



# TEGELS

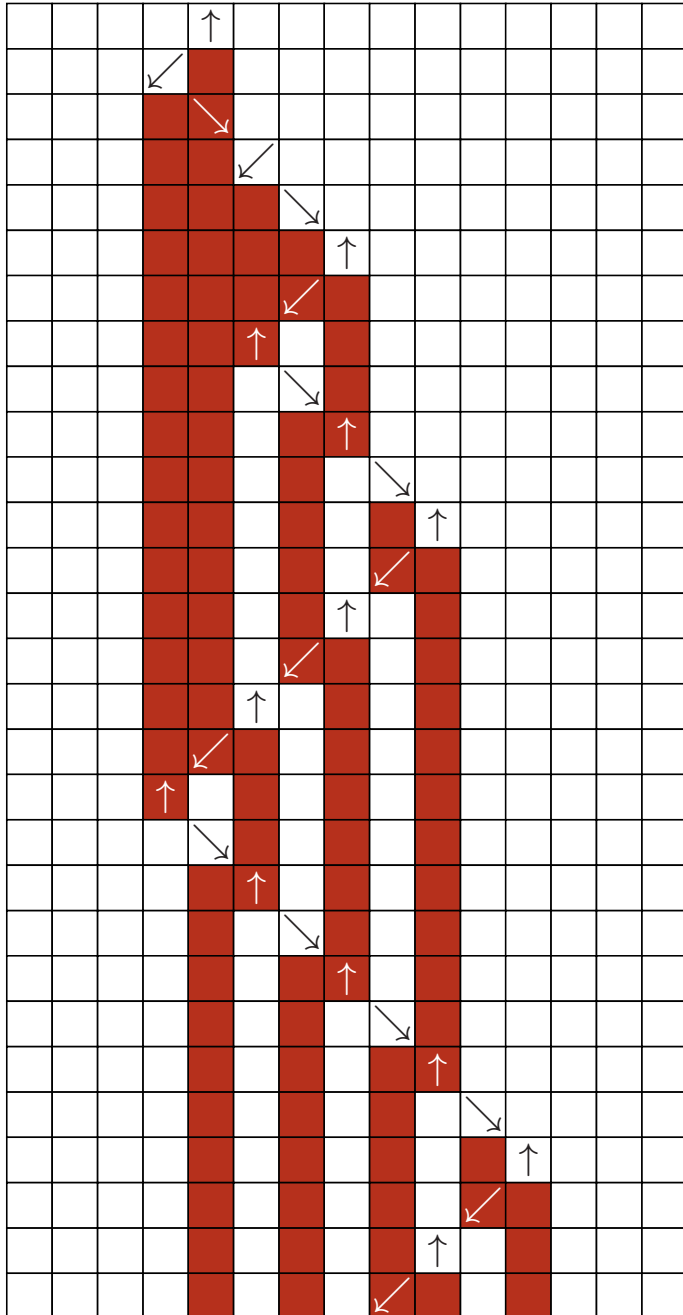
van patronen tot berekeningen

Hendrik Jan Hoogeboom

Universiteit Leiden, Informatica

[www.liacs.nl/home/hoogeboo/praatjes/tegels/](http://www.liacs.nl/home/hoogeboo/praatjes/tegels/)



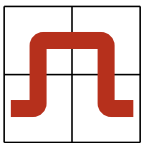
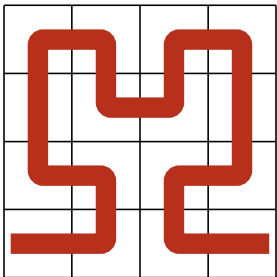
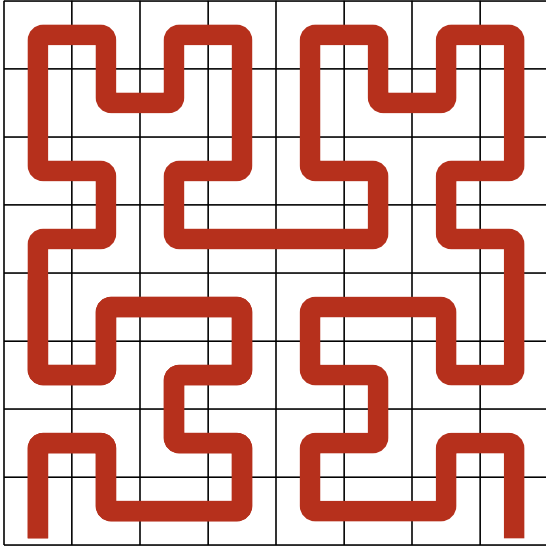


wiskunde: regelmaat, patronen

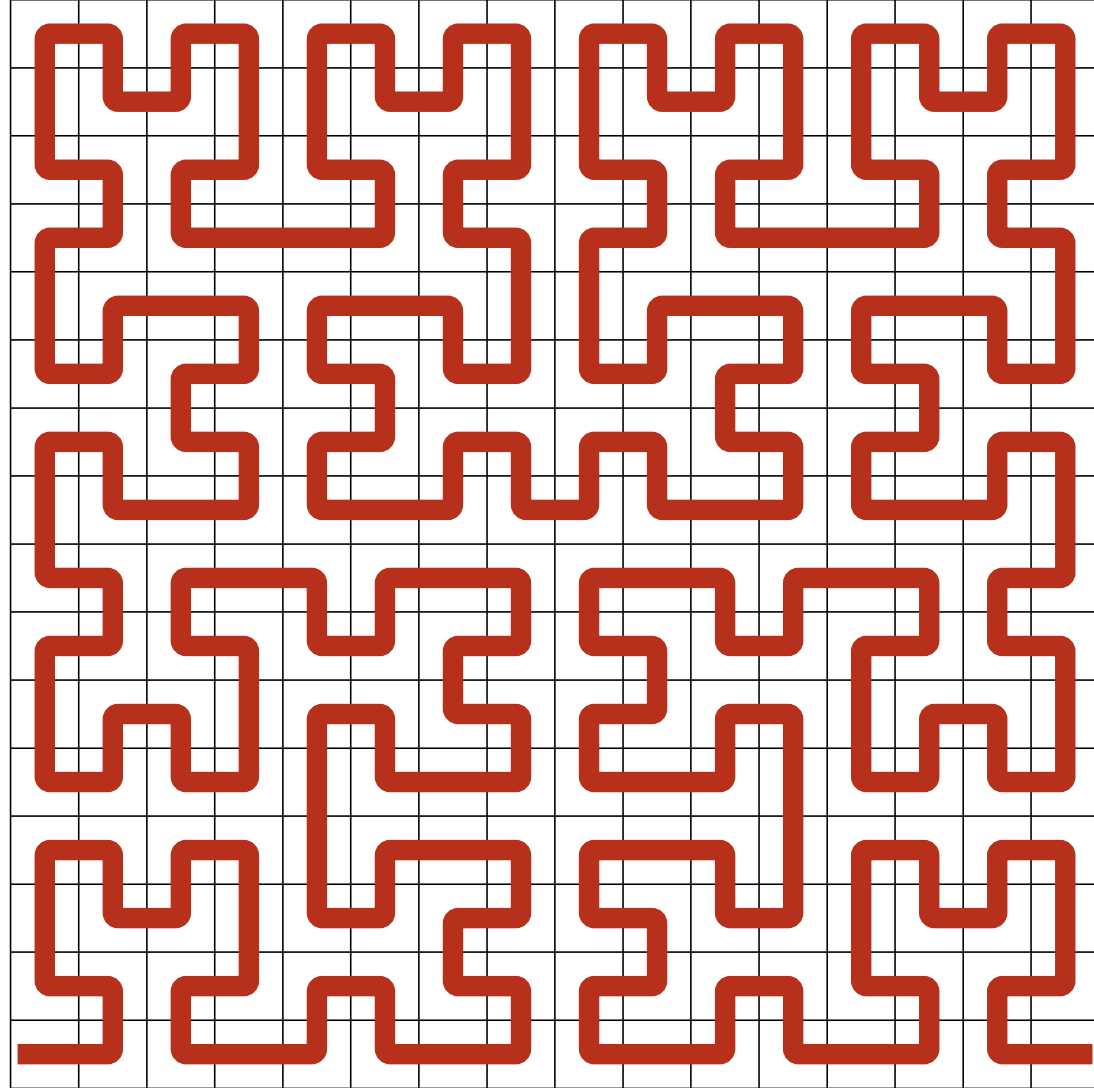
informatica: berekeningen

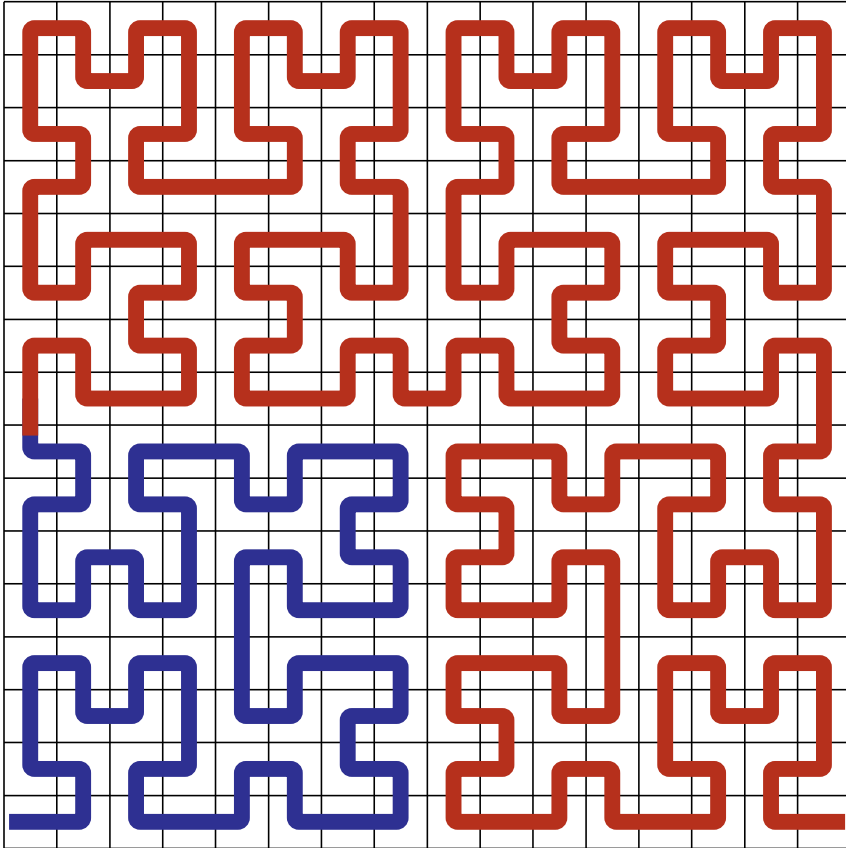
modellen:

