Some more solutions to exercises in

John C. Martin: Introduction to Languages and The Theory of Computation

fourth edition

3.51(a)

$r^0(i,j)$	$j=1$	2	3
$i=1$	Λ	b	a
2	a	Λ	b
3	b	a	Λ

$r^1(i,j)$	$j=1$	2	3
$i=1$	$\Lambda+\Lambda\Lambda^*\Lambda$	$b+\Lambda\Lambda^*b$	$a+\Lambda\Lambda^*a$
2	$a+a\Lambda^*\Lambda$	$\Lambda+a\Lambda^*b$	$b+a\Lambda^*a$
3	$b+b\Lambda^*\Lambda$	$a+b\Lambda^*b$	$\Lambda+b\Lambda^*a$

$r^1(i,j)$ simplified	$j=1$	2	3
$i=1$	Λ	b	a
2	a	$\Lambda+ab$	$b+aa$
3	b	$a+bb$	$\Lambda+ba$

$r^2(i,j)$	$j=1$	2	3
$i=1$	$\Lambda+b(\Lambda+ab)^*a$	$b+b(\Lambda+ab)^*(\Lambda+ab)$	$a+b(\Lambda+ab)^*(b+aa)$
2	$a+(\Lambda+ab)(\Lambda+ab)^*a$	$\Lambda+ab+(\Lambda+ab)(\Lambda+ab)^*(\Lambda+ab)$	$b+aa+(\Lambda+ab)(\Lambda+ab)^*(b+aa)$
3	$b+(a+bb)(\Lambda+ab)^*a$	$a+bb+(a+bb)(\Lambda+ab)^*(\Lambda+ab)$	$\Lambda+ba+(a+bb)(\Lambda+ab)^*(b+aa)$

$r^2(i,j)$ simplified	$j=1$	2	3
$i=1$	$\Lambda+b(ab)^*a$	$b(ab)^*$	$a+b(ab)^*(b+aa)$
2	$(ab)^*a$	$(ab)^*$	$(ab)^*(b+aa)$
3	$b+(a+bb)(ab)^*a$	$(a+bb)(ab)^*$	$\Lambda+ba+(a+bb)(ab)^*(b+aa)$

$$\begin{aligned} r^3(1,3) &= a+b(ab)^*(b+aa) \\ &\quad +(a+b(ab)^*(b+aa))(\Lambda+ba+(a+bb)(ab)^*(b+aa))^*(\Lambda+ba+(a+bb)(ab)^*(b+aa)) \\ &= (a+b(ab)^*(b+aa))(ba+(a+bb)(ab)^*(b+aa))^* \end{aligned}$$

For simplifying the expressions we used the following equalities, for arbitrary regular expressions r, r_1, r_2:

$$\begin{aligned} \Lambda^* &= \Lambda \\ \Lambda r &= r\Lambda = r \\ r+r &= r \\ (\Lambda+r)^* &= r^* \\ r^*(\Lambda+r) &= (\Lambda+r)r^* = r^* \\ r_1+r_1(r_2)^* &= r_1(r_2)^* \\ r_1+(r_2)^*r_1 &= (r_2)^*r_1 \\ \Lambda+r^* &= r^* \\ r+r^* &= r^* \end{aligned}$$