

### Exercise 3.21.

Consider the following transition table for an NFA with states 1–5, initial state 1 and input alphabet  $\{a, b\}$ . There are no  $\Lambda$ -transitions:

$q$	$\delta(q, a)$	$\delta(q, b)$
1	$\{1, 2\}$	$\{1\}$
2	$\{3\}$	$\{3\}$
3	$\{4\}$	$\{4\}$
4	$\{5\}$	$\emptyset$
5	$\emptyset$	$\{5\}$

**a.** Draw a transition diagram of the NFA (note that the accepting states are not specified).

**b.** Calculate  $\delta^*(1, ab)$ .

Hint: first calculate  $\delta^*(1, \Lambda)$ , then  $\delta^*(1, a)$ , then  $\delta^*(1, ab)$ .

**c.** Calculate  $\delta^*(1, abaab)$ .

### Exercise 3.24.

Let  $M = (Q, \Sigma, q_0, A, \delta)$  be an NFA with no  $\Lambda$ -transitions. Show that for every  $q \in Q$  and every  $\sigma \in \Sigma$ ,  $\delta^*(q, \sigma) = \delta(q, \sigma)$ .

### Exercise 3.33.

Given an example of a regular language  $L$  containing  $\Lambda$  that cannot be accepted by any NFA having only one accepting state and no  $\Lambda$ -transitions, and show that your answer is correct.

### Exercise 3.22.

A transition table is given for an NFA with seven states.

$q$	$\delta(q, a)$	$\delta(q, b)$	$\delta(q, \Lambda)$
1	$\emptyset$	$\emptyset$	$\{2\}$
2	$\{3\}$	$\emptyset$	$\{5\}$
3	$\emptyset$	$\{4\}$	$\emptyset$
4	$\{4\}$	$\emptyset$	$\{1\}$
5	$\emptyset$	$\{6, 7\}$	$\emptyset$
6	$\{5\}$	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	$\{1\}$

Find:

**d.**  $\delta^*(1, ba)$

Hint: first calculate  $\delta^*(1, \Lambda)$ , then  $\delta^*(1, b)$ , then  $\delta^*(1, ba)$ .

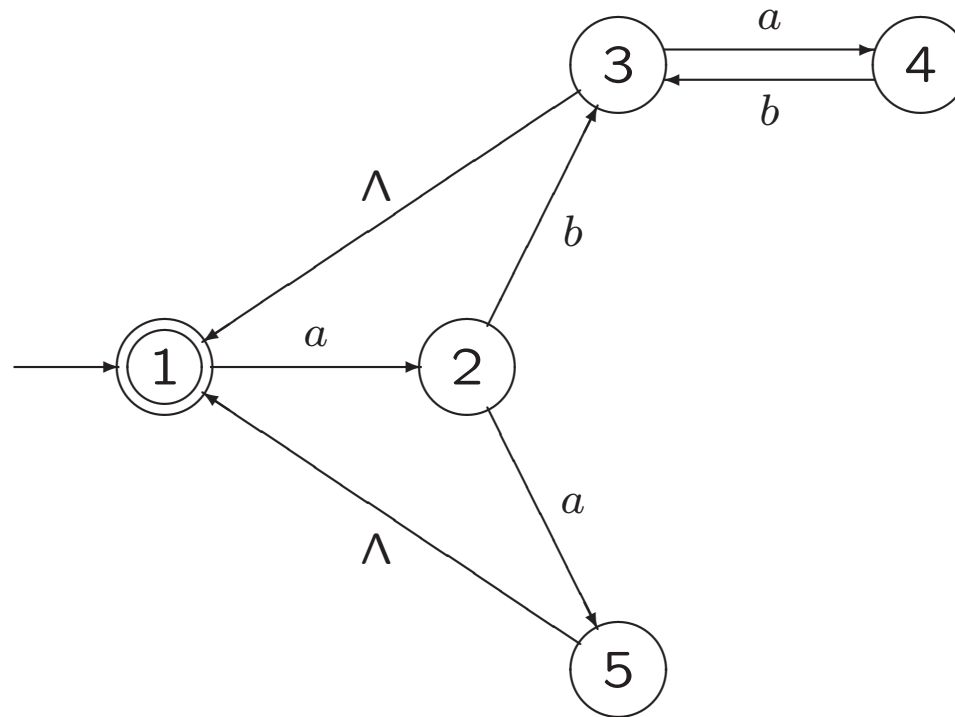
**e.**  $\delta^*(1, ab)$

**f.**  $\delta^*(1, ababa)$

### Exercise 3.37.

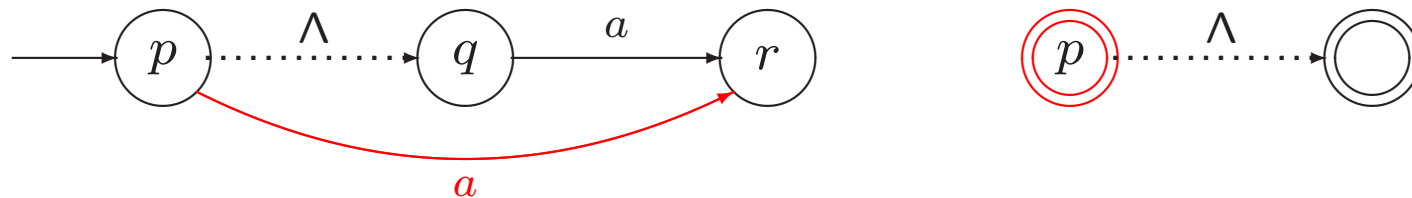
For each part below, use the algorithm from the lecture to draw an NFA with no  $\Lambda$ -transitions accepting the same language as the NFA pictured.

b.