- ➡ Wang tiles
- ➡ Cellular automata
- ➡ Turing machines

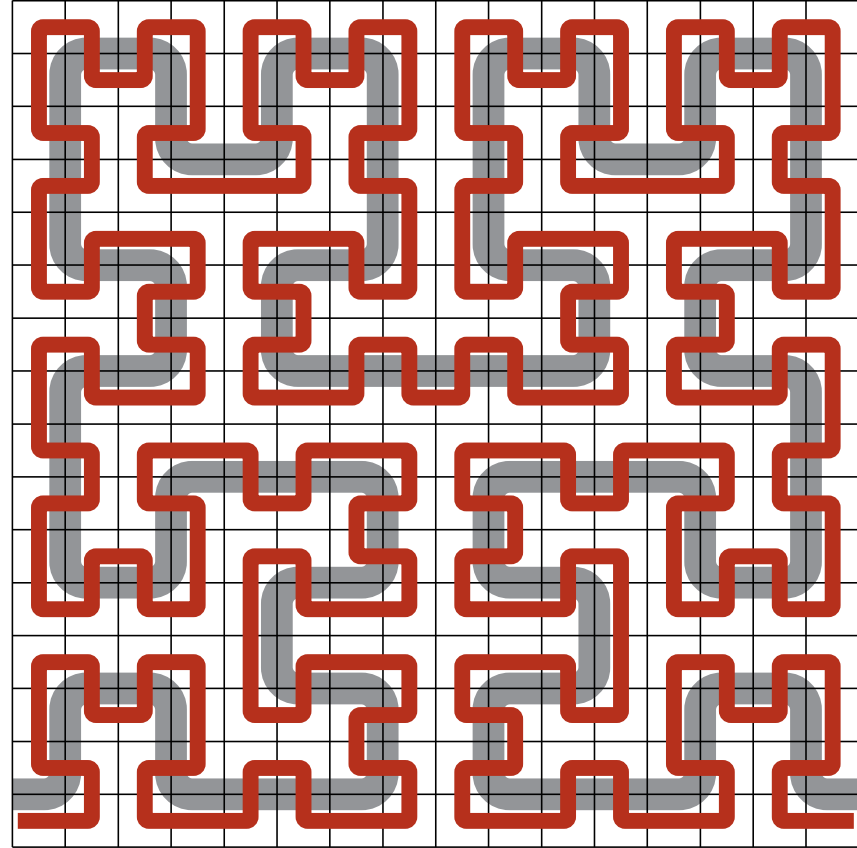


zes 'tegels':  
twee recht, vier bochten

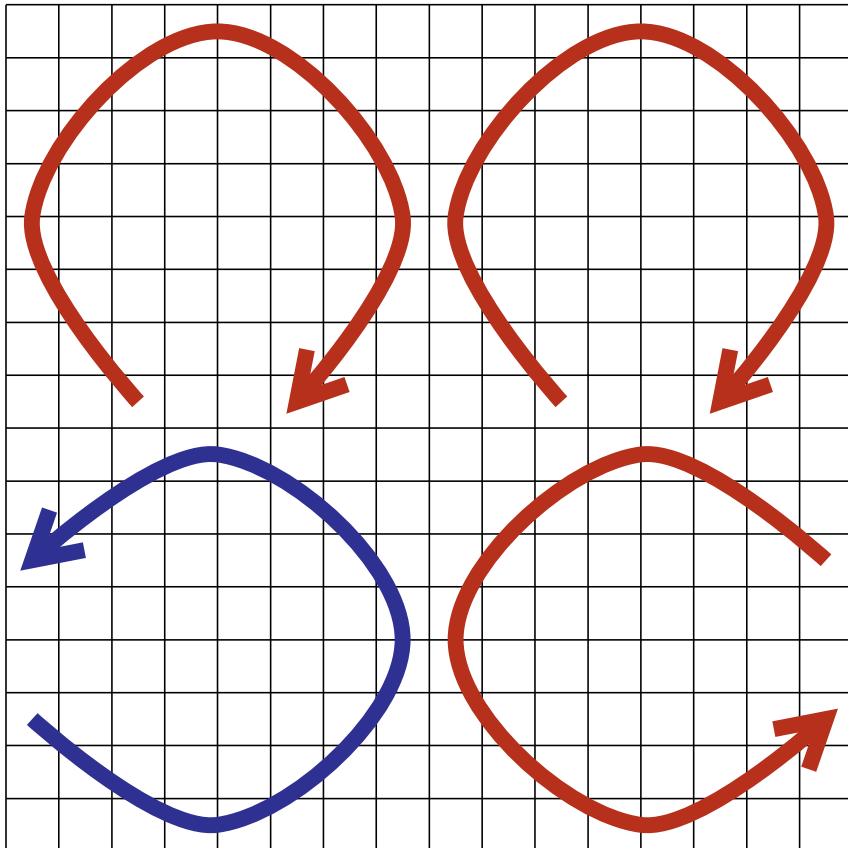




vier kopieën



verfijning



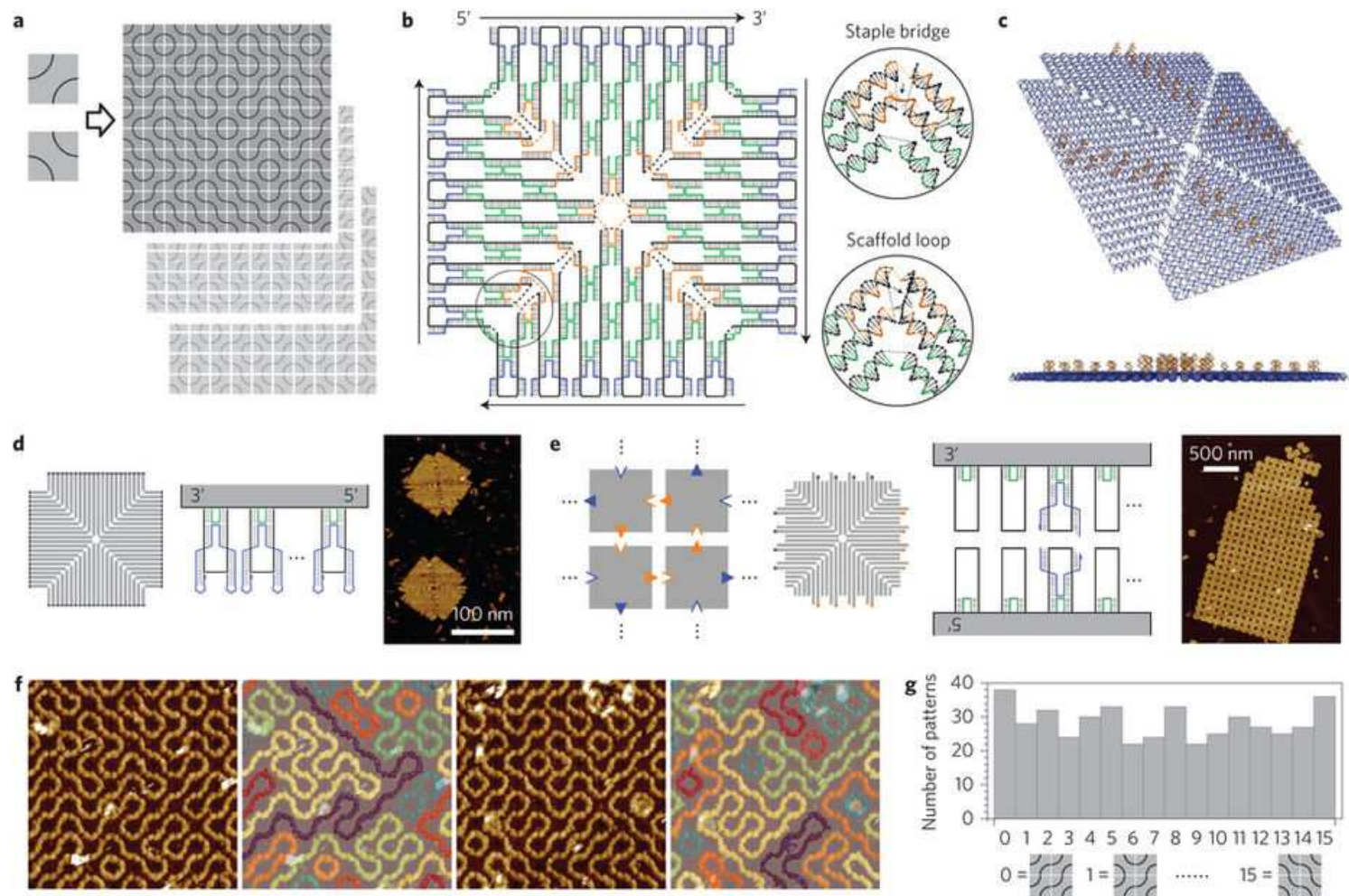
wikipedia

```
to starthilbert :size :level
  hilbert (:size / power 2 (:level-1)) :level 1
end
```

```
to hilbert :size :level :parity
  if :level = 0 [stop]
  right 90 * :parity
  hilbert :size (:level-1) (:parity * -1)
  forward :size
  right -90 * :parity
  hilbert :size (:level-1) :parity
  forward :size
  hilbert :size (:level-1) :parity
  right -90 * :parity
  forward :size
  hilbert :size (:level-1) (:parity * -1)
  right 90 * :parity
end
```

aanroep

```
starthilbert 200 5
```



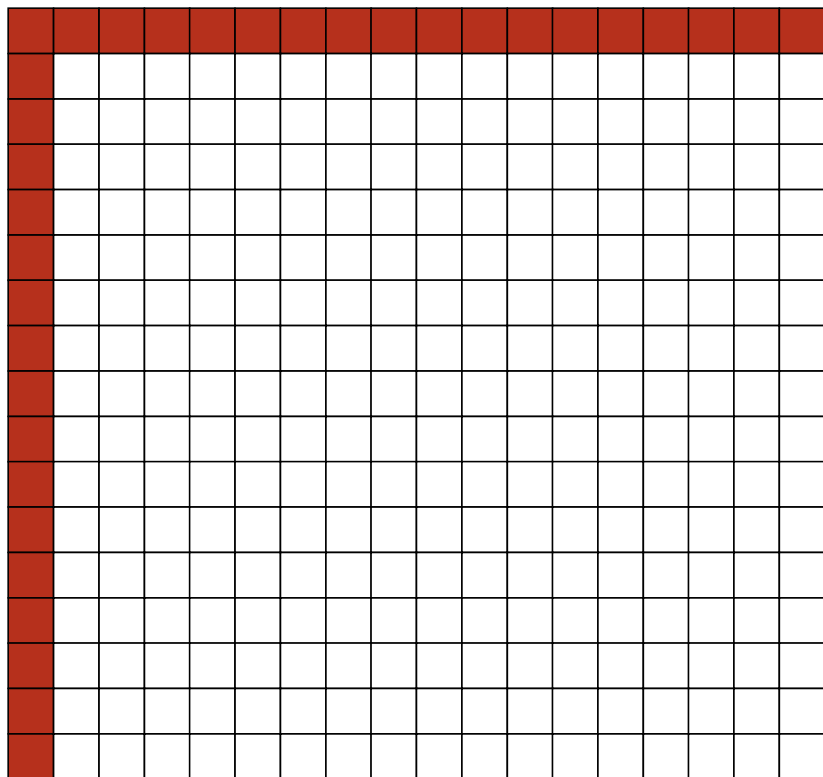
Programmable disorder in random DNA tilings

Tikhomirov, Petersen & Qian – Nature Nanotechnology (2017)

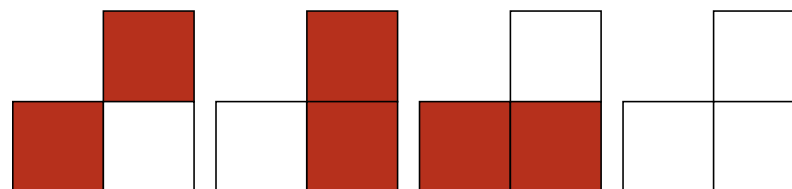
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## ■ Zelf proberen

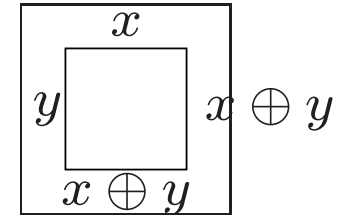
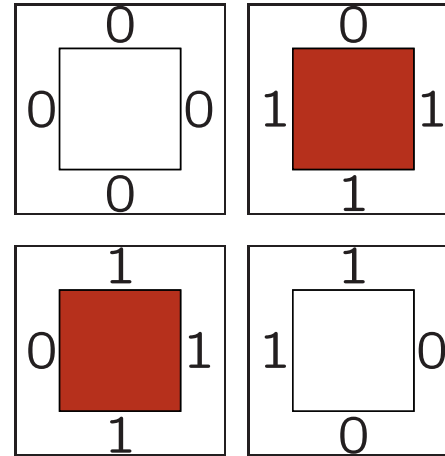
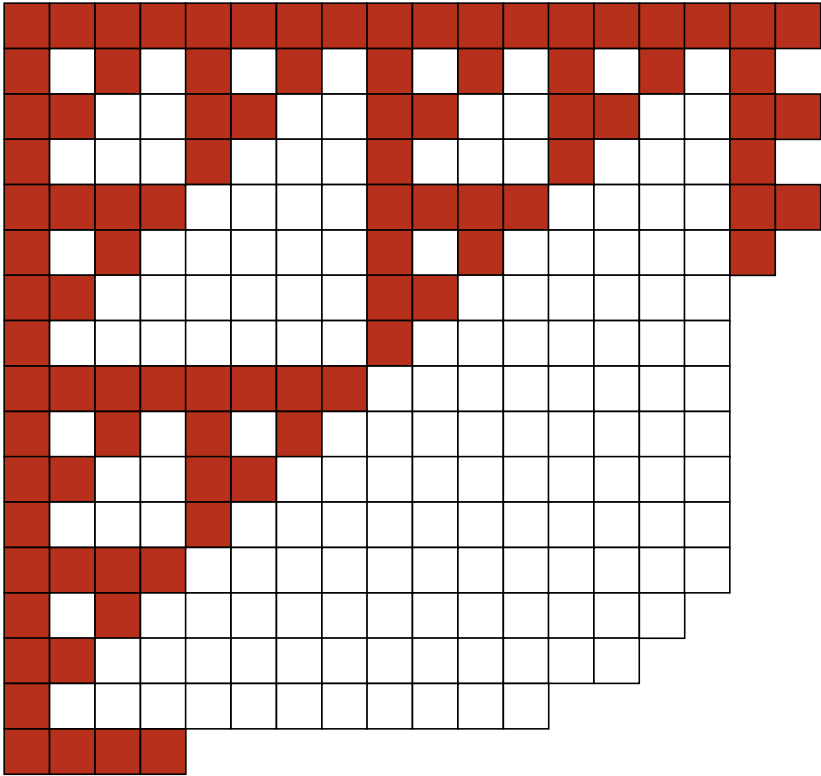
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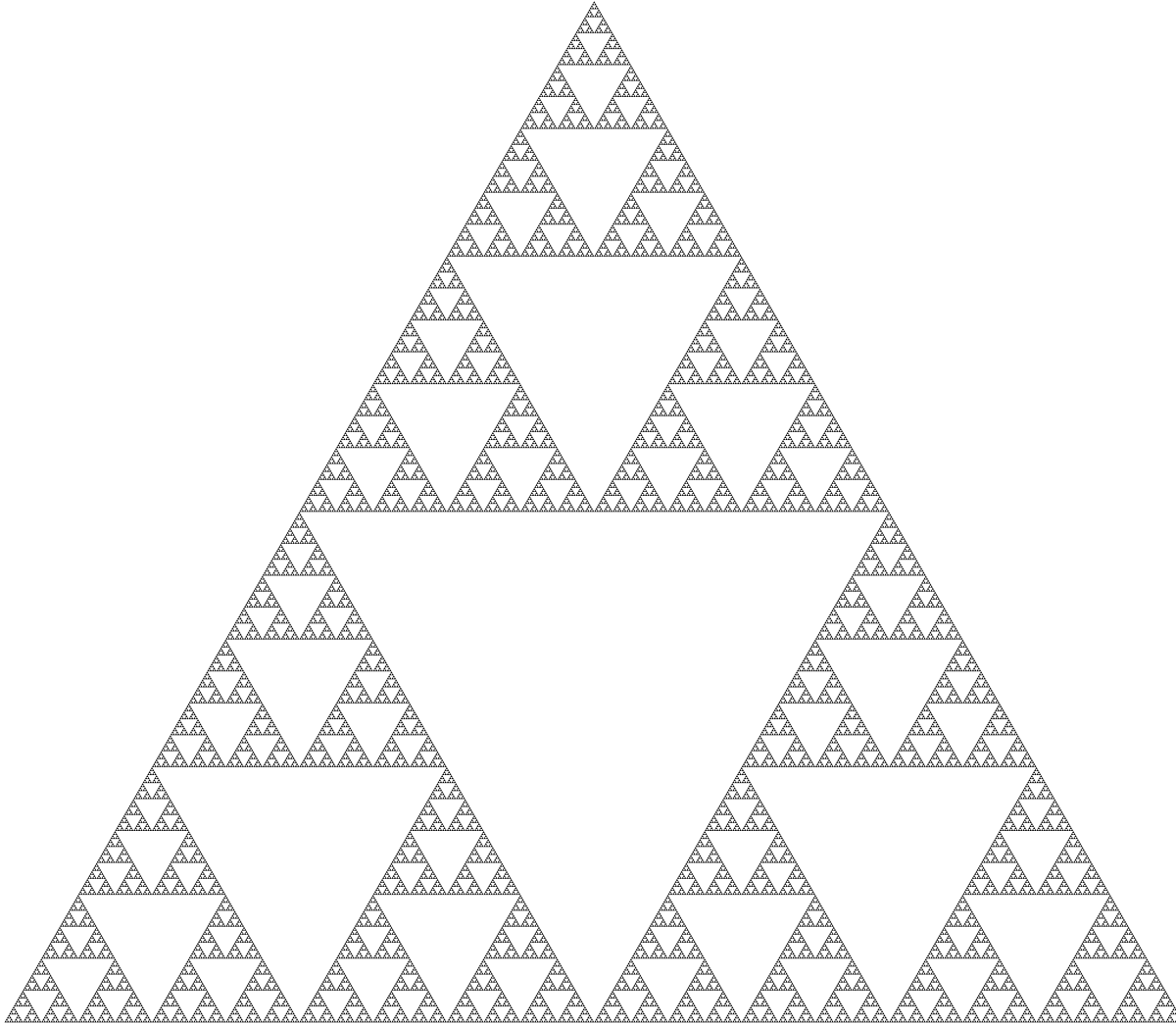
de buren bepalen de kleur







