Fundamentele Informatica 3

voorjaar 2019

Rudy van Vliet

kamer 140 Snellius, tel. 071-527 2876

rvvliet(at)liacs(dot)nl

college 3, 18 februari 2019

7. Turing Machines

7.4. Combining Turing Machines

7.5. Multitape Turing Machines

7.7. Nondeterministic Turing Machines

7.4. Combining Turing Machines
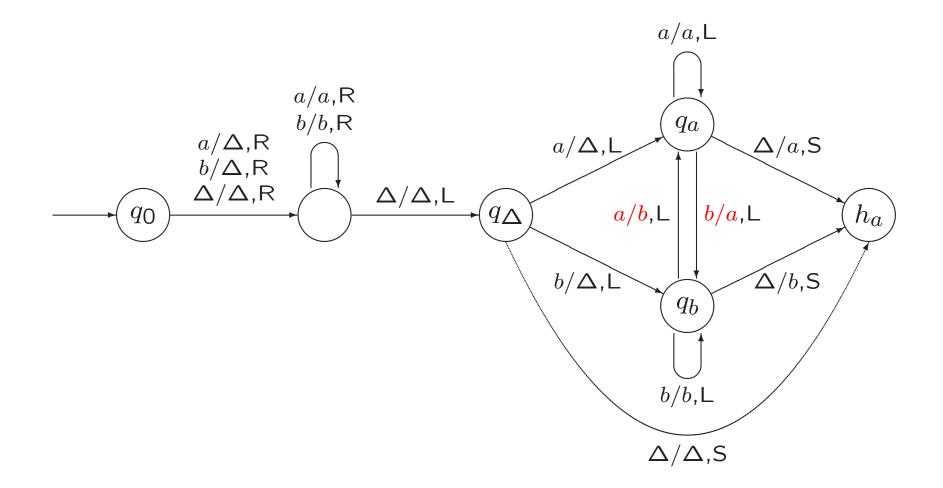
Example.

A TM for $f(x) = a^{n_a(x)}$

$x = aababba$

Example.

A TM for $f(x) = a^{n_a(x)}$

$x = aababba$

$$\underline{\triangle}\ a\ a\ b\ a\ b\ b\ a$$
$$\triangle\ a\ a\ \underline{\triangle}\ a\ b\ b\ a$$
$$\triangle\ a\ a\ \underline{a}\ b\ b\ a\ \triangle$$
$$\triangle\ a\ a\ a\ \underline{\triangle}\ b\ a\ \triangle$$
$$\triangle\ a\ a\ a\ \underline{b}\ a\ \triangle\triangle$$
$$\triangle\ a\ a\ a\ \underline{\triangle}\ a\ \triangle\triangle$$
$$\triangle\ a\ a\ a\ \underline{a}\ \triangle\triangle\triangle$$
$$\underline{\triangle}\ a\ a\ a\ a\ \triangle\triangle\triangle$$

a/a,L

a/a,R
b/b,R

a/Δ,R
b/Δ,R
Δ/Δ,R

Δ/Δ,L

q_0

q_Δ

a/Δ,L

q_a

Δ/a,S

a/b,L b/a,L

h_a

b/Δ,L

q_b

Δ/b,S

b/b,L

Δ/Δ,S

4

Example 7.20. Inserting and Deleting a Symbol

Delete: from $y\underline{\sigma}z$ to $y\underline{z}$

Insert(σ): from $y\underline{z}$ to $y\underline{\sigma}z$

N.B.: z does not contain blanks

TM T_1 computes f

TM T_2 computes g

TM T_1T_2 computes . . .

