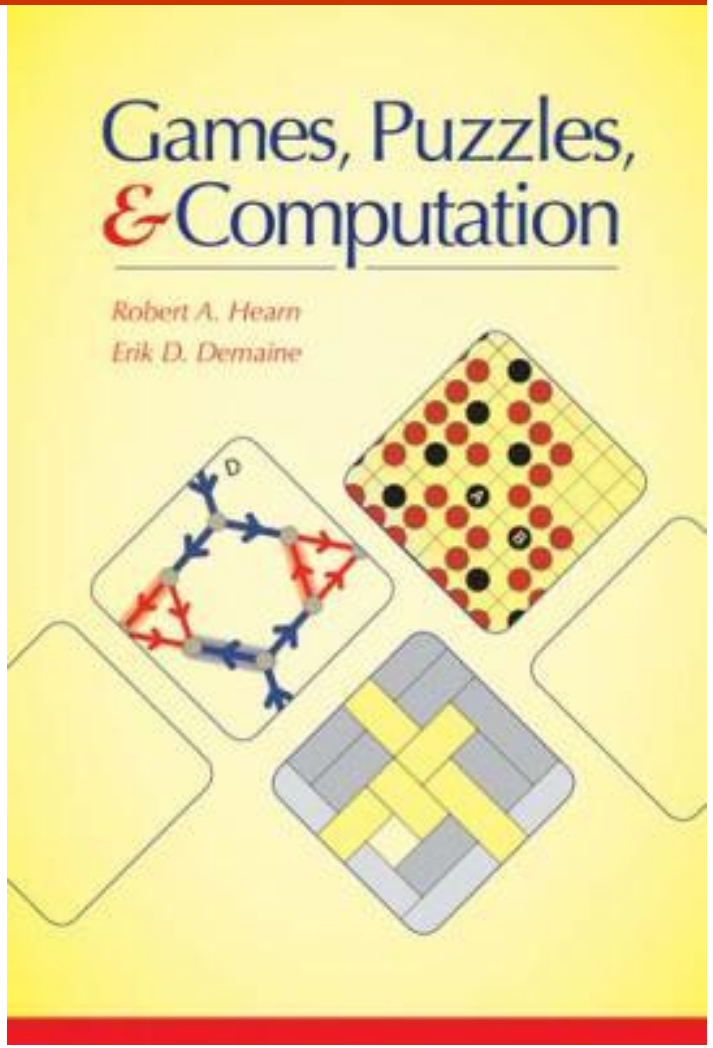
reference

Games, Puzzles, & Computation

Robert A. Hearn
Erik D. Demaine



E. Demaine and R.A. Hearn. Constraint Logic: A Uniform Framework for Modeling Computation as Games. In: Proceedings of the 23rd Annual IEEE Conference on Computational Complexity, June 2008. http://www.dartmouth.edu/~rah/constraint-logic.pdf

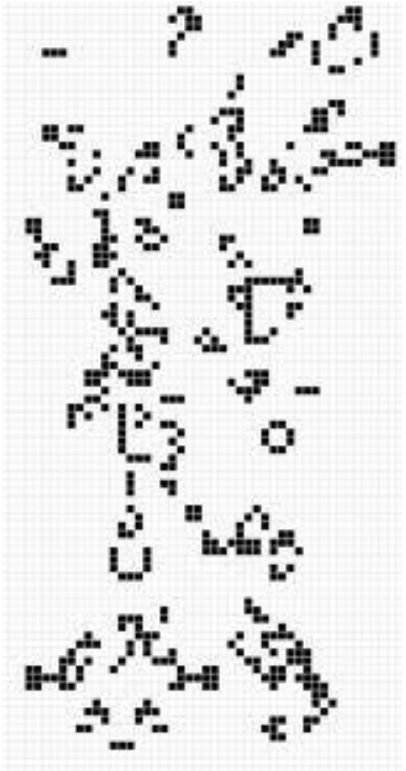
R.A. Hearn. Games, Puzzles, and Computation PhD thesis, MIT, 2006. http://www.dartmouth.edu/~rah/

1.1 what is a game?

complexity of

- board games (2p) *chess*
- puzzles (1p) *rush hour*
- simulation (0p) *game of life*
- teams