$x$	$y$	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0



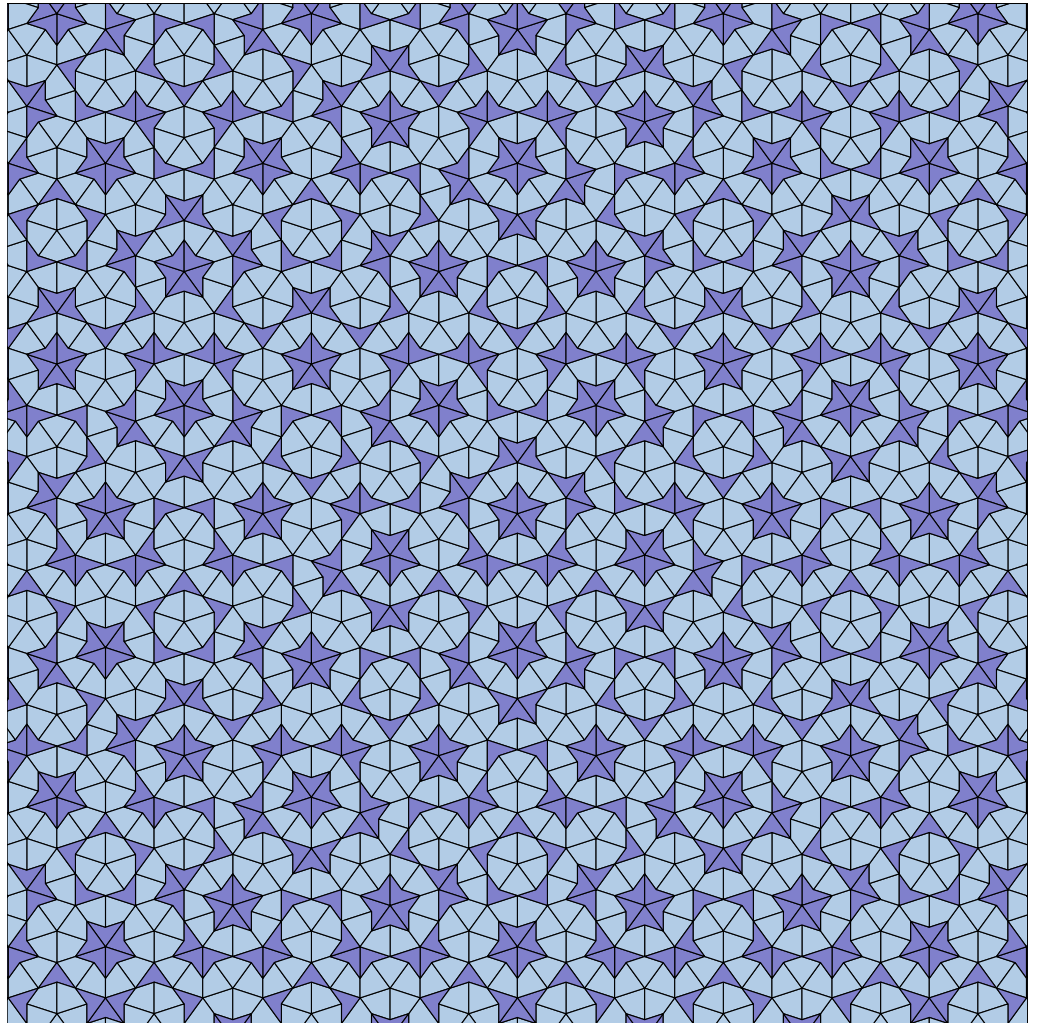
Qef's Website  
wikipedia



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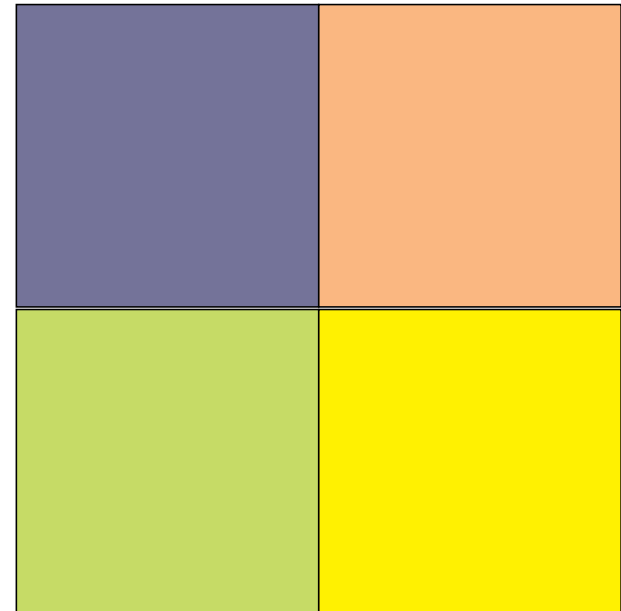
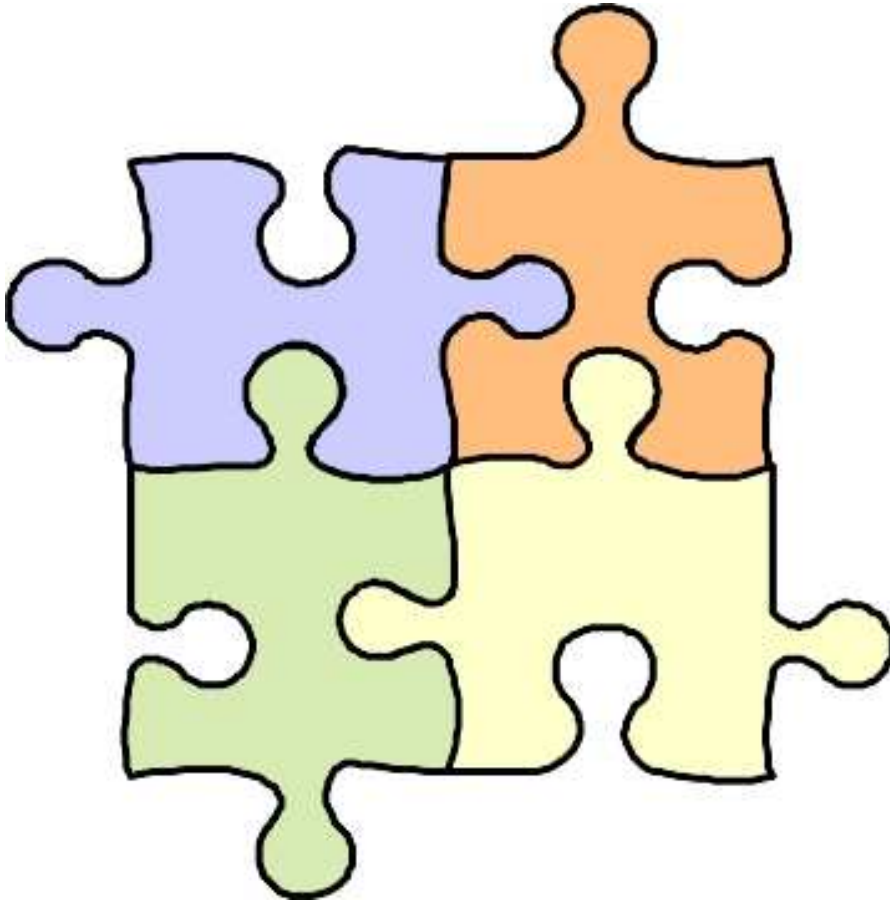
## ■ Spelregels

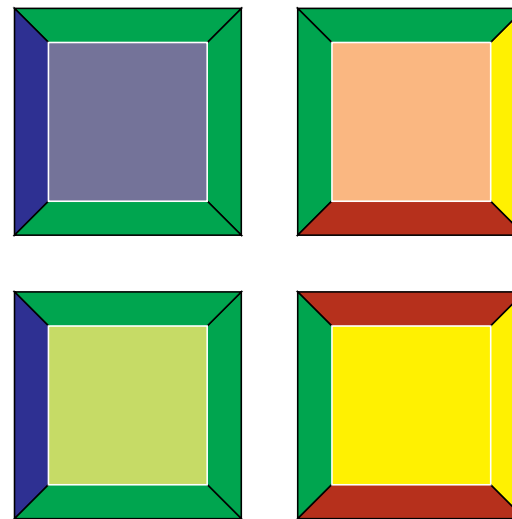
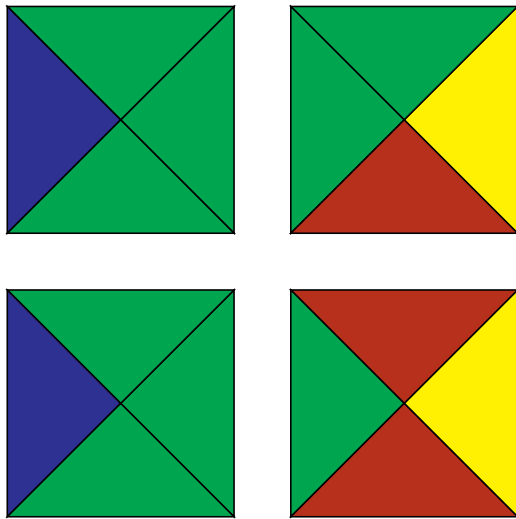
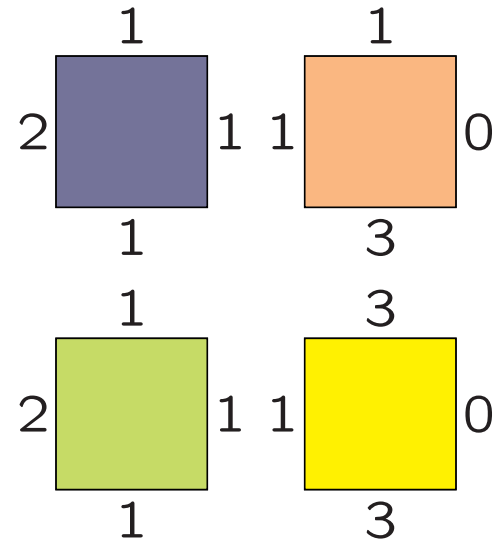
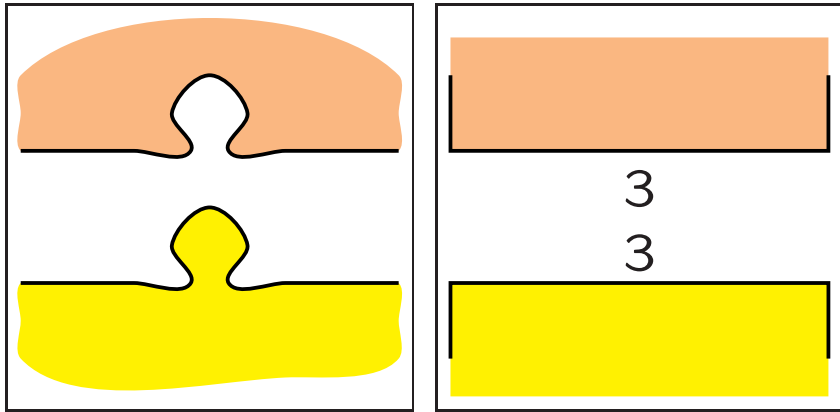
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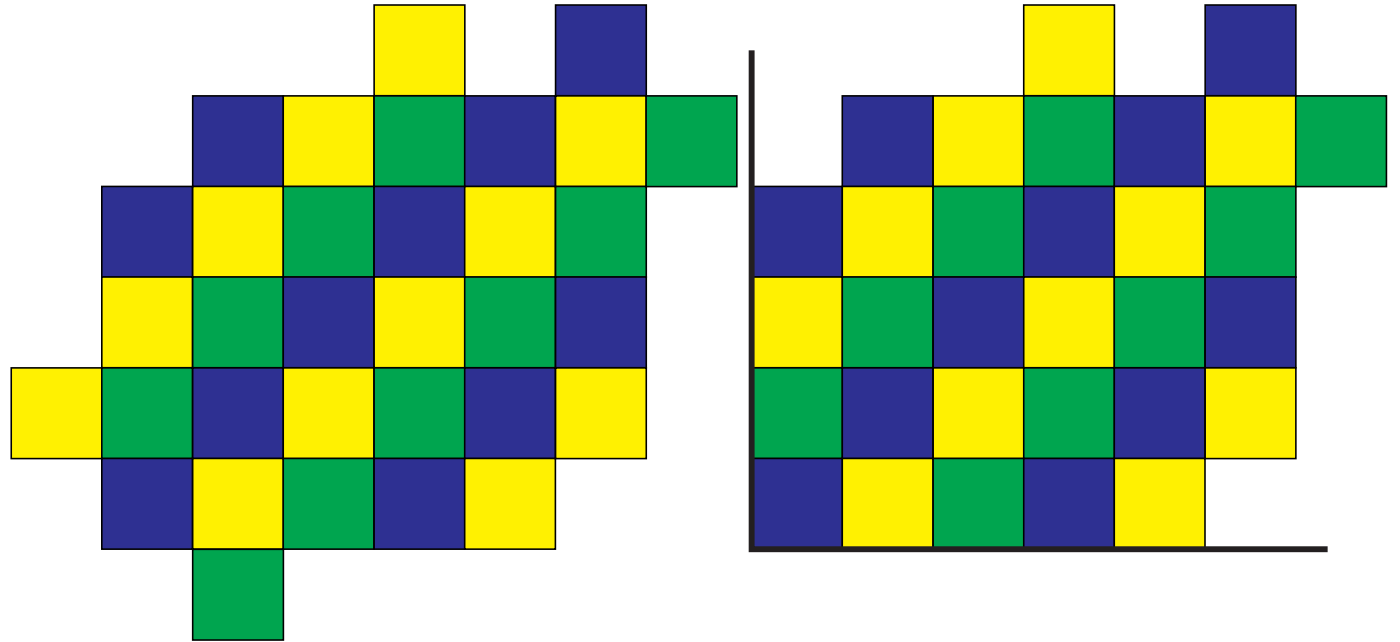
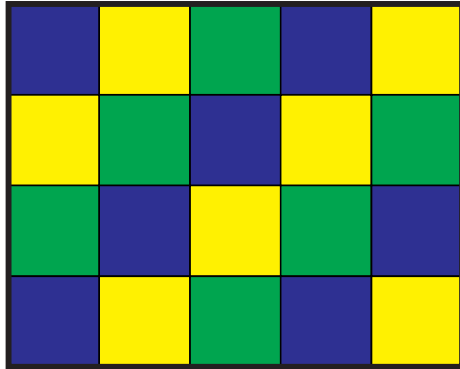


