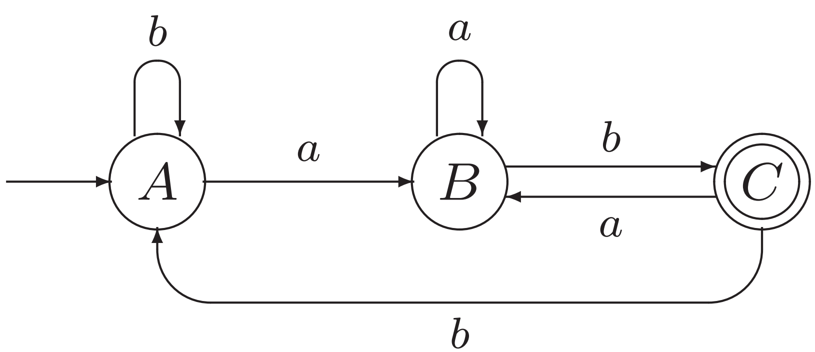
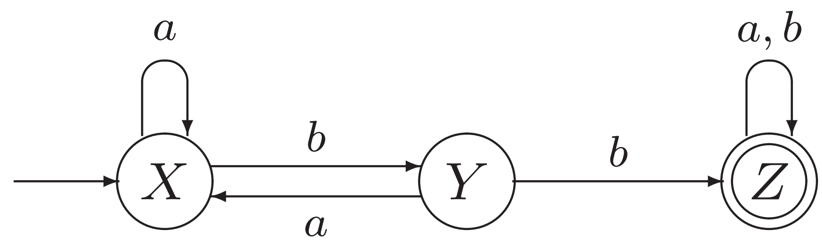
Exercise 2.10. Let M_1 and M_2 be the FAs pictured below, accepting languages L_1 and L_2, respectively.





Draw FAs accepting the following languages.

a. $L_1 \cup L_2$

b. $L_1 \cap L_2$

c. $L_1 - L_2$

Exercise 2.22. For each of the following languages, use the pumping lemma to show that it cannot be accepted by an FA.

a. $L = \{a^i b a^{2i} \mid i \geq 0\}$

b. $L = \{a^i b^j a^k \mid k > i + j\}$

d. $L = \{a^i b^j \mid \ j \text{ is a multiple of } i\ \}$

e. $L = \{x \in \{a, b\}^* \mid n_a(x) < 2n_b(x)\}$

f. $L = \{x \in \{a, b\}^* \mid \text{ no prefix of } x \text{ has more } b\text{'s than } a\text{'s } \}$

h. $L = \{ww \mid w \in \{a, b\}^*\}$

Exercise.

Use the pumping lemma to show that the following language cannot be accepted by an FA.

$$L = \{(ab)^i a^i \ \mid \ i \geq 0\}$$

Exercise 2.24.

Prove the following generalization of the pumping lemma, which can sometimes make it unnecessary to break the proof into cases.

If L can be accepted by an FA,
then there is an integer n
such that for any $x \in L$ with $|x| \geq n$
and for any way of writing x as $x_1 x_2 x_3$ with $|x_2| = n$,
there are strings u, v and w such that

a. $x_2 = uvw$

b. $|v| \geq 1$

c. For every $m \geq 0$, $x_1 u v^m w x_3 \in L$

Exercise 2.26.

The pumping lemma says that
if M accepts a language L,
and if n is the number of states of M,
then for every $x \in L$ satisfying $|x| \geq n$, . . .

Show that the statement provides no information if L is finite:
If M accepts a finite language L,
and n is the number of states of M,
then L can contain no strings of length n or greater.