LIACS — Leiden University

Examination Logic — 19 December 2016, 10:00 - 13:00

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1. **[0,5 point]** Show that $p \vee q, \neg p \vee \neg q \models q \to \neg p$ using a truth table.

2. **[1,5 points]** Give a proof by means of natural deduction of the following sequents:
   a) $p \to q \vdash \neg p \vee q$
   b) $(p \to r) \vee (q \to r) \vdash (p \wedge q) \to r$
   c) $p \to \neg p, \neg p \to p \vdash \bot$

3. **[1,5 points]** Use mathematical induction to prove that $1 + 2^2 + ... + 2^{n-1} = 2^n - 3$ for all integers $n \geq 3$.

4. **[1,5 points]** Find which of the following formula is valid by computing the conjunctive normal form. Explain your answer.
   a) $(p \wedge \neg q) \vee (p \wedge q)$.
   b) $\neg(p \wedge \neg q) \wedge (q \vee \neg p)$.
   c) $((p \to q) \vee p) \wedge (p \vee \neg(r \wedge \neg r \wedge q))$.

5. **[1,5 points]** Apply the marking algorithm to check if the following Horn formulas are satisfiable:
   a) $(\top \to q) \wedge ((p \wedge q) \to r) \wedge (q \to p)$.
   b) $(\top \to p) \wedge ((p \wedge q) \to r) \wedge (p \to q) \wedge ((r \wedge p) \to q)$.
   c) $(\top \to p) \wedge (p \to q) \wedge ((p \wedge q) \to r) \wedge (q \to \bot) \wedge (\top \to r)$.

6. **[2 points]** Show the validity by means of natural deduction of the following sequents:
   a) $\forall x\, P(x) \vdash P(a) \to P(b)$.
   b) $a = b \wedge \neg P(a,b) \vdash \neg \forall x\, P(x,x)$.
   c) $\vdash \forall x \forall y (x = x \vee x = y)$.
   d) $\vdash \neg \exists x \neg (x = x)$.

7. **[1,5 points]** Consider the predicate formula $\forall x \exists y\, (P(x,y) \to f(x,c) = y)$, where c is a constant, P is a binary predicate and f is a binary function. Find a model which makes the formula true.

The final score is given by the sum of the points obtained.