Penrose (1974)

© Franz Gähler, Bielefeld







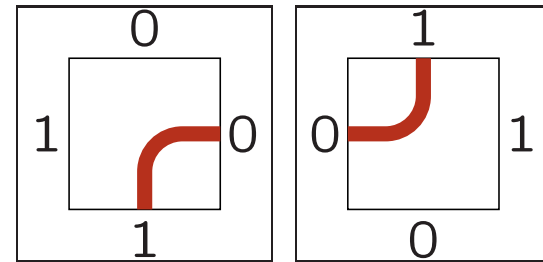
- rechthoek
- (half) vlak
- kwadrant



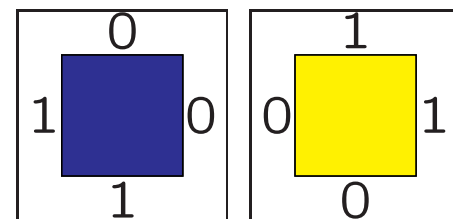
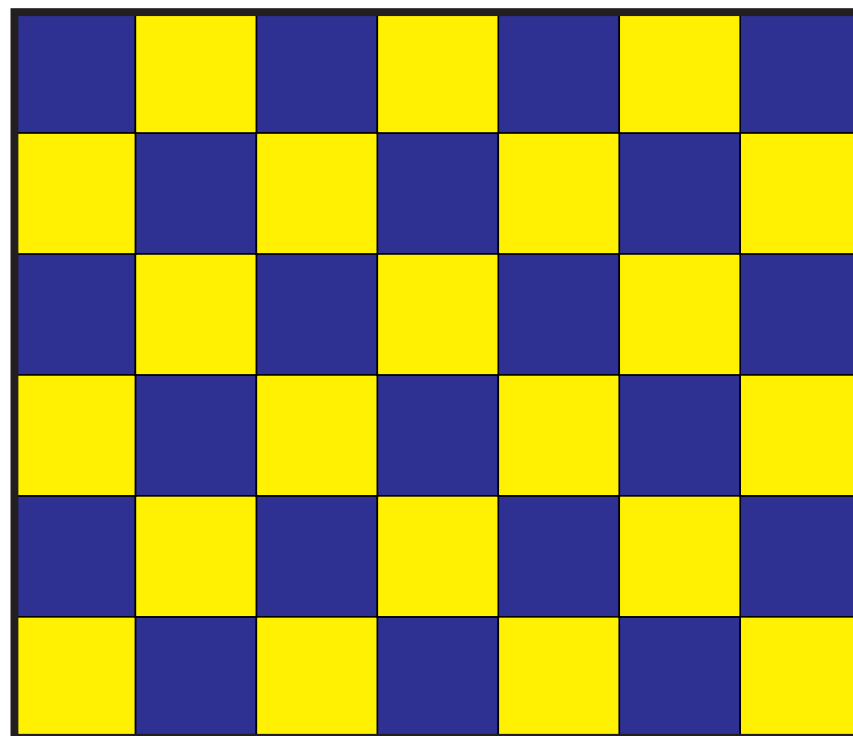
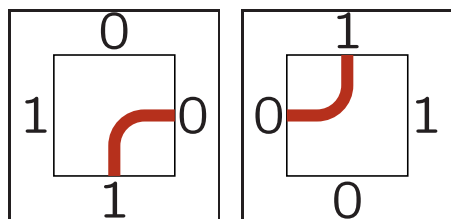
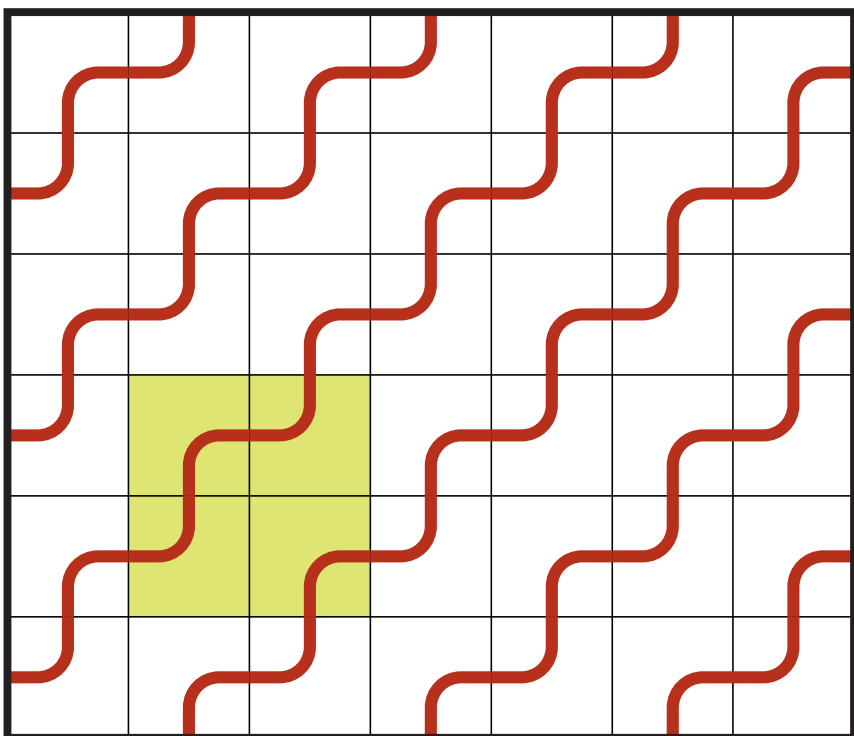
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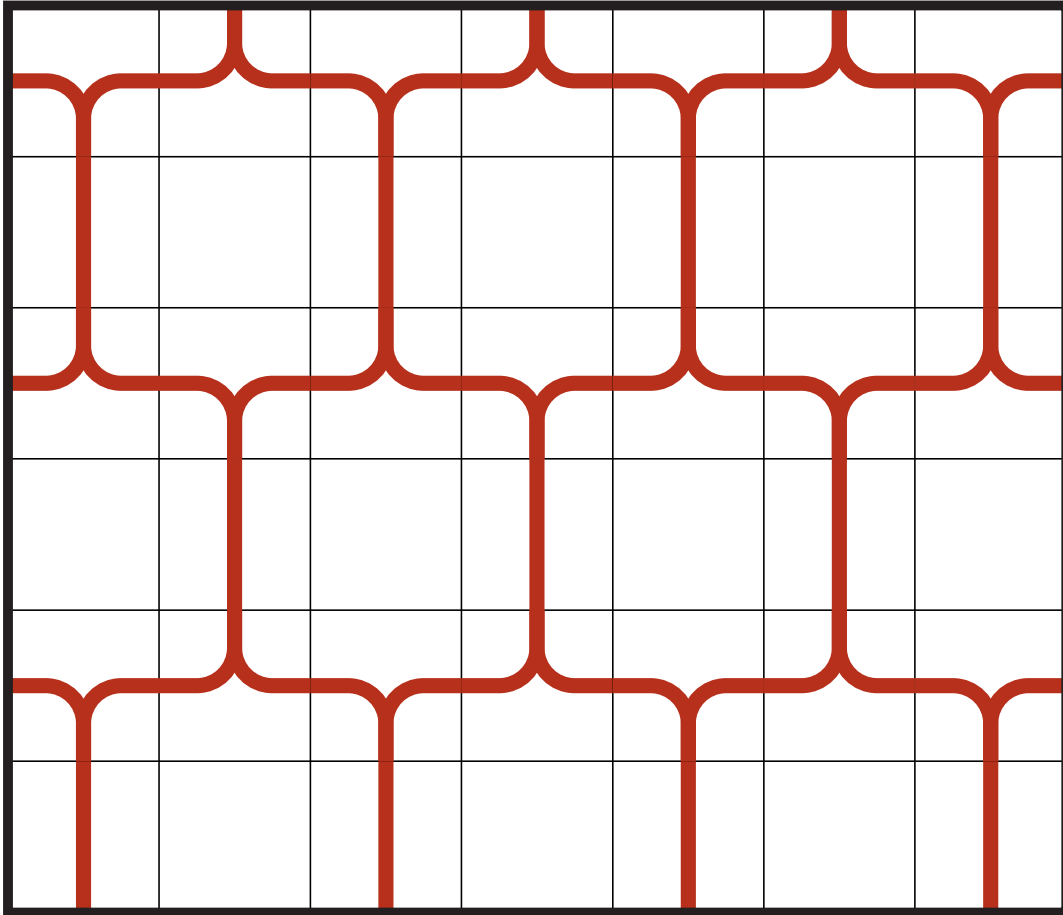
## ■ Patronen

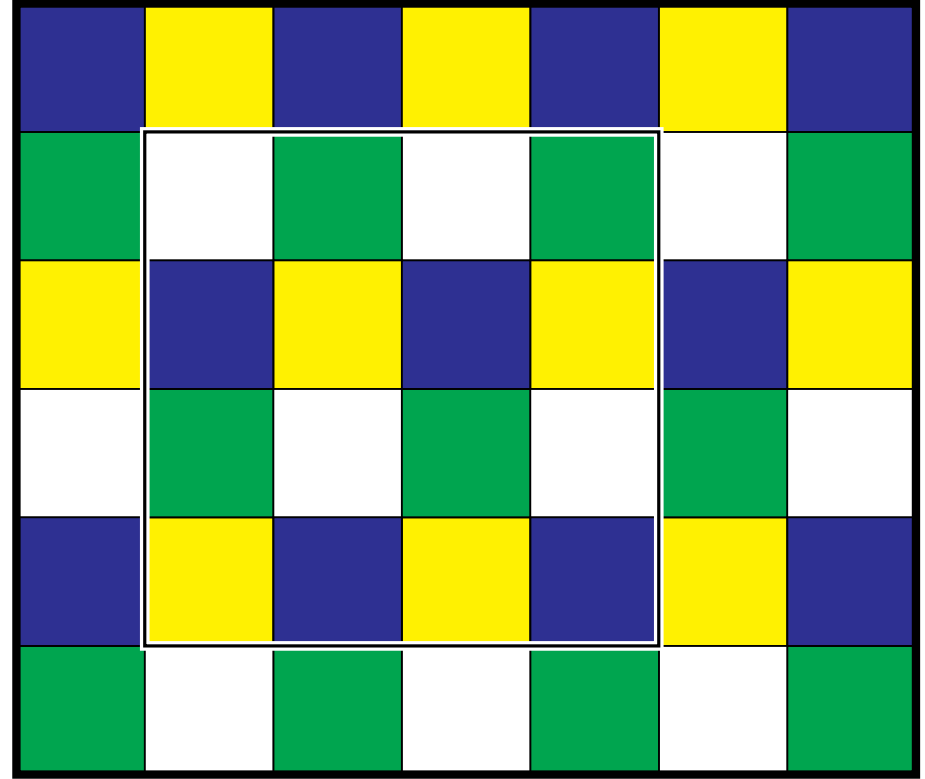
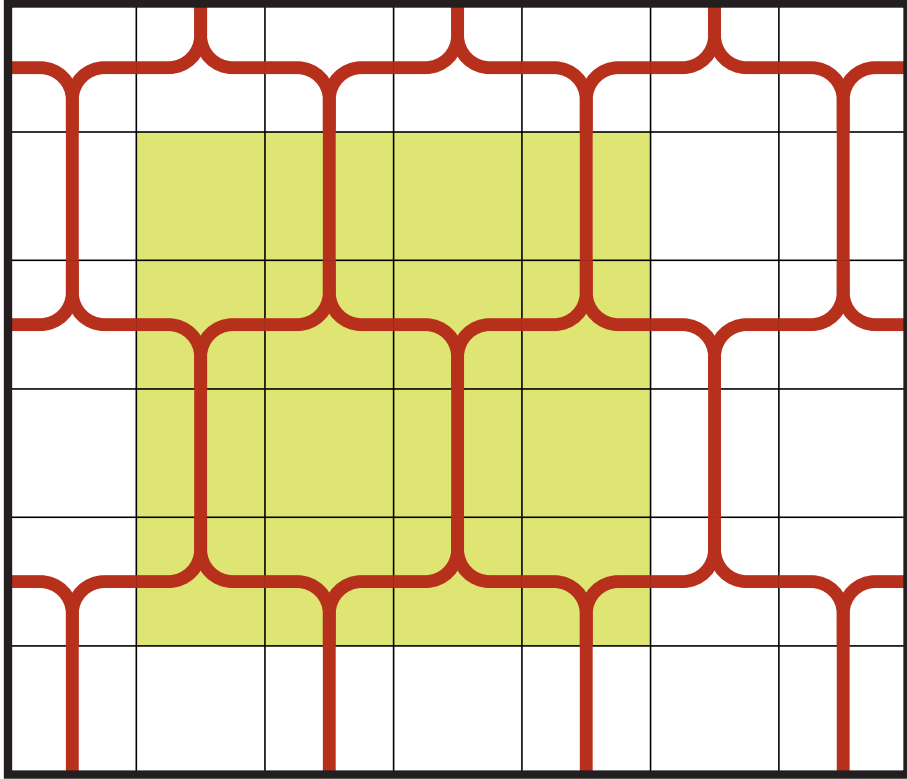
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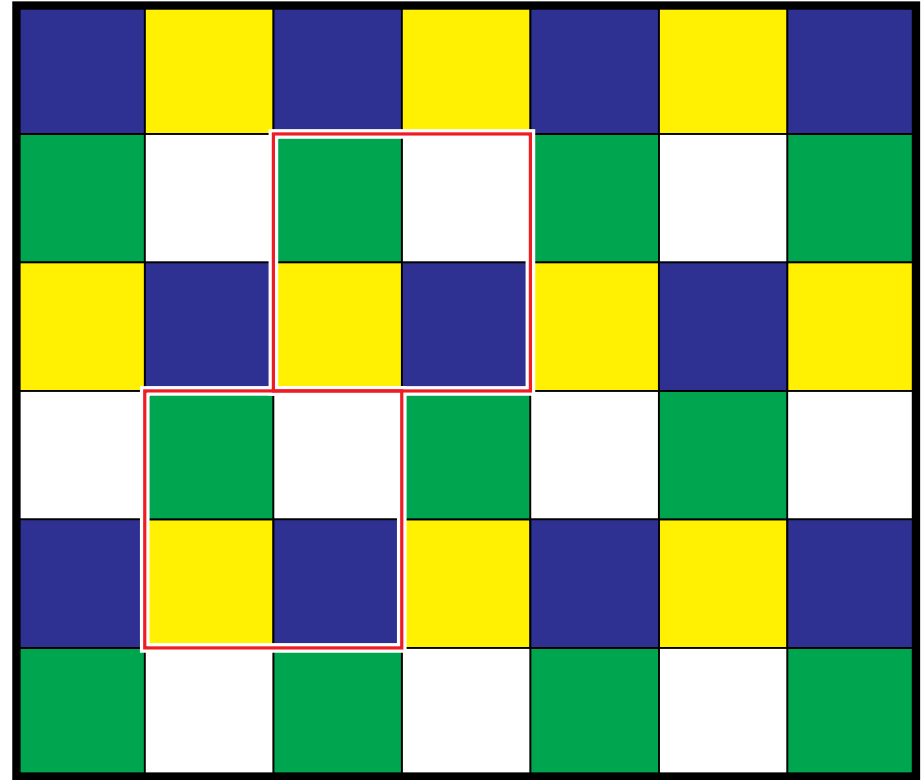
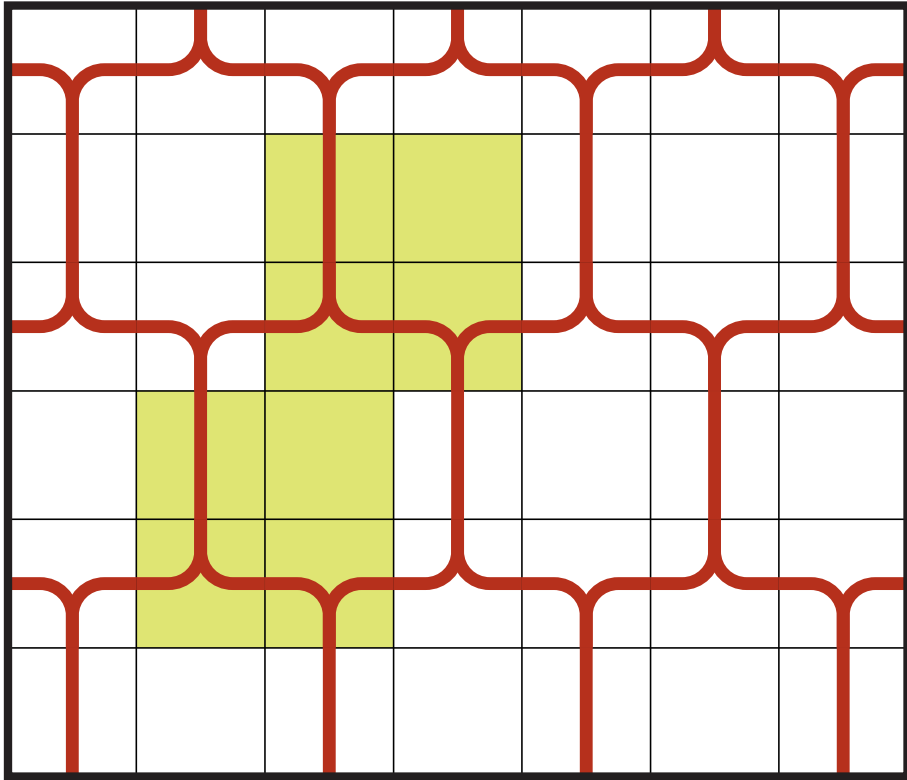


