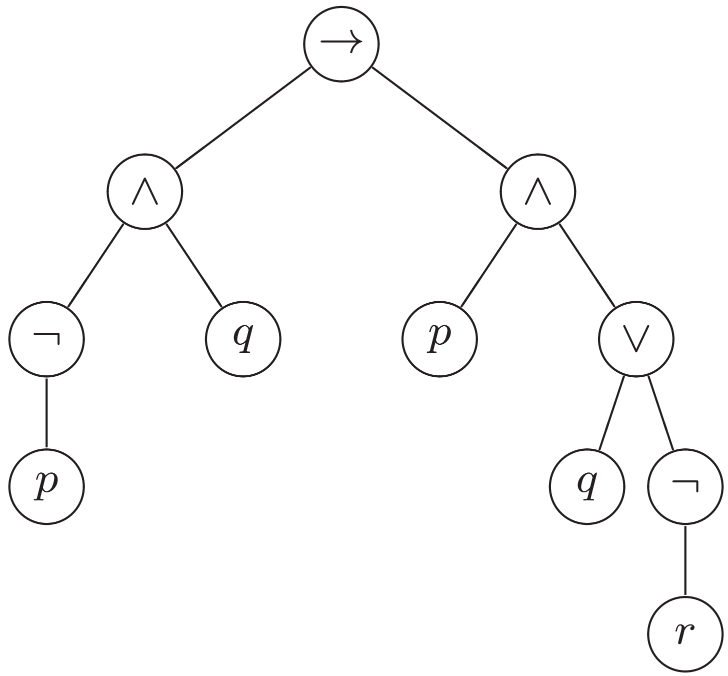


'Inorder' walk

$$(((\neg p) \wedge q) \rightarrow (p \wedge (q \vee (\neg r))))$$

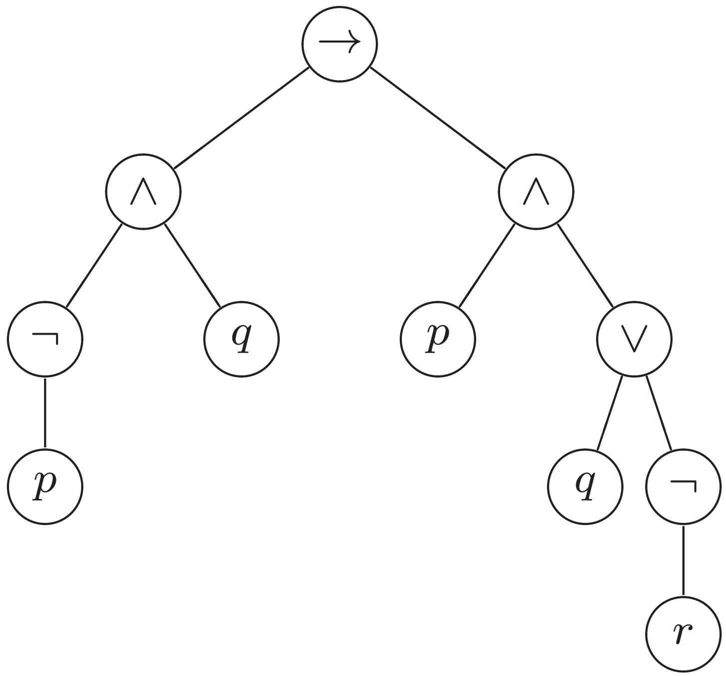
Subformulas. . .

$$(((\neg p) \wedge q) \rightarrow (p \wedge (q \vee (\neg r))))$$



Subformulas. . .

$$(((\neg p) \land q) \to (p \land (q \lor (\neg r))))$$



Subformulas

p

q

r

$(\neg p)$

$((\neg p) \land q)$

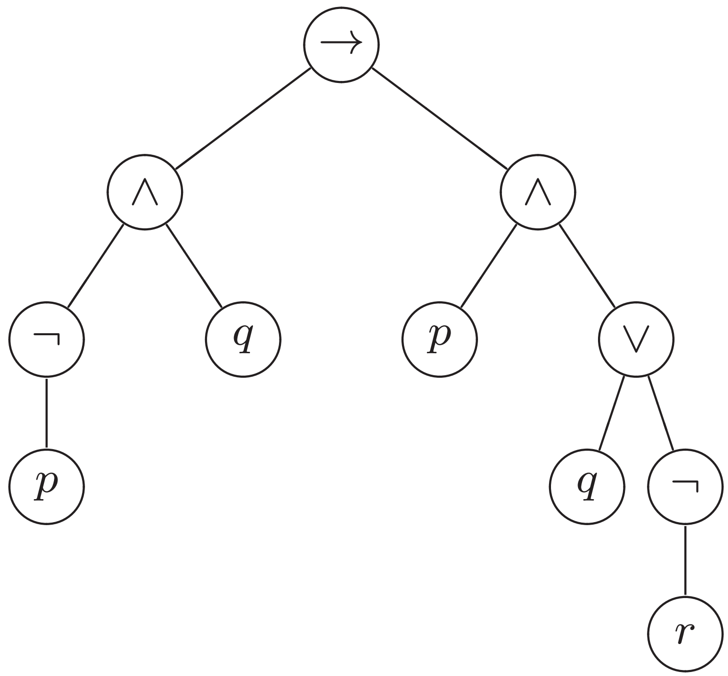
$(\neg r)$

$(q \lor (\neg r))$

$(p \land (q \lor (\neg r)))$

$(((\neg p) \land q) \to (p \land (q \lor (\neg r))))$

Requirements on tree



1.4. Semantics of propositional logic

Definition 1.28.
1. The set of truth values contains two elements T and F, where T represents 'true' and F represents 'false'.
2. A *valuation* of *model* of a formula ϕ is an assignment of each propositional atom in ϕ to a truth value.

1.4. Semantics of propositional logic

Definition 1.28.
1. The set of truth values contains two elements T and F, where T represents 'true' and F represents 'false'.
2. A *valuation* of *model* of a formula ϕ is an assignment of each propositional atom in ϕ to a truth value.

Example 1.29. $\phi = p \lor \neg q$
p: F
q: T

Truth table for conjunction

ϕ	ψ	$\phi \wedge \psi$
$\ldots$	$\ldots$	$\ldots$

Truth table for conjunction

ϕ	ψ	$\phi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F