LIACS
Leiden University

Examination Logic
15 June 2015, 14:00 - 17:00

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1. **[1 point]** Draw the parse tree of the formula $p \wedge ((q \rightarrow \neg\neg p) \rightarrow \neg(q \vee p))$ and list *all* its subformulas.

2. **[2 points]** Give a proof in natural deduction for each of the following sequents:
   a) $\neg p \vee q, \neg p \rightarrow q \vdash q$
   b) $p \rightarrow (\neg p \wedge q) \vdash \neg p$
   c) $p \wedge q, \neg(p \wedge r) \vdash p \wedge \neg r$
   d) $\vdash (p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q))$

3. **[1,5 points]** Apply the marking algorithm to find a valuation witness for the satisfiability of the following Horn formulas:
   a) $(\top \rightarrow p) \wedge (p \wedge q \rightarrow r) \wedge (p \rightarrow q) \wedge (q \wedge r \rightarrow s) \wedge (r \rightarrow \bot)$
   b) $(p \wedge q \rightarrow r) \wedge (q \rightarrow p) \wedge (r \wedge p \rightarrow q) \wedge (r \rightarrow s) \wedge (\top \rightarrow p)$
   c) $(p \wedge q \wedge r \rightarrow s) \wedge (p \wedge q \rightarrow r) \wedge (r \rightarrow q) \wedge (p \rightarrow \bot) \wedge (\top \rightarrow r)$

4. **[1 point]** Draw the DAG corresponding to the formula $\neg(\neg q \wedge \neg p) \wedge ((p \wedge \neg q) \wedge \neg q)$ and use a SAT solver to give a witness for its satisfiability.

5. **[1,5 points]** Let ϕ be the formula $\exists x(x=y \rightarrow \forall y(y=z \wedge x=z))$ where x,y,z are three variables. Draw the parse tree of ϕ and compute, when possible, the following substitutions:
   - $\phi[f(v)/y]$
   - $\phi[f(y)/y]$
   - $\phi[f(v)/z]$.

   Here f is a function symbol of arity 1 and v is a variable.

6. **[1 points]** Find a model for each of the following sequent showing that it is not valid.
   a) $\exists x P(x), \exists x Q(x) \vdash \exists x(P(x) \wedge Q(x))$, where P and Q are predicates of arity 1.
   b) $\forall x \forall y(\neg x=y \rightarrow (P(x) \wedge P(y))) \vdash \forall x P(x)$, where P is a predicate of arity 1.

7. **[2 points]** Show the validity of each of the following sequent by means of a proof in natural deduction, where P, Q, are predicates of arity 1, and R is a predicate of arity 2:
   a) $\forall x \forall y(x=y \rightarrow R(x,y)) \vdash \forall x R(x,x)$,
   b) $\exists x(P(x) \wedge \neg Q(x)), \exists x(\neg P(x) \wedge Q(x)) \vdash \exists x \exists y(P(x) \wedge Q(y))$.

The final score is given by the sum of the points obtained.