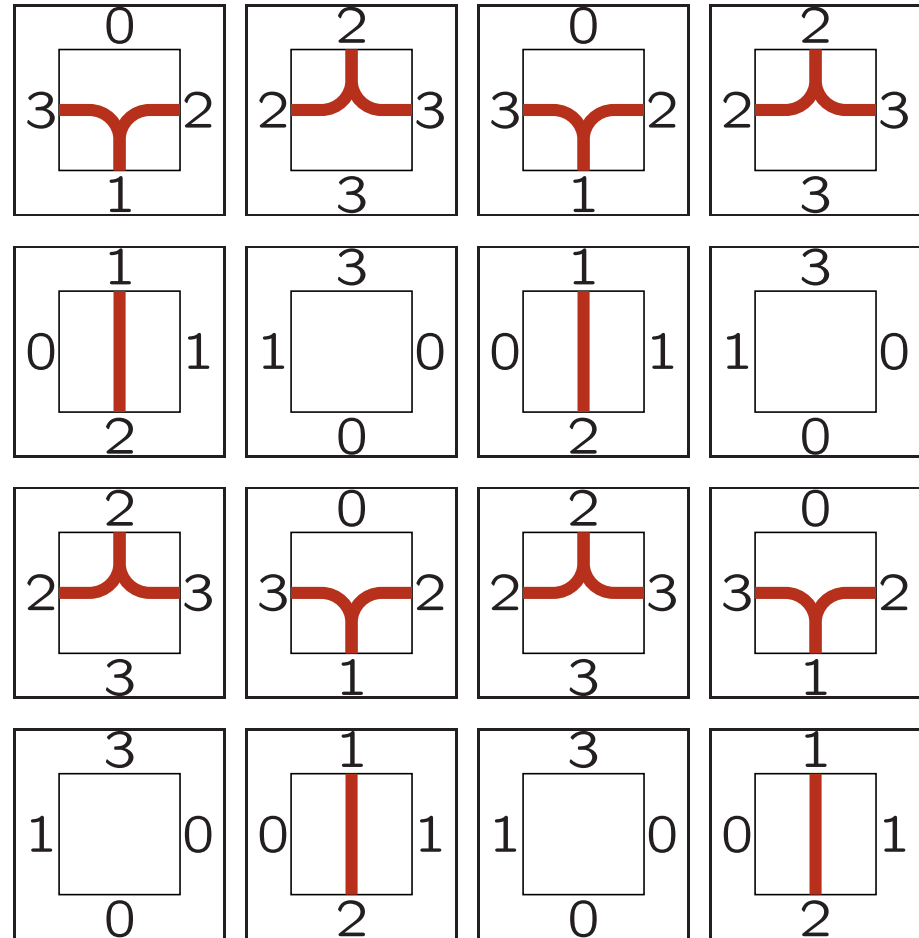
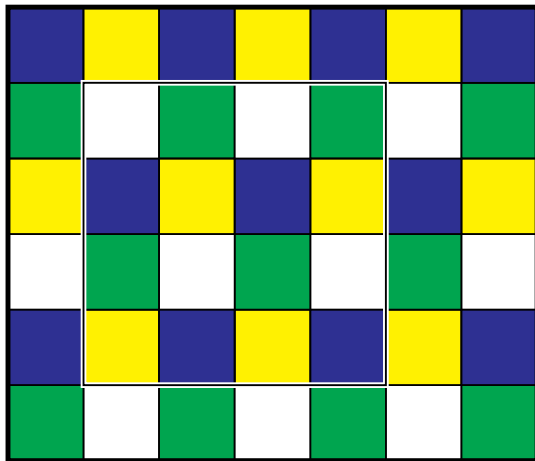
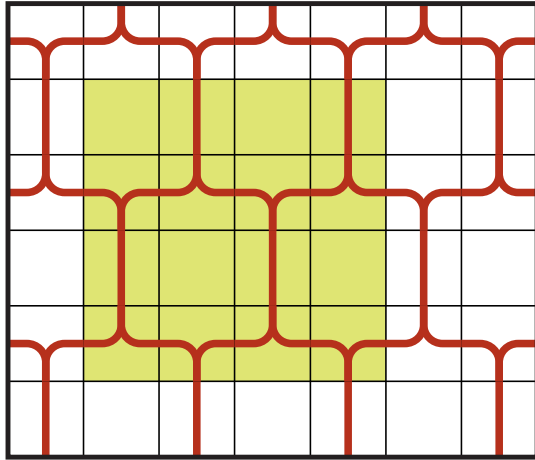
twee tegels



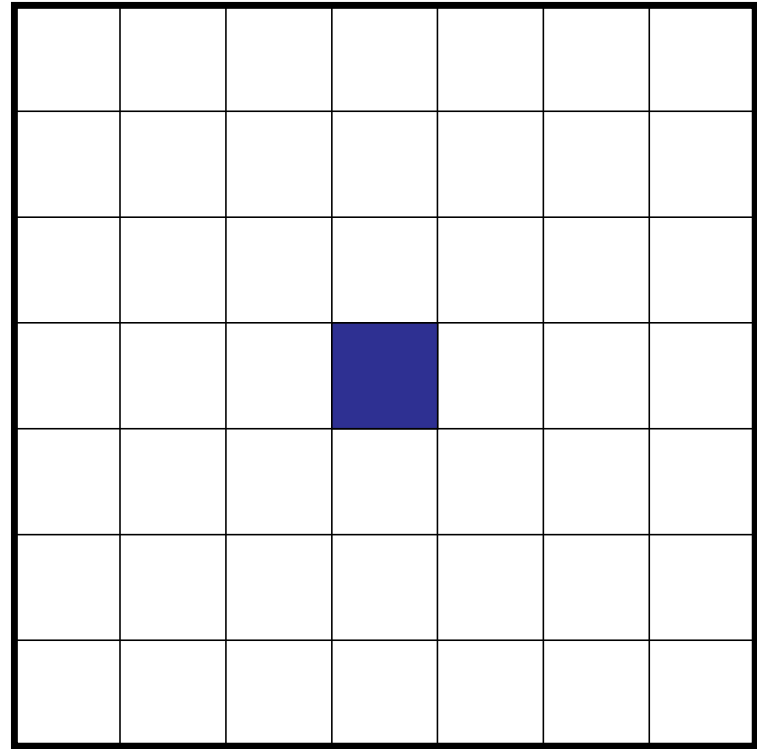
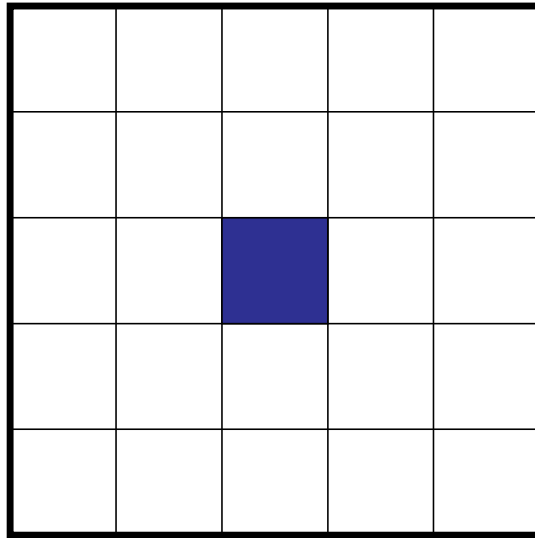
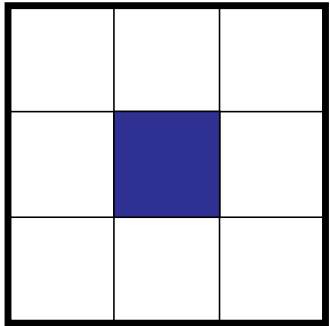






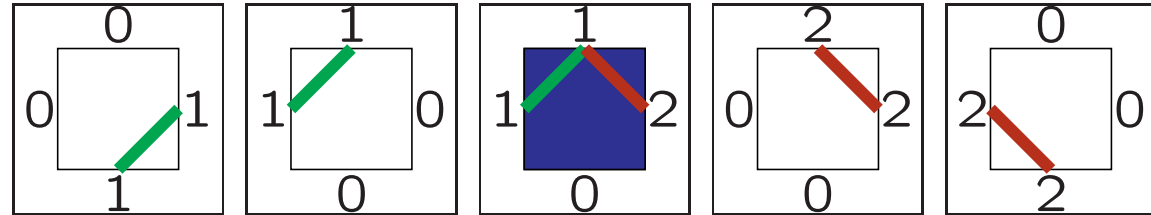
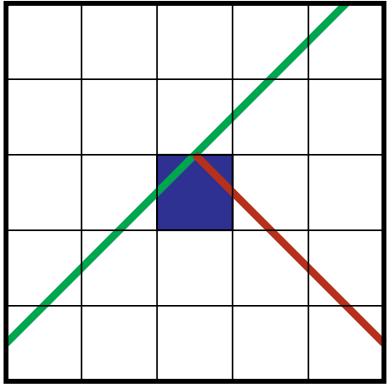


vier tegels  
(en eventueel randjes)