- bounded state
- moves
- players, goal



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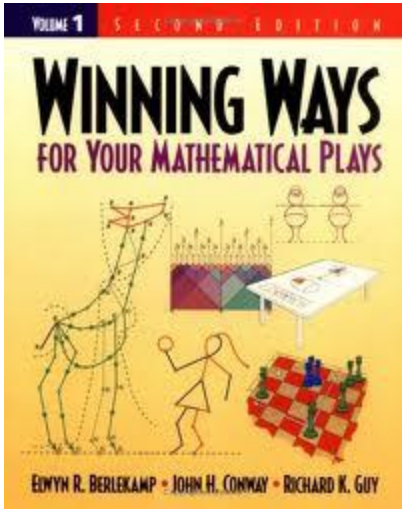
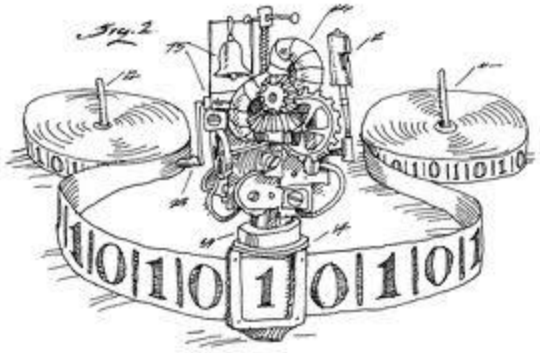


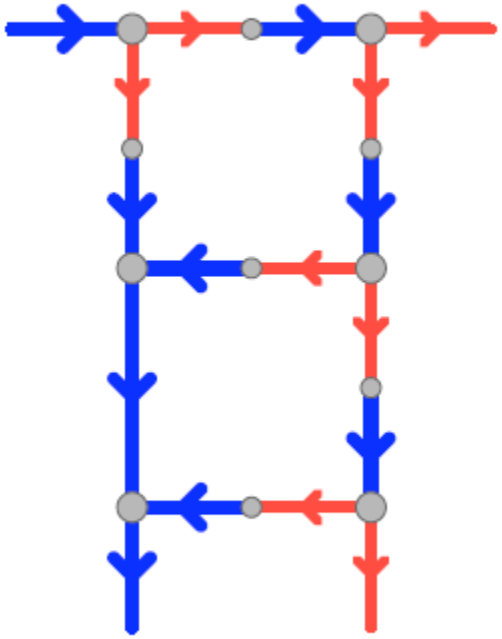
combinatorial game theory
- algorithms
- mathematical theory

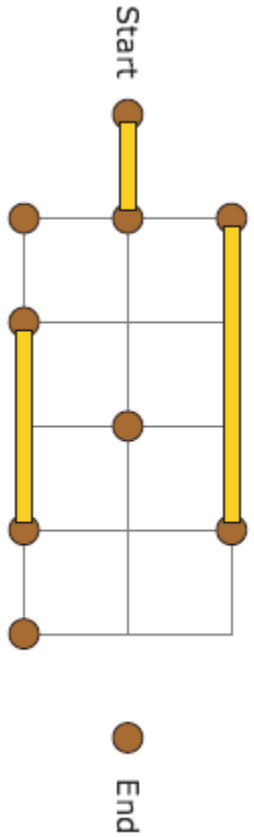
economic game theory
- von Neumann, Nash
- strategy, optimization expected profit

computational complexity
- models of computation
- turing machine

outline







PSPACE - QBF
NP - 3SAT

part I
constraint logic
'graph games'
NCL

part II
games in particular
plank puzzle
river crossing

1.2 computational complexity classes

(turing machine) resources

time

- P — polynomial
- EXPTIME — exponential $2^{p(n)}$

space

- PSPACE — $P \subseteq PSPACE$
- EXPSPACE

nondeterminism $\exists$

- NP vs. P
- PSPACE = NPSPACE

alternation $\exists\forall\exists\forall\ldots$

$$P \subseteq NP \subseteq PSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

X-complete

- X-hard vs. in X

complexity theory

$$P \subseteq NP \subseteq PSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

