

### Exercise 2.33.

Let  $x$  be a string of length  $n$  in  $\{a, b\}^*$ , and let  $L = \{x\}$ .

How many equivalence classes does  $\equiv_L$  have? Describe them.

Hint: first draw an FA accepting  $L$ .

### Exercise 2.36.

For a certain language  $L \subseteq \{a,b\}^*$ ,  $\equiv_L$  has exactly four equivalence classes. They are  $[\Lambda]$ ,  $[a]$ ,  $[ab]$  and  $[b]$ .

It is also true that the three strings  $a$ ,  $aa$ , and  $abb$  are all equivalent,  
and that the two strings  $b$  and  $aba$  are equivalent.

Finally,  $ab \in L$ , but  $\Lambda$  and  $a$  are not in  $L$ , and  $b$  is not even a prefix of any element of  $L$ .

Draw an FA accepting  $L$ .

### Exercise 2.37.

Suppose  $L \subseteq \{a, b\}^*$  and  $\equiv_L$  has three equivalence classes. Suppose they can be described as the three sets  $[a]$ ,  $[aa]$ , and  $[aaa]$ , and also as the three sets  $[b]$ ,  $[bb]$ , and  $[bbb]$ .

How many possibilities are there for the language  $L$ ? For each one, draw a transition diagram for an FA accepting it.

### Exercise 2.38.

In each part, find every possible language  $L \subseteq \{a, b\}^*$  for which the equivalence classes of  $\equiv_L$  are the given three sets.

**a.**       $\{a, b\}^* \{b\},$        $\{a, b\}^* \{ba\},$        $\{\Lambda, a\} \cup \{a, b\}^* \{aa\}$

## Exercise 2.40.

Consider the language

$$L = AEqB = \{x \in \{a, b\}^* \mid n_a(x) = n_b(x)\}$$

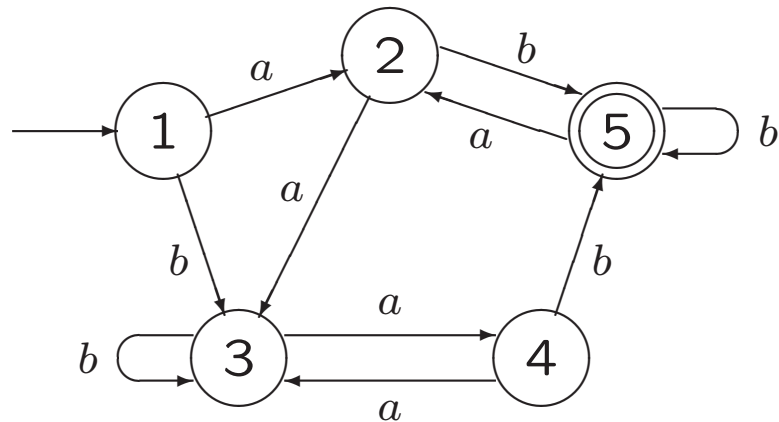
Let  $x$  and  $y$  be arbitrary elements of  $\{a, b\}^*$  (not necessarily in  $L$ )

- a. Show that if  $n_a(x) - n_b(x) = n_a(y) - n_b(y)$ , then  $x \equiv_L y$ .
- b. Show that if  $n_a(x) - n_b(x) \neq n_a(y) - n_b(y)$ , then  $x$  and  $y$  are  $L$ -distinguishable.
- c. Describe all the equivalence classes of  $\equiv_L$ .

### Exercise 2.55.

For each of the FAs below, use the minimization algorithm described in Section 2.6 to find a minimum-state FA accepting the same language. (It's possible that the given FA is already minimal.)

a.



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For each of the FAs below, use the minimization algorithm described in Section 2.6 to find a minimum-state FA accepting the same language. (It's possible that the given FA is already minimal.)

C.