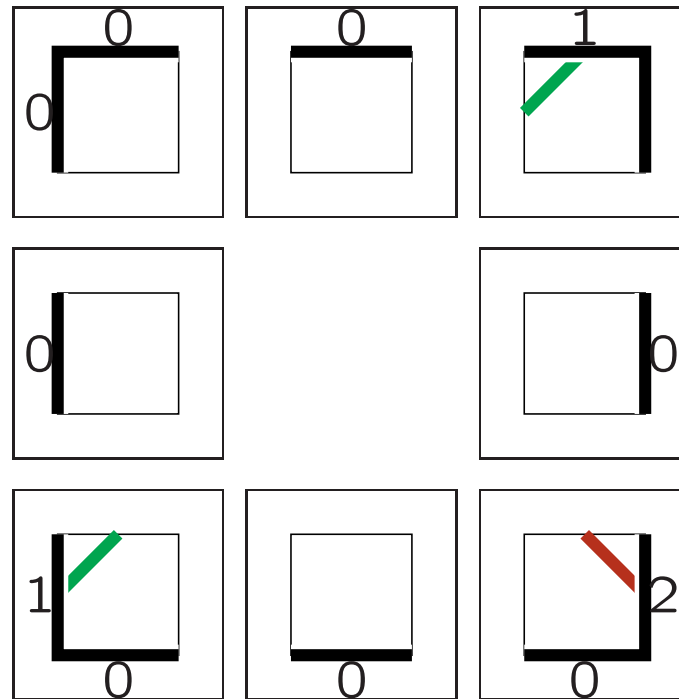


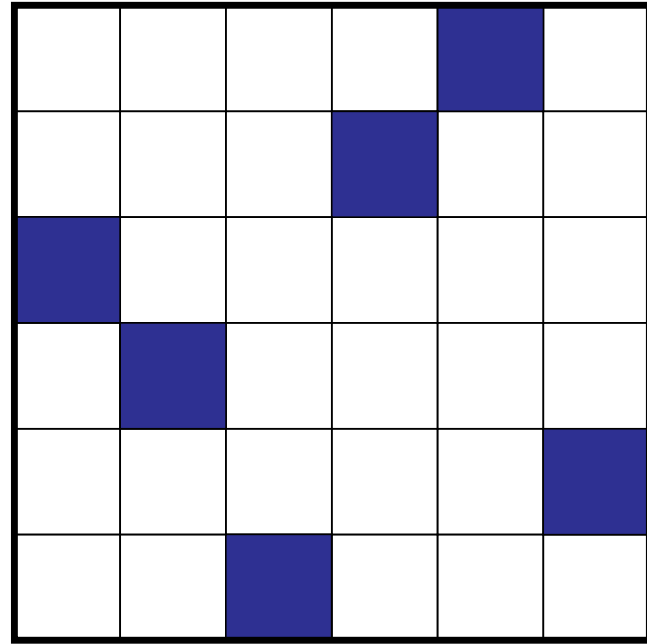
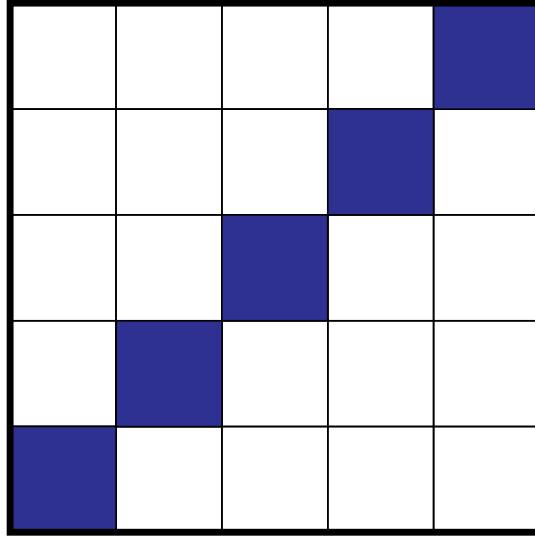
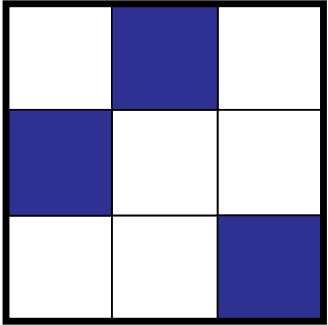


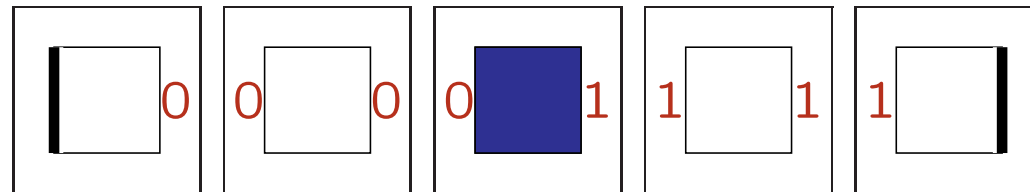
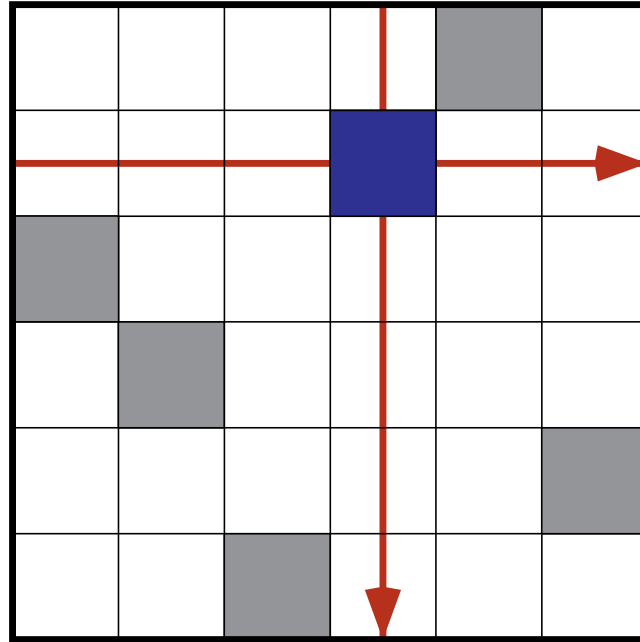


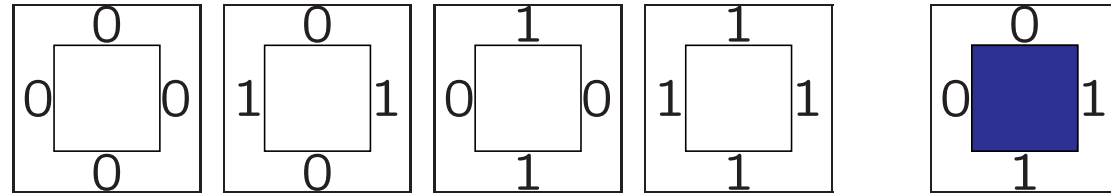
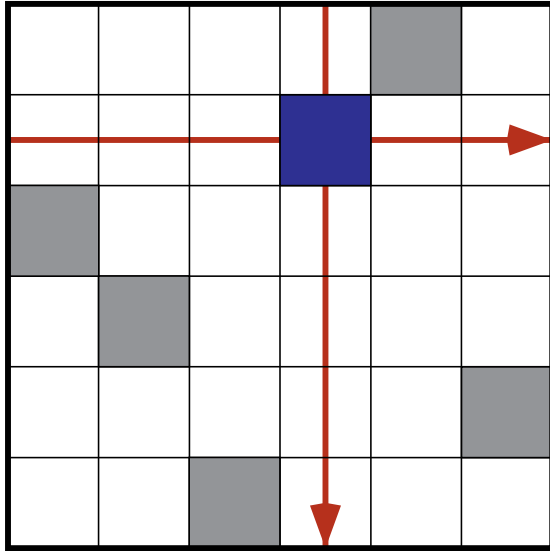


randen

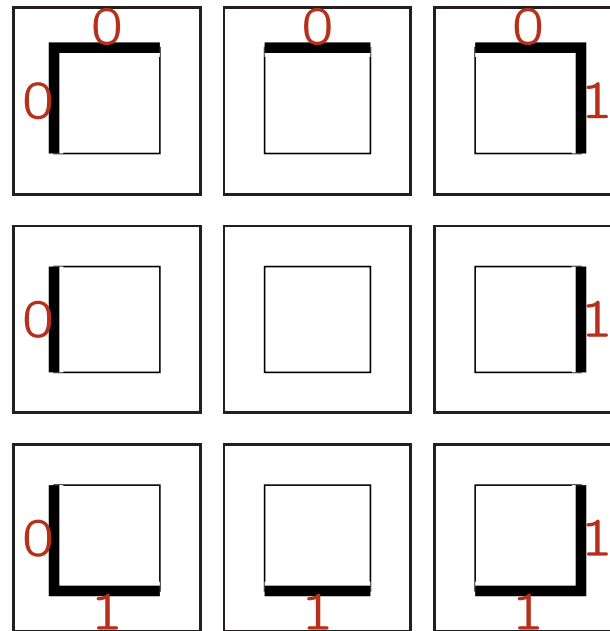






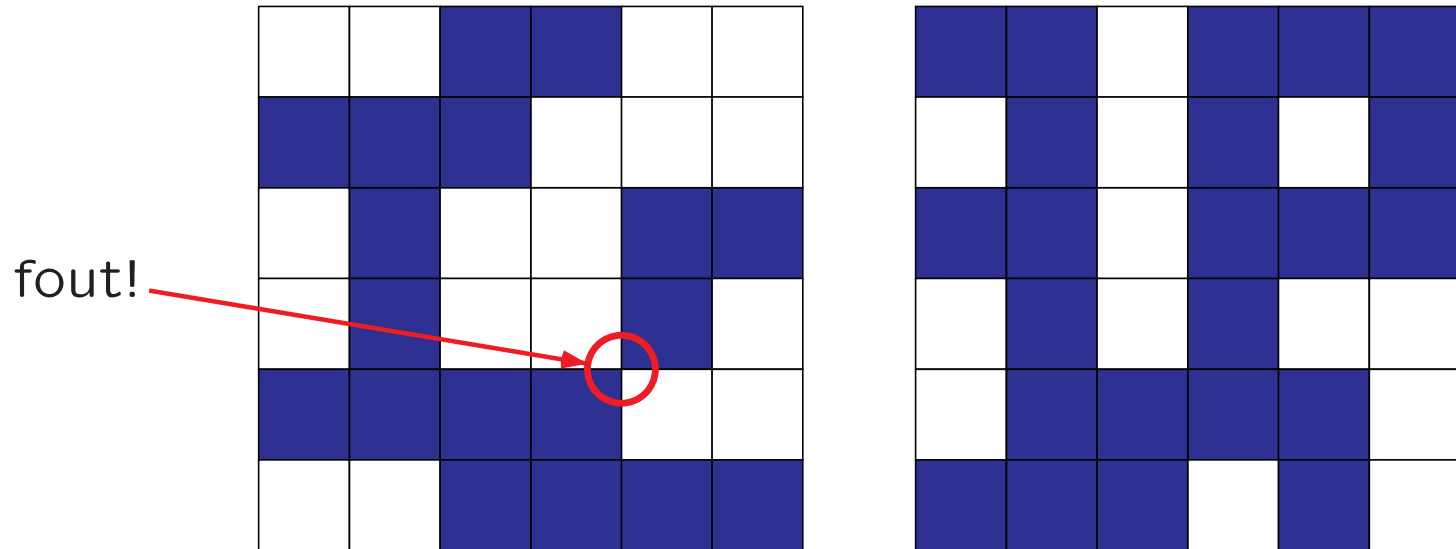


links/boven 0 rechts/onder 1

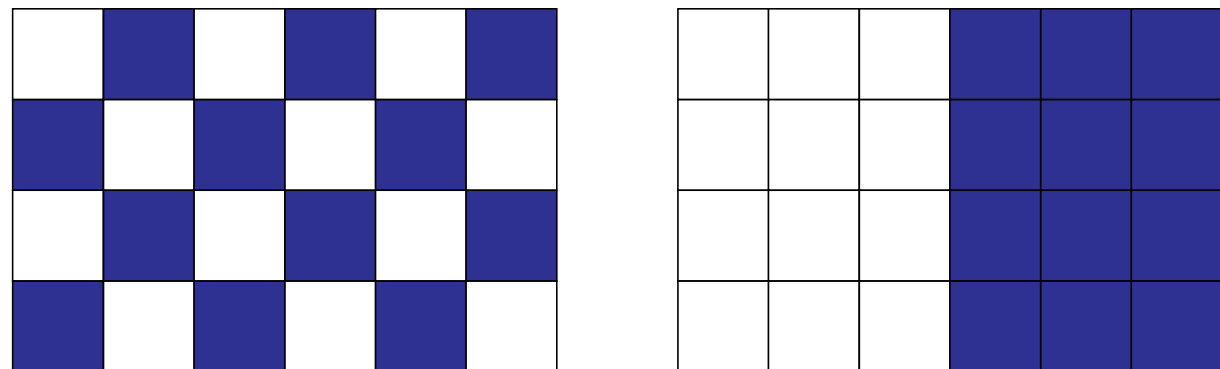


Hoeveel tegels bij twee torens per rij/kolom?

- alle blauwe tegels zijn verbonden (*moeilijk*)



- evenveel van beide kleuren (*heel moeilijk*)



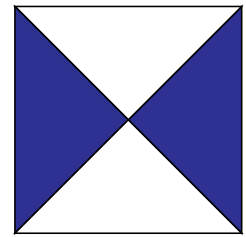
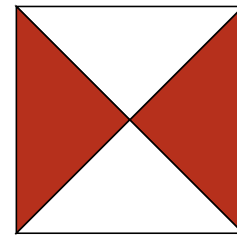
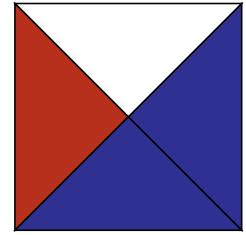
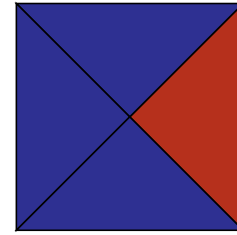
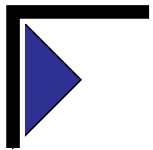