$$NSPACE(s(n)) \subseteq TIME(2^{\mathcal{O}(s(n))})$$

$$NSPACE(s(n)) \subseteq SPACE(s^2(n))$$

Savitch's theorem

space & time hierarchy

$$[N]P \subset [N]EXPTIME$$

$$PSPACE \subset EXPSPACE$$

constraint logic

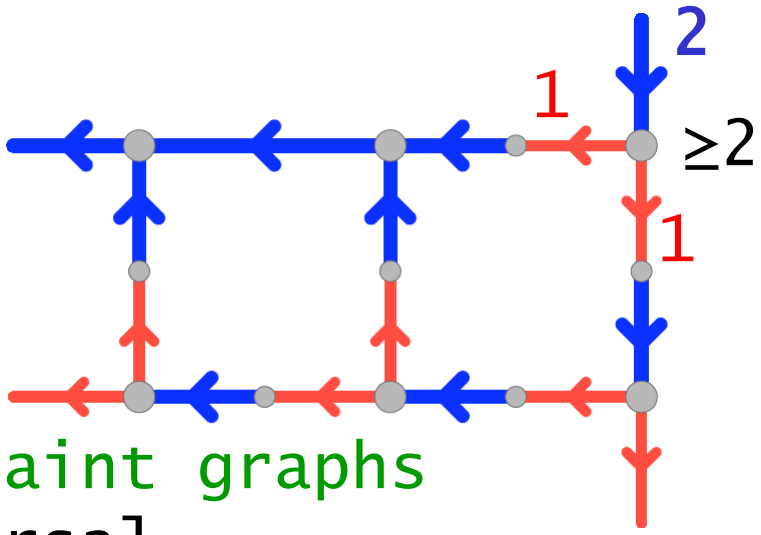
constraint graphs

directed ‘oriented’