$$T_1 \longrightarrow T_2$$

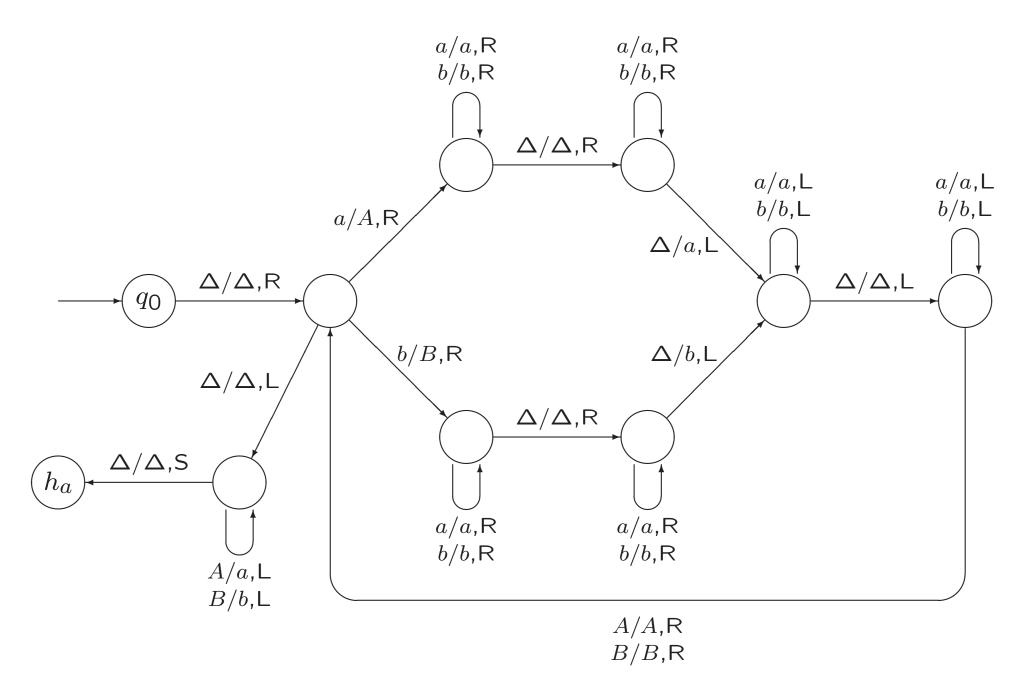**Example 7.17.** Finding the Next Blank or the Previous Blank

NB

PB

Example 7.18. Copying a String

Copy: from $\underline{\triangle}x$ to $\underline{\triangle}x\triangle x$

$x = abaa$

A slide from lecture 2

Example 7.10. The Reverse of a String

$$
\begin{aligned}
&\underline{\triangle}\,a\,a\,b\,a\,b \\
&\triangle A\,a\,b\,a\,b \\
&\triangle A\,a\,b\,a\,A \\
&\triangle B\,a\,b\,a\,A \\
&\triangle B A\,b\,a\,A \\
&\triangle B A\,b\,A A \\
&\triangle B A\,b\,A A \\
&\triangle B A B A A \\
&\underline{\triangle}\,b\,a\,b\,a\,a
\end{aligned}
$$

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\triangle}x\triangle y$ if $x = y$,
    and reject if $x \neq y$

Exercise 7.17.

For each case below, draw a TM that computes the indicated function.

e. $E : \{a, b\}^* \times \{a, b\}^* \to \{0, 1\}$
   defined by $E(x, y) = 1$ if $x = y$, $\qquad E(x, y) = 0$ otherwise.

Example 7.25. Accepting the Language of ...

$$Copy \rightarrow NB \rightarrow R \rightarrow PB \rightarrow Equal$$

Example 7.25. Accepting the Language of Palindromes

$$Copy \rightarrow NB \rightarrow R \rightarrow PB \rightarrow Equal$$
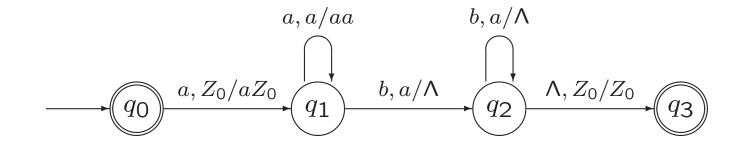
Example 7.21. Erasing the Tape

From the current position to the right

Many notations for composition

7.5. Multitape Turing Machines

Example 5.3. A PDA Accepting the Language *AnBn*

$$AnBn = \{a^i b^i \mid i \geq 0\}$$

Part of a slide from exercise class 1

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

With two tapes. . .

Exercise 7.4.

For each of the following languages, draw a transition diagram
for a Turing machine that accepts that language.