## Exercise.

Our construction:



## $\Lambda$ -removal

Given NFA  $M = (Q, \Sigma, \delta, q_0, A)$ ,

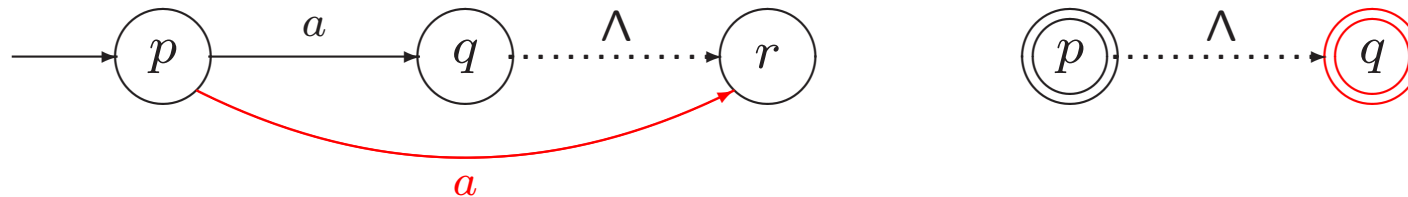
construct NFA  $M_1 = (Q, \Sigma, \delta_1, q_0, A_1)$  without  $\Lambda$ -transitions:

- whenever  $q \in \Lambda_M(\{p\})$  and  $r \in \delta(q, a)$ , add  $r$  to  $\delta_1(p, a)$
- whenever  $\Lambda_M(\{p\}) \cap A \neq \emptyset$ , add  $p$  to  $A_1$ .

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## Exercise. (ctd.)

Is it possible to invert the construction:



### $\Lambda$ -removal

Given NFA  $M = (Q, \Sigma, \delta, q_0, A)$ ,

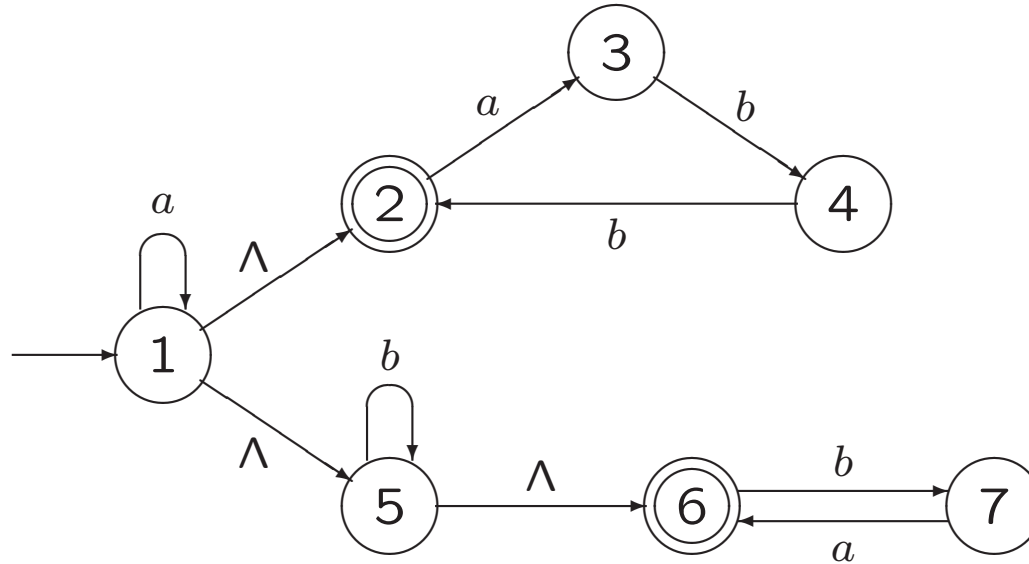
construct NFA  $M_1 = (Q, \Sigma, \delta_1, q_0, A_1)$  without  $\Lambda$ -transitions:

- whenever  $q \in \delta(p, a)$  and  $r \in \Lambda_M(\{q\})$ , add  $r$  to  $\delta_1(p, a)$
- whenever  $p \in A$  and  $q \in \Lambda_M(\{p\})$ , add  $q$  to  $A_1$ .

### Exercise 3.40.

For each part below, draw an FA accepting the same language as the NFA shown.

a.





### Exercise 3.32.

Let  $M = (Q, \Sigma, q_0, A, \delta)$  be an NFA accepting a language  $L$ . Assume that there are no transitions to  $q_0$ , that  $A$  has only one element,  $q_f$ , and that there are not transitions from  $q_f$ .

**a.** Let  $M_1$  be obtained from  $M$  by adding  $\Lambda$ -transitions from  $q_0$  to every state that is reachable from  $q_0$  in  $M$ .

(If  $p$  and  $q$  are states,  $q$  is reachable from  $p$  if there is a string  $x \in \Sigma^*$  such that  $q \in \delta^*(p, x)$ .)

Describe (in terms of  $L$ ) the language accepted by  $M_1$ .

**b.** Let  $M_2$  be obtained from  $M$  by adding  $\Lambda$ -transitions to  $q_f$  from every state from which  $q_f$  is reachable in  $M$ .

Describe (in terms of  $L$ ) the language accepted by  $M_2$ .

**c.** Let  $M_3$  be obtained from  $M$  by adding both the  $\Lambda$ -transitions in (a) and those in (b).

Describe (in terms of  $L$ ) the language accepted by  $M_3$ .