Huiswerk. . .

Vanmiddag: geen (compleet) werkcollege

Vragenuur: vrijdag 16 december, 13.15-15.00, (waarschijnlijk) zaal 412

Tentamen: maandag 19 december, 09.00-12.00

From lecture 13:

Example

AnBnCn is intersection of two context-free languages.

[M] E 6.10

Hence, CFL is not closed under intersection

$L_1 \cap L_2 = (L_1' \cup L_2')'$

Hence, CFL is not closed under complement

Example

Complement of *AnBnCn* is context-free.

[M] E 6.12

Example

Complement of *AnBnCn* is context-free.

$AnBnCn = L_1 \cap L_2 \cap L_3$, with
$L_1 = \{a^i b^j c^k \mid i \leq j\}$
$L_2 = \{a^i b^j c^k \mid j \leq k\}$
$L_3 = \{a^i b^j c^k \mid k \leq i\}$
[M] E 6.12

Example

Complement of $\{x \in \{a, b, c\}^* \mid n_a(x) = n_b(x) = n_c(x)\}$ is context-free.

$\{x \in \{a, b, c\}^* \mid n_a(x) = n_b(x) = n_c(x)\} = A_1 \cap A_2 \cap A_3$, with
$A_1 = \{x \in \{a, b, c\}^* \mid n_a(x) \leq n_b(x)\}$
$A_2 = \{x \in \{a, b, c\}^* \mid n_b(x) \leq n_c(x)\}$
$A_3 = \{x \in \{a, b, c\}^* \mid n_c(x) \leq n_a(x)\}$
[M] E 6.12

Example

$L_1 = \{\ a^{2n} b^n \mid n \geq 1\ \}^*$

$a^{16} b^8 a^8 b^4 a^4 b^2 a^2 b^1$

$L_2 = a^* \{\ b^n a^n \mid n \geq 1\ \}^* \{\ b\ \}$

From lecture 2:

FA $M_i = (Q_i, \Sigma, q_i, A_i, \delta_i) \quad i = 1, 2$

Product construction

construct FA $M = (Q, \Sigma, q_0, A, \delta)$ such that

– $Q = Q_1 \times Q_2$

– $q_0 = (q_1, q_2)$

– $\delta(\ (p, q), \sigma) = (\ \delta_1(p, \sigma), \delta_2(q, \sigma)\ )$

– A as needed

Theorem (2.15 Parallel simulation)

– $A = \{(p, q) \mid p \in A_1$ *or* $q \in A_2\}$, *then* $L(M) = L(M_1) \cup L(M_2)$

– $A = \{(p, q) \mid p \in A_1$ *and* $q \in A_2\}$, *then* $L(M) = L(M_1) \cap L(M_2)$

– $A = \{(p, q) \mid p \in A_1$ *and* $q \notin A_2\}$, *then* $L(M) = L(M_1) - L(M_2)$

Proof. . .

[M] Sect 2.2

Theorem

If L_1 is a CFL, and L_2 in REG, then $L_1 \cap L_2$ is CFL.

[M] Thm 6.13

product construction

PDA $M_1 = (Q_1, \Sigma, \Gamma, q_1, Z_1, A_1, \delta_1)$
FA $M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$

$Q = Q_1 \times Q_2 \qquad q_0 = \langle q_1, q_2 \rangle \qquad A = A_1 \times A_2$

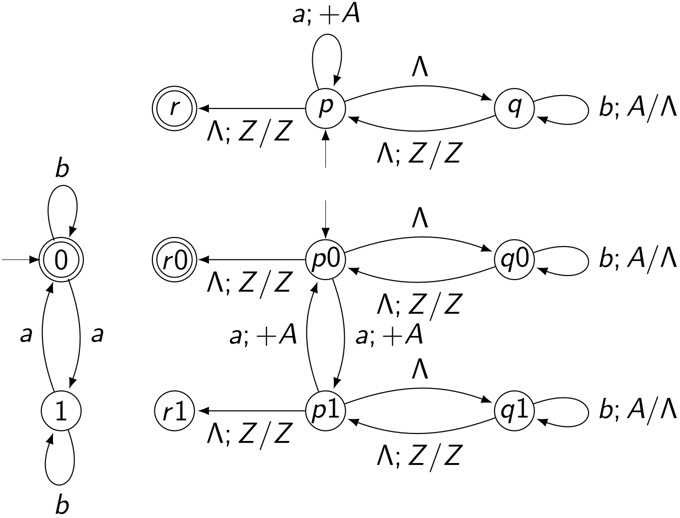
$\delta(\langle p_1, q_1 \rangle, \sigma, X) \ni (\langle p_2, q_2 \rangle, \alpha)$
whenever $\delta_1(p_1, \sigma, X) \ni (p_2, \alpha)$ and $\delta_2(q_1, \sigma) = q_2$

$\delta(\langle p_1, q \rangle, \Lambda, X) \ni (\langle p_2, q \rangle, \alpha)$
whenever $\delta_1(p_1, \Lambda, X) \ni (p_2, \alpha)$ and $q \in Q_2$

The inductive proof that this construction works does not have to be known for the exam.

Also CFG proof

$\{ \ a^n b^n \mid n \geq 1 \ \}^* \cap \{ \ w \in \{a, b\}^* \mid n_a(x) \text{ even } \}$



Non-determinism of PDA

- enables $L(M_1) \cup L(M_2)$
- 'prevents' $L(M_1)'$ (also Λ-transitions)

If L is accepted by DPDA without Λ-transitions, then so is L'

Even: if L is accepted by DPDA, then so is L'

Hence, if L is CFL and L' is not, then there is no DPDA for L
Not reversed (see Pal)

"given a CFL L, does it have property ... ?" yes/no
input CFG G

Given CFG G [G_1 and G_2]
– and given a string x, is $x \in L(G)$? membership problem
convert G to ChNF, and try all derivations of length $2|x| - 1$
(special case if $x = \Lambda$)
Cocke, Younger, and Kasami (1967): n^3 (with DP)
Earley (1970): n^3 (and n^2 if G is unambiguous)

– is $L(G) \neq \varnothing$? non-emptiness

is S useful?

pumping lemma

– is $L(G)$ infinite?

pumping lemma

– is $L(G_1) \cap L(G_2)$ nonempty? [M] Thm 9.20
– is $L(G) = \Sigma^*$? [M] Thm 9.23
– is $L(G_1) \subseteq L(G_2)$?

$$L(G) = \Sigma^*, \text{ if and only if } \Sigma^* \subseteq L(G)$$

All undecidable

Given context-free L and regular R

– is $R \subseteq L$?

– is $L \subseteq R$?

ABOVE

$R \subseteq L$?

Special case $R = \Sigma^*$

$\Sigma^* \subseteq L$ iff $L = \Sigma^*$ undecidable

$L \subseteq R$?

iff $L \cap R' = \varnothing$

regular languages are closed under complement

CFL closed under intersection with regular languages

emptiness context-free decidable

Section 7

Course Computability

- Turing machines
- Recursively enumerable languages / recursive languages
- Unrestricted grammars
- Undecidability

END.

Thanks to HJH for the slides