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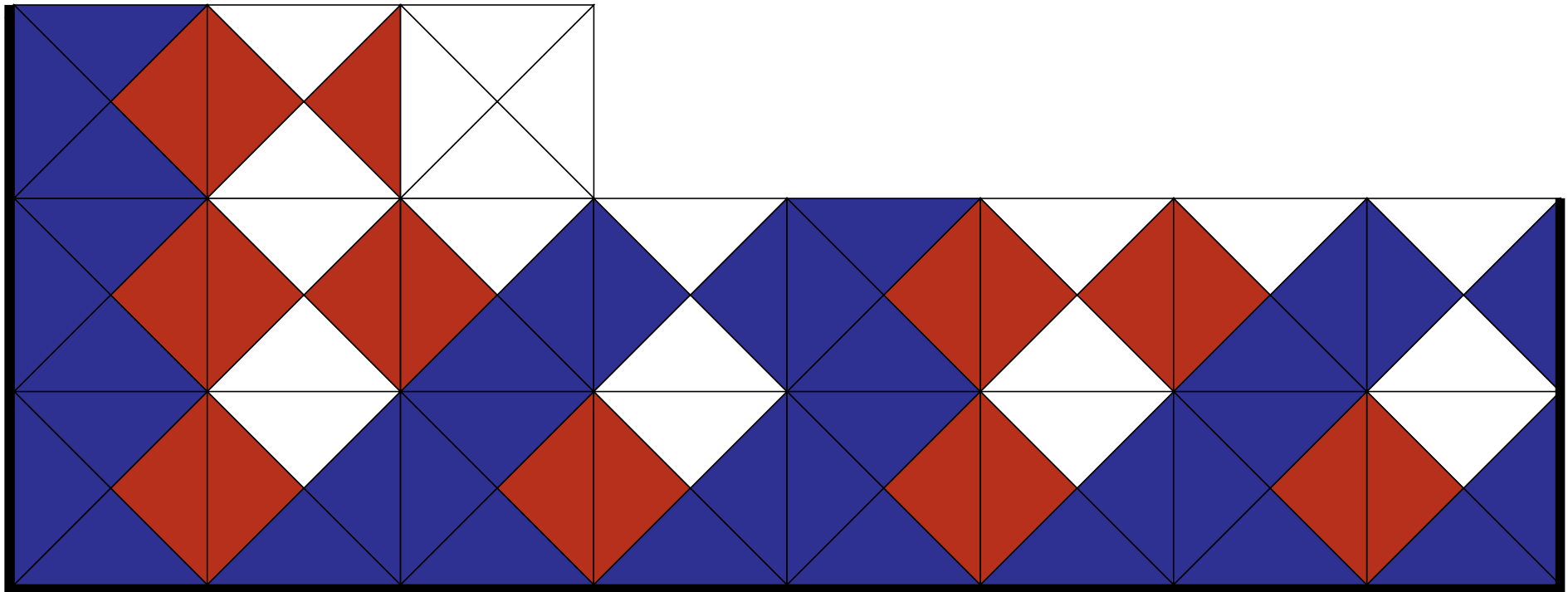
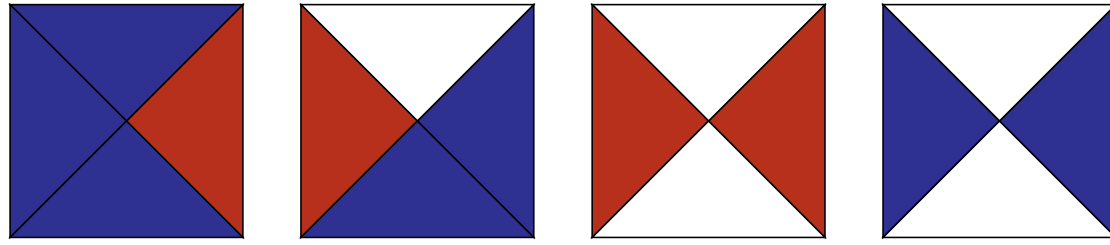
## ■ Rekenen

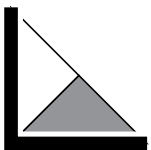
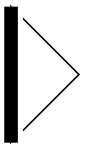
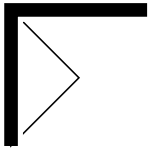
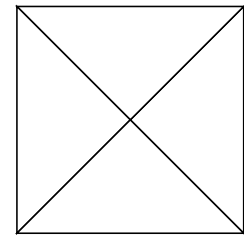
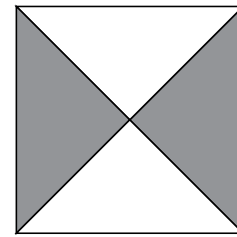
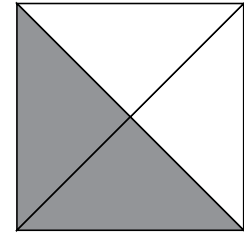
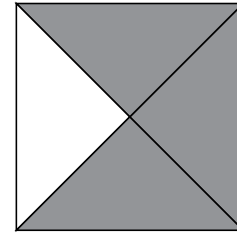
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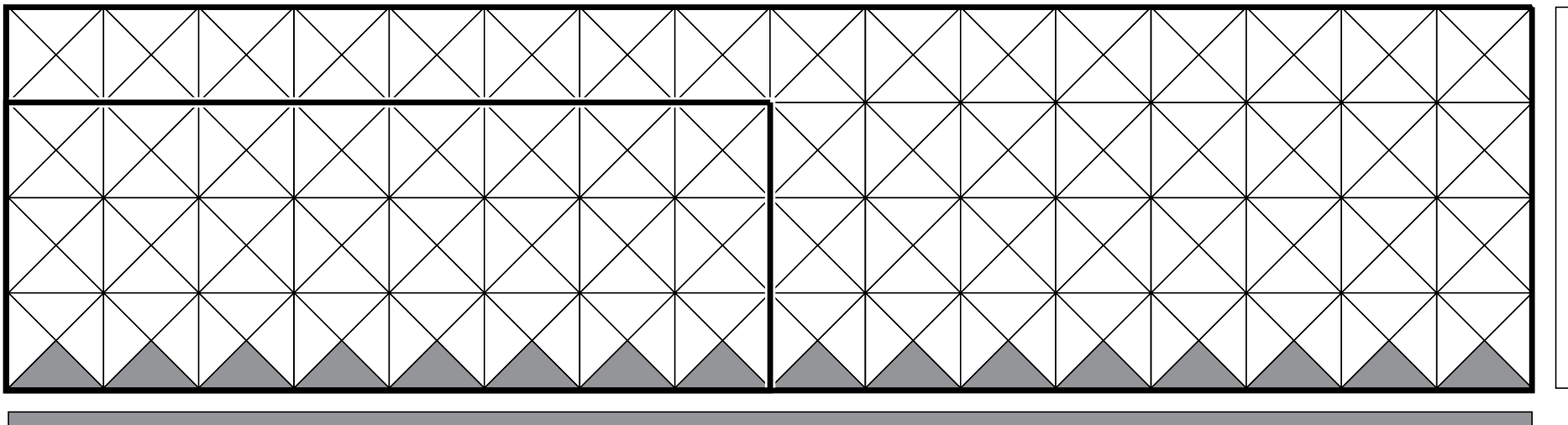
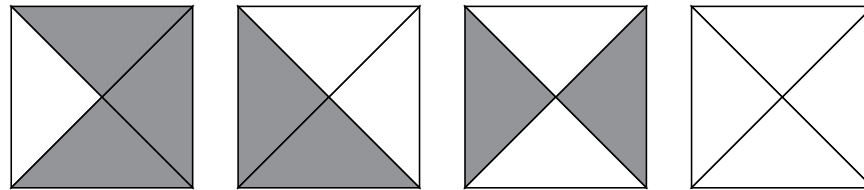
blauw onder en rechts  
begin linksonder

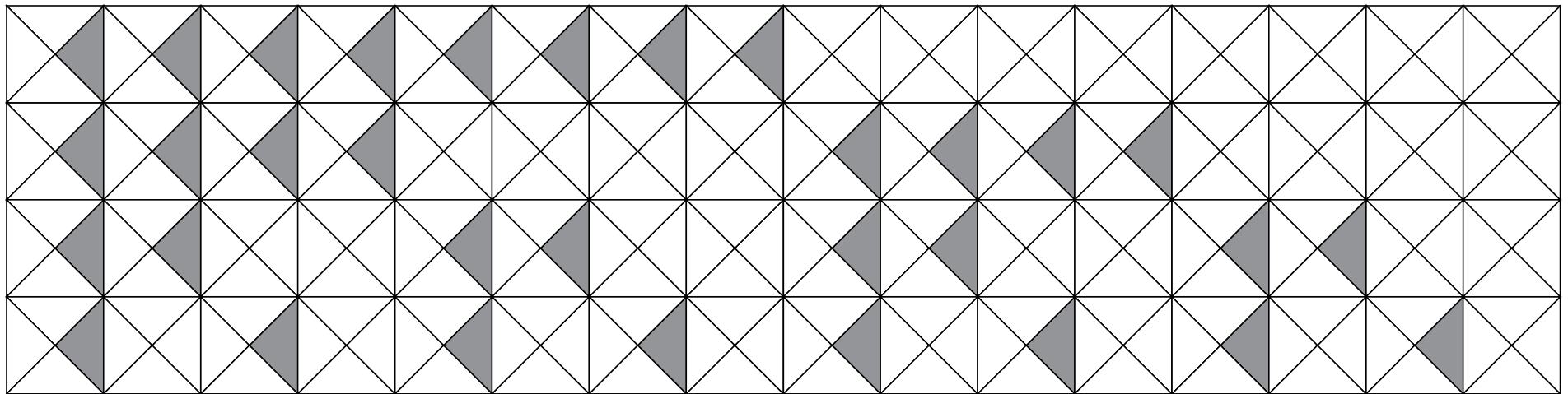
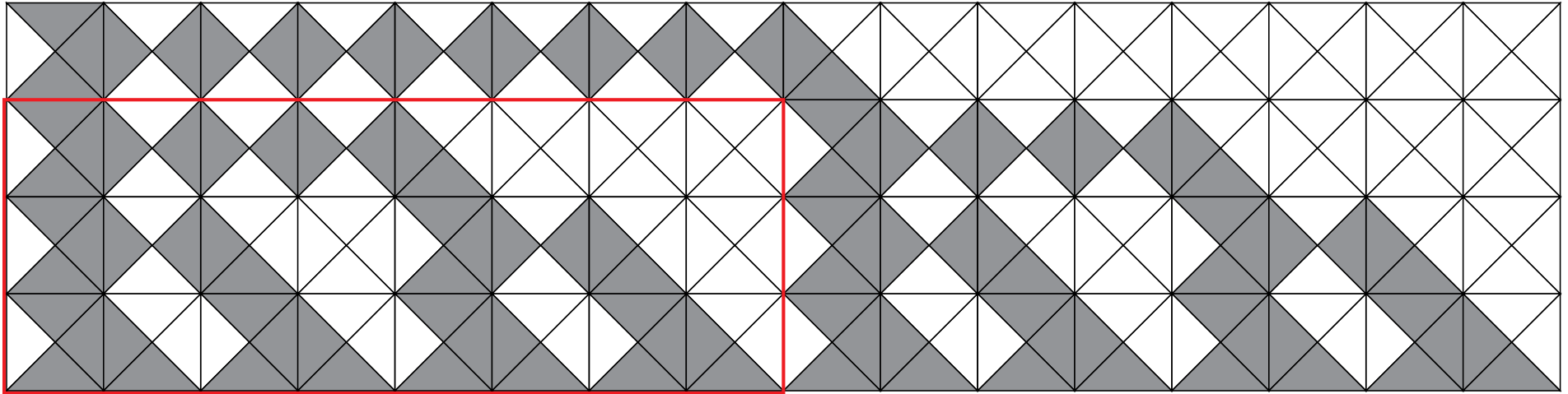


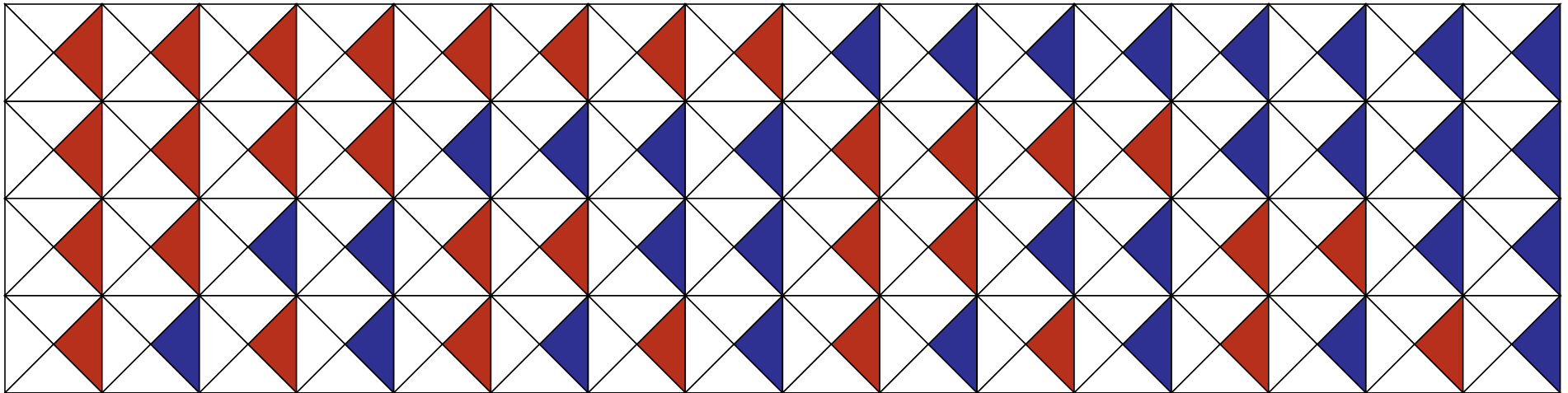
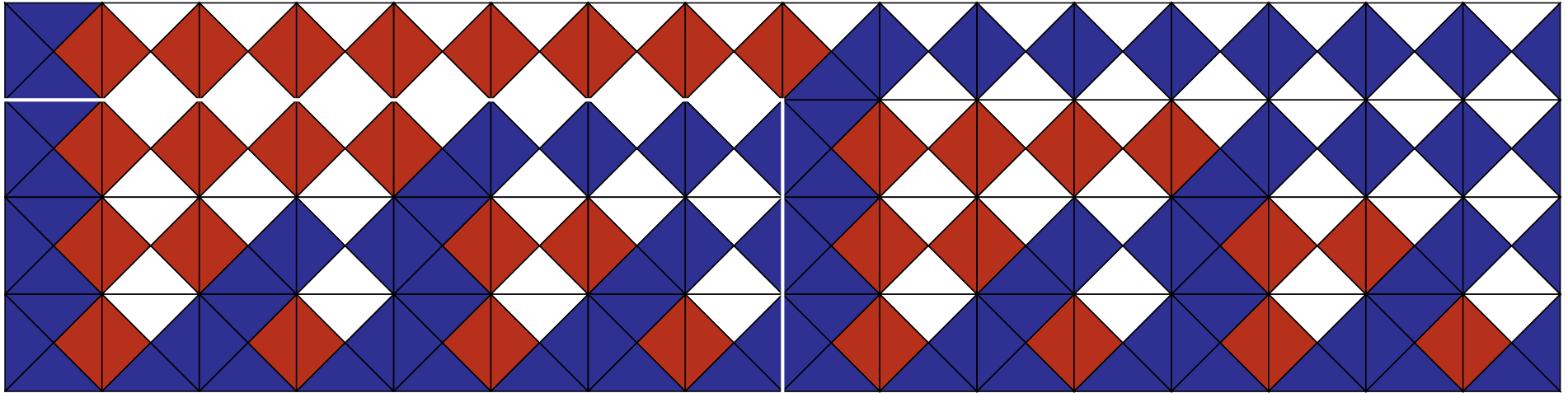