edge weight 1,2

inflow constraints

legal configuration



game/computation on constraint graphs

move: legal edge reversal

goal: reverse given edge

p.9

NCL - nondet constraint logic

instance: constraint graph G, edge e

question: sequence which reverses e

BOUNDED NCL

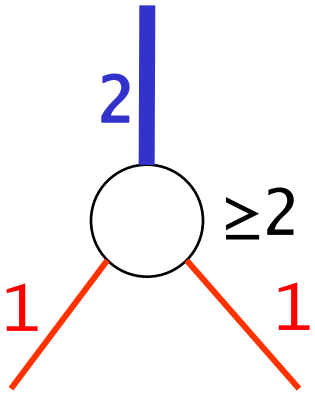
… reverses each edge at most once

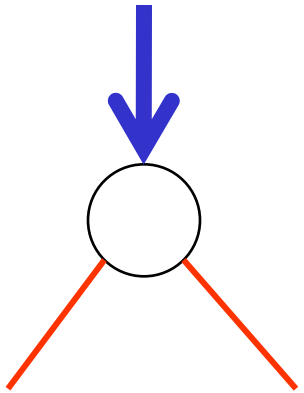
p.56

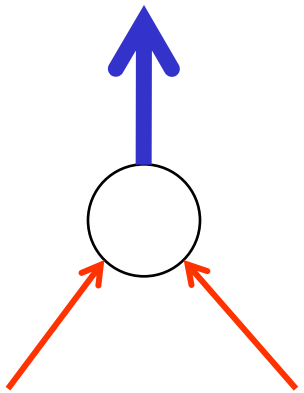
implementing gates

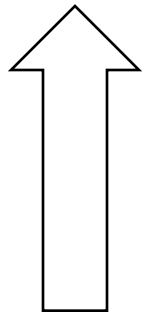
intuitive meaning of vertices

AND

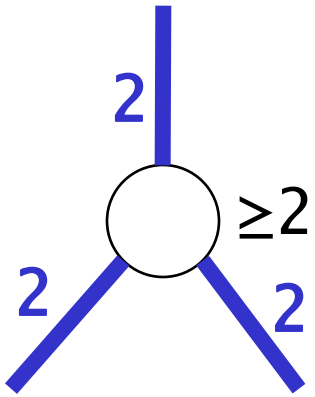


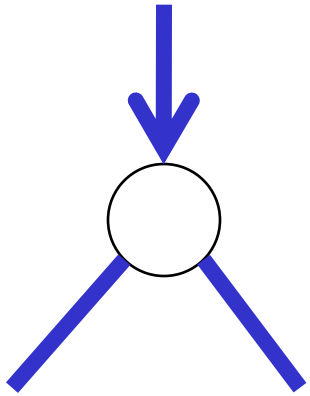


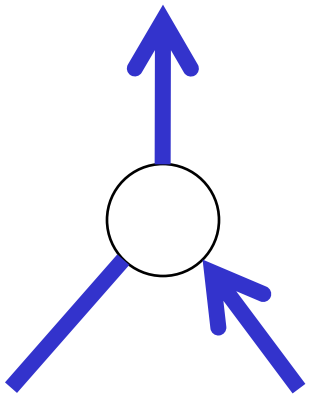




OR

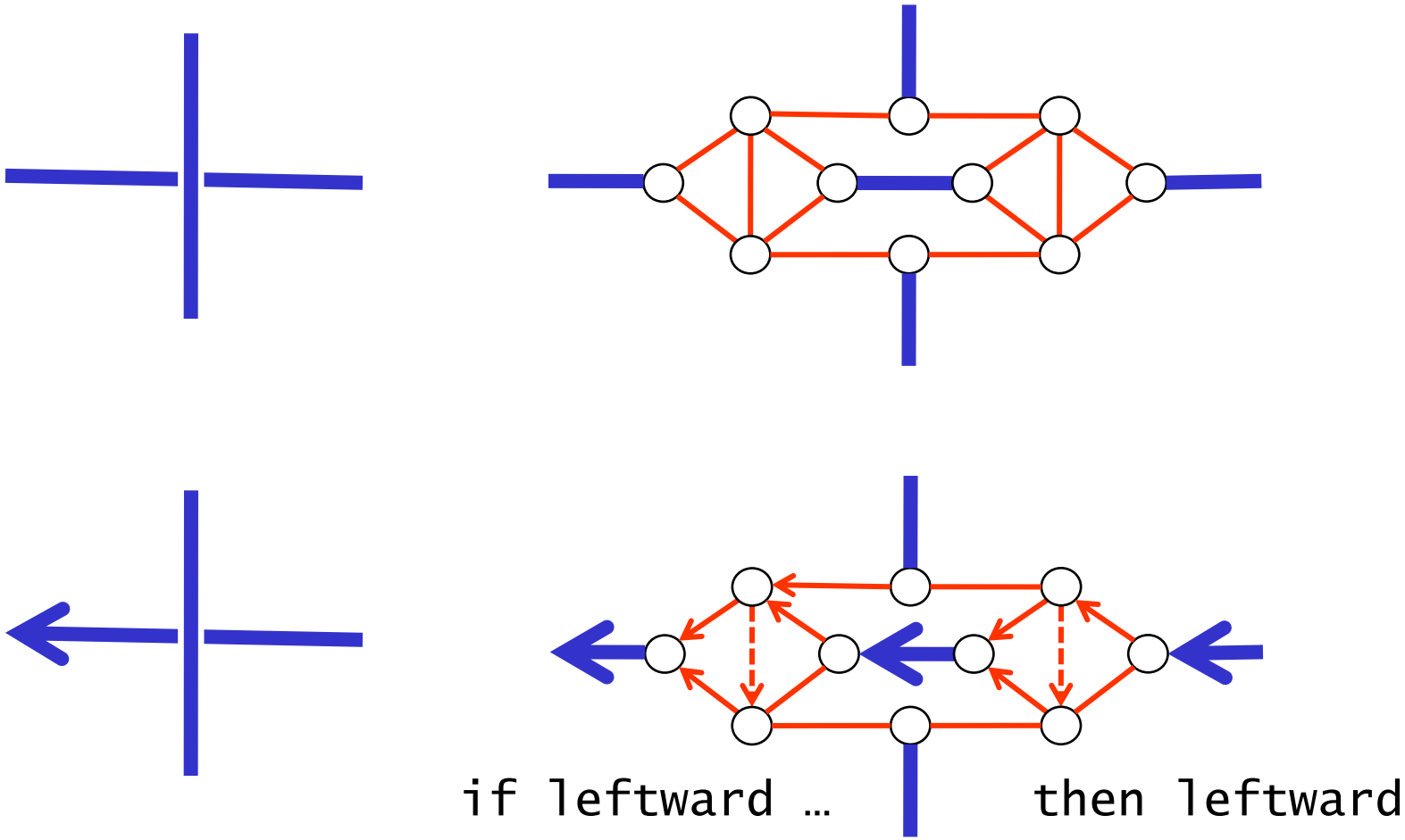






planar crossover gadget

formal proof Lemma 5.10



game categories

game categories and their natural complexities

rush hour
Theorem 9.20
sliding blocks
Theorem 9.8

unbounded	**PSPACE**	**PSPACE**	**EXPTIME**	**undecid**
bounded	**P**	**NP**	**PSPACE**	**NEXPTIME**
	zero pl.	*one pl.*	*two pl.*	*team*
	simulat.	*puzzle*		*imperfect*
		nondeterm.	*alternat.*	*informat.*

peg solitaire
Table A.7 p.174

I. games in general
5. one-player games (puzzles)

Thm 5.9 NCL is PSPACE-complete

p.66 via QBF

Thm 5.12 … , even for *planar graphs* using restricted vertex types

II. games in particular
9. one-player games (puzzles)

Thm 9.11 Plank puzzles are PSPACE-complete

via NCL

p.125