Exercise 4.34.

Show that the CFG with productions

$$S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$$

is ambiguous.

Exercise 4.36.

In each case below, decide whether the grammar is ambiguous or not, and prove your answer.

b. $S \rightarrow SS \mid bS \mid a$

c. $S \rightarrow SaS \mid b$

e. $S \rightarrow TT \qquad T \rightarrow aT \mid Ta \mid b$

f. $S \rightarrow aSa \mid bSb \mid aAb \mid bAa \qquad A \rightarrow aAa \mid bAb \mid a \mid b \mid \wedge$

g. $S \rightarrow aT \mid bT \mid \wedge \qquad T \rightarrow aS \mid bS$

Exercise 4.38.

In each case below, show that the grammar is ambiguous, and find an equivalent unambiguous grammar.

a. $S \to SS \mid a \mid b$

b. $S \to ABA \qquad A \to aA \mid \wedge \qquad B \to bB \mid \wedge$

c. $S \to aSb \mid aaSb \mid \wedge$

d. $S \to aSb \mid abS \mid \wedge$

Exercise 4.45.

Use induction to prove that the CFG below is unambiguous.

b. The CFG with productions $S \to (S)S \mid \Lambda$

Exercise.

Let G be a context-free grammar with start variable S and the following productions:

$$S \rightarrow aSbS \mid bSaS \mid \wedge$$

a. Show that $L(G) = AEqB = \{x \in \{a, b\}^* \mid n_a(x) = n_b(x)\}$

b. Is G ambiguous? Motivate your answer.

Exercise 4.53.

c. In each case, given the context-free grammar G, find an equivalent CFG with no useless variables.

ii. G has productions

$$S \to AB \mid AC \qquad A \to aAb \mid bAa \mid a \qquad B \to bbA \mid aaB \mid AB$$
$$C \to abCa \mid aDb \qquad D \to bD \mid aC$$

Exercise 4.50.

In each case, given the context-free grammar G, find a CFG G' in Chomsky normal form that generates the language $L(G) - \{\Lambda\}$.

a. G has productions

$$S \to ABA \qquad A \to aA \mid \Lambda \qquad B \to bB \mid \Lambda$$

b. G has productions

$$S \to aSa \mid bSb \mid \Lambda \qquad A \to aBb \mid bBa \qquad B \to aB \mid bB \mid \Lambda$$

Exercise 4.54.

In each case below, given the context-free grammar G, find a CFG G_1 in Chomsky normal form generating $L(G) - \{\Lambda\}$.

a. G has productions $\qquad S \to SS \mid (S) \mid \Lambda$

b. G has productions $\qquad S \to S(S) \mid \Lambda$

c. G has productions

$$S \to AaA \mid CA \mid BaB \qquad A \to aaBa \mid CDA \mid aa \mid DC$$
$$B \to bB \mid bAB \mid bb \mid aS \qquad C \to Ca \mid bC \mid D \qquad D \to bD \mid \Lambda$$

Exercise 4.48.

Show that the nullable variables defined by Definition 4.26 are precisely those variables A for which $A \Rightarrow^* \Lambda$.