Homework 1 Automata Theory 2023

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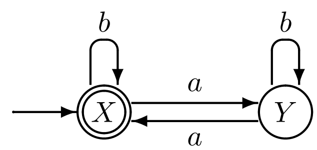
Deadline for submission: Monday 9 October 2023, 23.59.
The assignment must be completed individually. A total of 100 points can be earned. Answers to be submitted via Brightspace. Submit a single file, e.g., a pdf or possibly a zip. Please include your name and student number in your submission. You may either type your answers or hand-write them. In the latter case, please hand in an easy-to-read scan / photos.

1. [60 pt] Consider the language $L_1 = \{(ab)^i a^j \mid i, j \geq 0\}$.

   (a) List the first six elements of L_1 in canonical (shortlex) order.

   (b) Draw a (deterministic) finite automaton M_1, such that $L(M_1) = L_1$ with at most five states. If your automaton has more than five states, use the minimization algorithm – *on scratch paper*, to reduce the number of states.

   (c) The language $L_2 = \{x \in \{a, b\}^* \mid n_a(x) \text{ is even}\}$ is accepted by the following finite automaton M_2:

   Use the product construction, Theorem 2.15 in the book, to obtain a finite automaton M_3 that accepts $L_1 \cap L_2$. Draw only the resulting finite automaton.

   (d) Minimize the finite automaton M_3 you obtain in (c), using Algorithm 2.40 from the book. Show the application of the algorithm with a table of unordered pairs as in Figure 2.42(b), indicating the order in which you mark the pairs. Also, indicate how you proceed in marking pairs, e.g., one vertical column at a time, and considering the pairs in each column from top to bottom. Finally, draw the resulting finite automaton.

2. [30 pt] Use the pumping lemma for regular languages, Theorem 2.29 in the book, to show that the language $L_3 = \{(ab)^i a^j b^k \mid i, j, k \geq 0 \text{ and } k < i + j\}$ cannot be accepted by a finite automaton.

   In other words: suppose that L_3 can be accepted by a finite automaton, choose a suitable string $x \in L_3$ and show that x cannot be pumped up or down, whatever decomposition uvw of x one would consider. Also, don't forget to draw the conclusion.

3. [10 pt] Let L_4 be the language $\{x \in \{a, b\}^* \mid x \text{ does not contain the substring } abba\}$

   (a) Find two distinct strings x and y in $\{a, b\}^*$ that are indistinguishable with respect to L_4.

   (b) Find three distinct strings x, y and z in $\{a, b\}^*$ that are pairwise L_4-distinguishable.