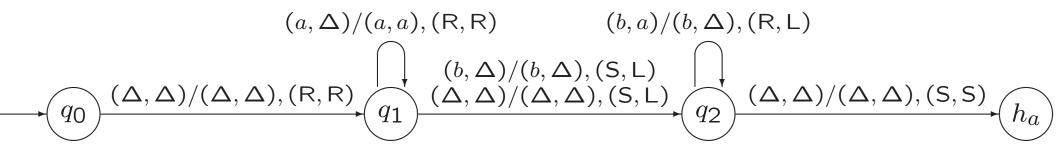**a.** $AnBn = \{a^i b^i \mid i \geq 0\}$

With two tapes:

$$(a, \Delta)/(a, a), (\mathsf{R}, \mathsf{R}) \qquad (b, a)/(b, \Delta), (\mathsf{R}, \mathsf{L})$$

$$(b, \Delta)/(b, \Delta), (\mathsf{S}, \mathsf{L})$$

$q_0 \xrightarrow{(\Delta, \Delta)/(\Delta, \Delta), (\mathsf{R}, \mathsf{R})} q_1 \xrightarrow{(\Delta, \Delta)/(\Delta, \Delta), (\mathsf{S}, \mathsf{L})} q_2 \xrightarrow{(\Delta, \Delta)/(\Delta, \Delta), (\mathsf{S}, \mathsf{S})} h_a$$

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

We could also use the portion of the tape to the right of the input, to simulate the stack of a deterministic pushdown automaton (works for any deterministic PDA!)

21

Example 7.24. Comparing Two Strings

Equal: accept $\triangle x \triangle y$ if $x = y$,
          and reject if $x \neq y$

2-tape TM. . .

Theorem 7.26. (informal)

For every 2-tape TM T, there is an ordinary 1-tape TM T_1, which for every input x,

- simulates the computation of T for x,

- accepts (rejects) x, if and only if T accepts (rejects) x,

- on acceptance, leaves the same output on its tape as T leaves on its first tape.

The proof of this result does not have to be known for the exam.

Corollary 7.27.

Every language that is accepted by a 2-tape TM can be accepted by an ordinary 1-tape TM,
and every function that is computed by a 2-tape TM can be computed by an ordinary TM.

This generalizes to k-tape TMs for $k \geq 3$.

7.7. Nondeterministic Turing Machines

A slide from lecture 2

Definition 7.1. Turing machines

A Turing machine (TM) is a 5-tuple $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where

Q is a finite set of states. The two *halt* states h_a and h_r are not elements of Q.

Σ, the input alphabet, and Γ, the tape alphabet, are both finite sets, with $\Sigma \subseteq \Gamma$. The *blank* symbol Δ is not an element of Γ.

q_0, the initial state, is an element of Q.

δ is the transition function:

$$\delta : Q \times (\Gamma \cup \{\Delta\}) \to (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\}) \times \{R, L, S\}$$

Nondeterministic Turing machine.

There may be more than one move for a state-symbol pair.

Same notation:

$$wpax \vdash_T yqbz \qquad wpax \vdash_T^* yqbz$$

A string x is accepted by T if

$$q_0 \Delta x \vdash_T^* w h_a y$$

for some strings $w, y \in (\Gamma \cup \{\Delta\})^*$.

NTM useful for accepting languages, for producing output, but not for computing function.

Example 7.28. The Set of Composite Natural Numbers.

Use *G2*

Example 7.28. The Set of Composite Natural Numbers.

$$NB \rightarrow G2 \rightarrow NB \rightarrow G2 \rightarrow PB \rightarrow M \rightarrow PB \rightarrow Equal$$

Take $x = 1^{15}$

Example 7.30. The Language of Prefixes of Elements of L.

Let $L = L(T)$. Then
$$P(L) = \{x \in \Sigma^* \mid xy \in L \text{ for some } y \in \Sigma^*\}$$

Example 7.30. The Language of Prefixes of Elements of L.

Let $L = L(T)$. Then

$$P(L) = \{x \in \Sigma^* \mid xy \in L \text{ for some } y \in \Sigma^*\}$$

Deterministic TM accepting $P(L)$ may execute following algorithm for input x:

$y = \Lambda$;
while (T does not accept xy)
    y is next string in Σ^* (in canonical order);
accept;

but. . .

Example 7.30. The Language of Prefixes of Elements of L.

Let $L = L(T)$. Then

$$P(L) = \{x \in \Sigma^* \mid xy \in L \text{ for some } y \in \Sigma^*\}$$

$$NB \rightarrow G \rightarrow Delete \rightarrow PB \rightarrow T$$