**Exercise 2.33.**

Let $x$ be a string of length $n$ in $\{a, b\}^*$, and let $L = \{x\}$.

How many equivalence classes does $\equiv_L$ have? Describe them.

Hint: first draw an FA accepting $L$.

**Exercise 2.36.**

For a certain language $L \subseteq \{a, b\}^*$, $\equiv_L$ has exactly four equivalence classes. They are $[\Lambda]$, $[a]$, $[ab]$ and $[b]$.

It is also true that the three strings $a$, $aa$, and $abb$ are all equivalent,
and that the two strings $b$ and $aba$ are equivalent.

Finally, $ab \in L$, but $\Lambda$ and $a$ are not in $L$, and $b$ is not even a prefix of any element of $L$.

Draw an FA accepting $L$.

**Exercise 2.37.**

Suppose $L \subseteq \{a, b\}^*$ and $\equiv_L$ has three equivalence classes. Suppose they can be described as the three sets $[a]$, $[aa]$, and $[aaa]$, and also as the three sets $[b]$, $[bb]$, and $[bbb]$.

How many possibilities are there for the language $L$? For each one, draw a transition diagram for an FA accepting it.

**Exercise 2.38.**

In each part, find every possible language $L \subseteq \{a, b\}^*$ for which the equivalence classes of $\equiv_L$ are the given three sets.

**a.** $\{a, b\}^*\{b\}$, $\{a, b\}^*\{ba\}$, $\{\Lambda, a\} \cup \{a, b\}^*\{aa\}$

**Exercise 2.40.**

Consider the language

$$L = AEqB = \{x \in \{a,b\}^* \mid n_a(x) = n_b(x)\}$$

Let $x$ and $y$ be arbitary elements of $\{a,b\}^*$ (not necessarily in $L$)

**a.** Show that if $n_a(x) - n_b(x) = n_a(y) - n_b(y)$, then $x \equiv_L y$.

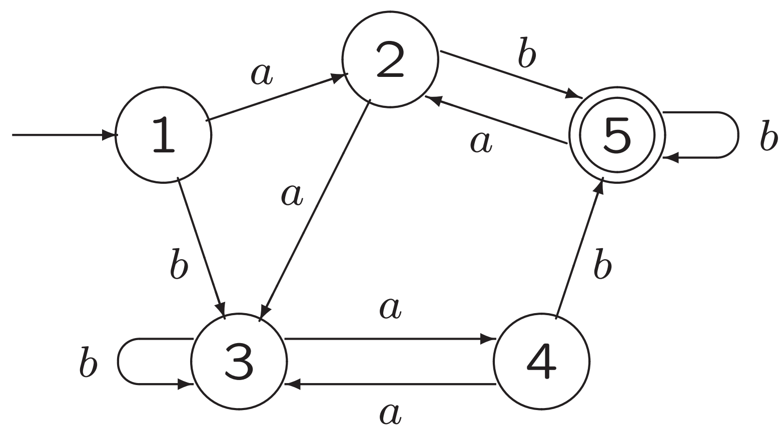
**b.** Show that if $n_a(x) - n_b(x) \neq n_a(y) - n_b(y)$, then $x$ and $y$ are $L$-distinguishable.

**c.** Describe all the equivalence classes of $\equiv_L$.

**Exercise 2.55.**

For each of the FAs below, use the minimization algorithm described in Section 2.6 to find a minimum-state FA accepting the same language. (It's possible that the given FA is already minimal.)

**a.**



**Exercise 2.55.**
For each of the FAs below, use the minimization algorithm described in Section 2.6 to find a minimum-state FA accepting the same language. (It's possible that the given FA is already minimal.)

**c.**