rechthoeken:  
8×3 & 16×4

links&rechts wit, onder zwart







tientallig: machten van tien

$$3418_{10} = 3 \cdot 10^3 + 4 \cdot 10^2 + 1 \cdot 10^1 + 8 \cdot 10^0$$

binair: machten van twee

32, 16, 8, 4, 2, 1

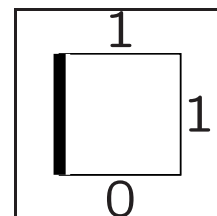
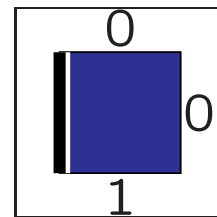
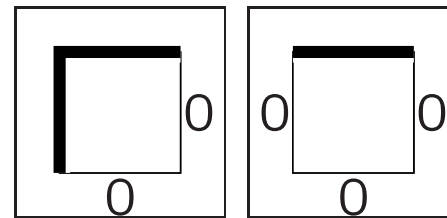
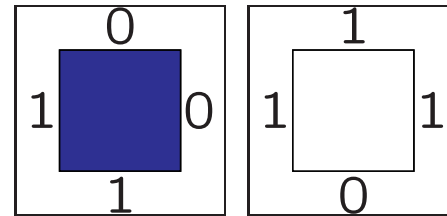
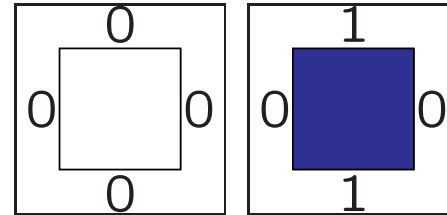
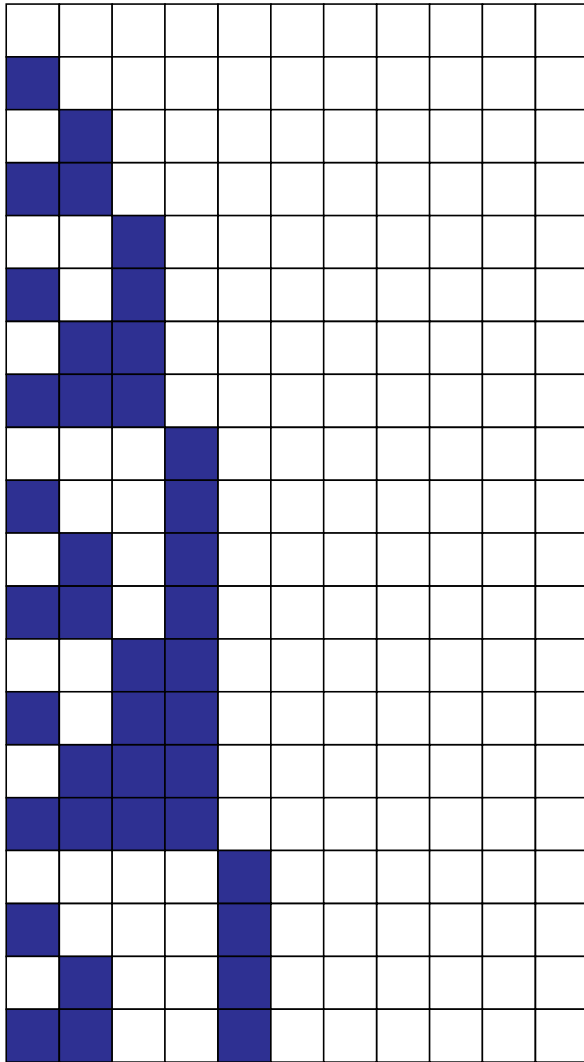
13	
6	1
3	0
1	1
0	1

$$1101_2 = 2^3 + 2^2 + 2^0 = 8 + 4 + 1 = 13$$

0, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010,  
1011, 1100, 1101, 1110, 1111, 10000, ...



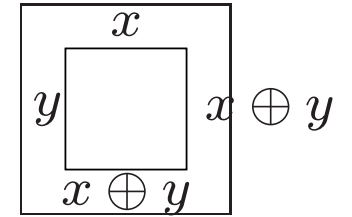
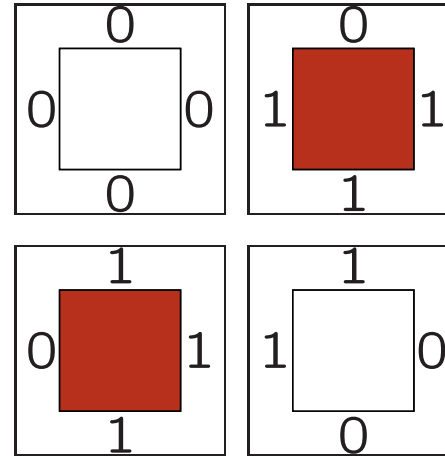
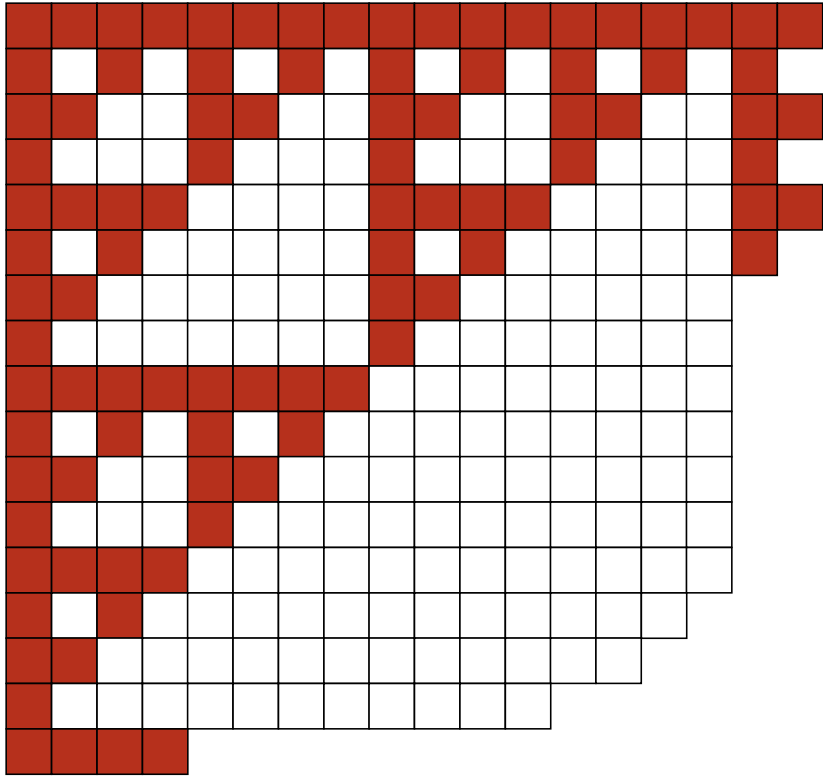




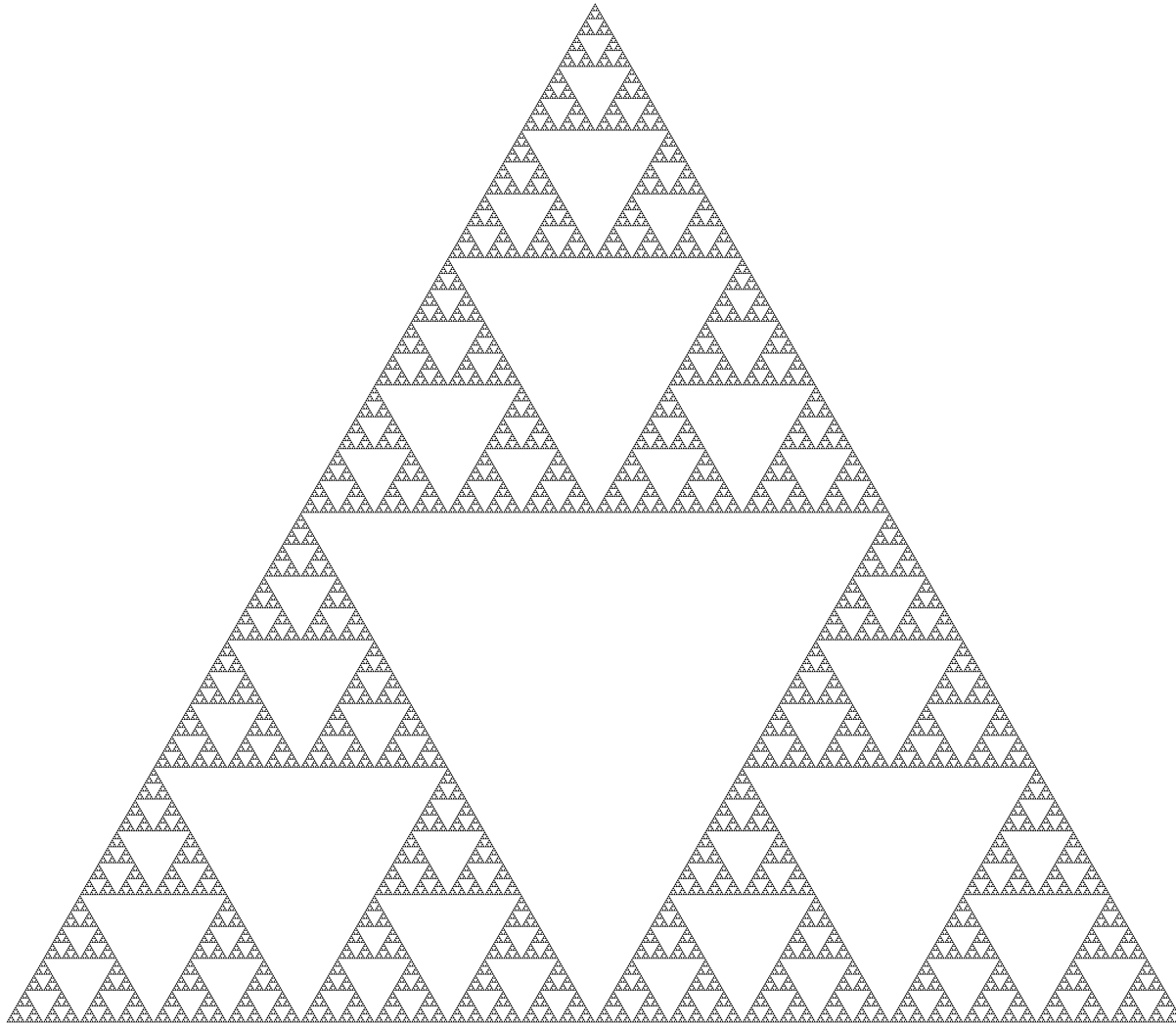
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## ■ Self-Assembly

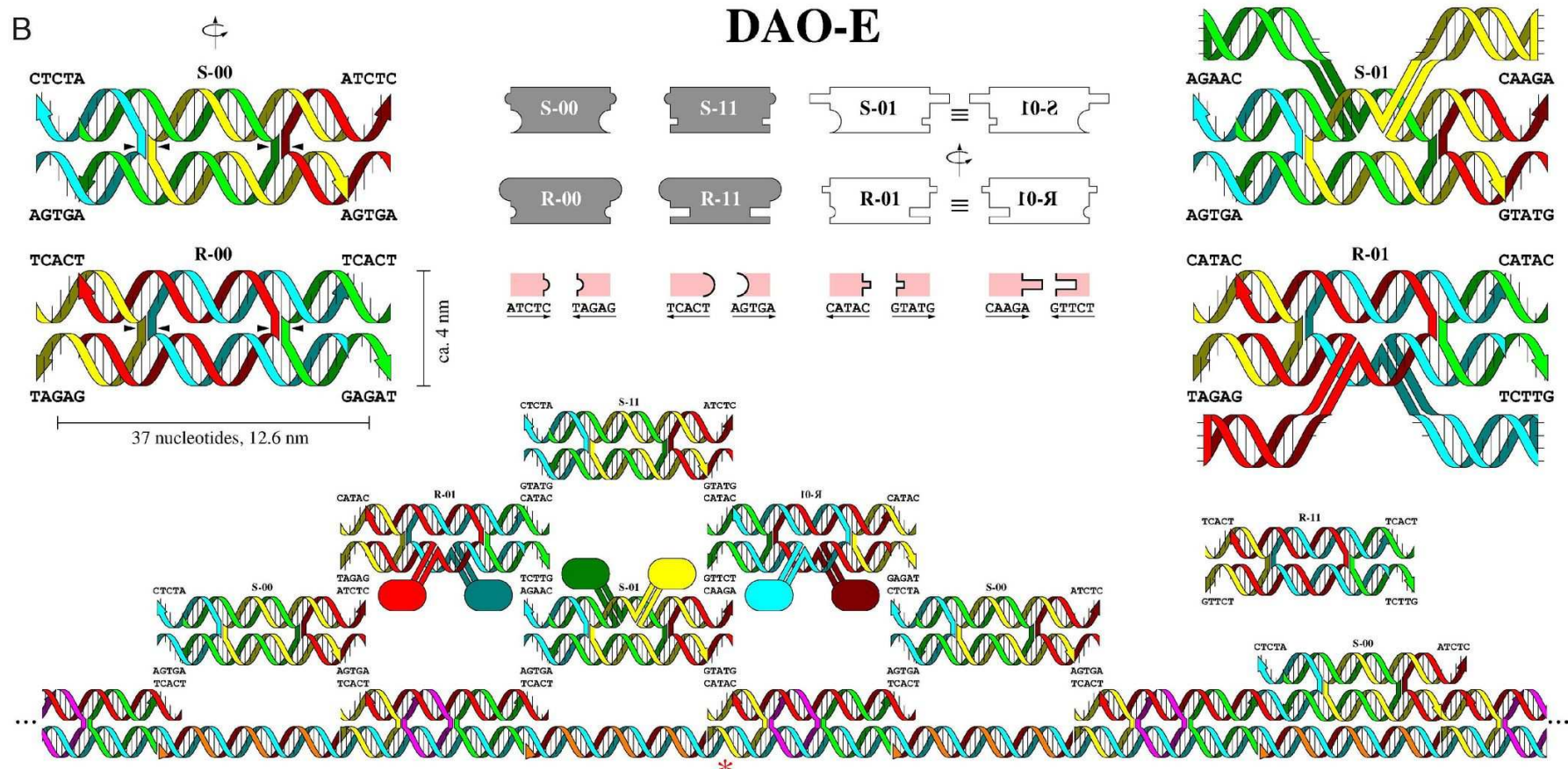
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$x$	$y$	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

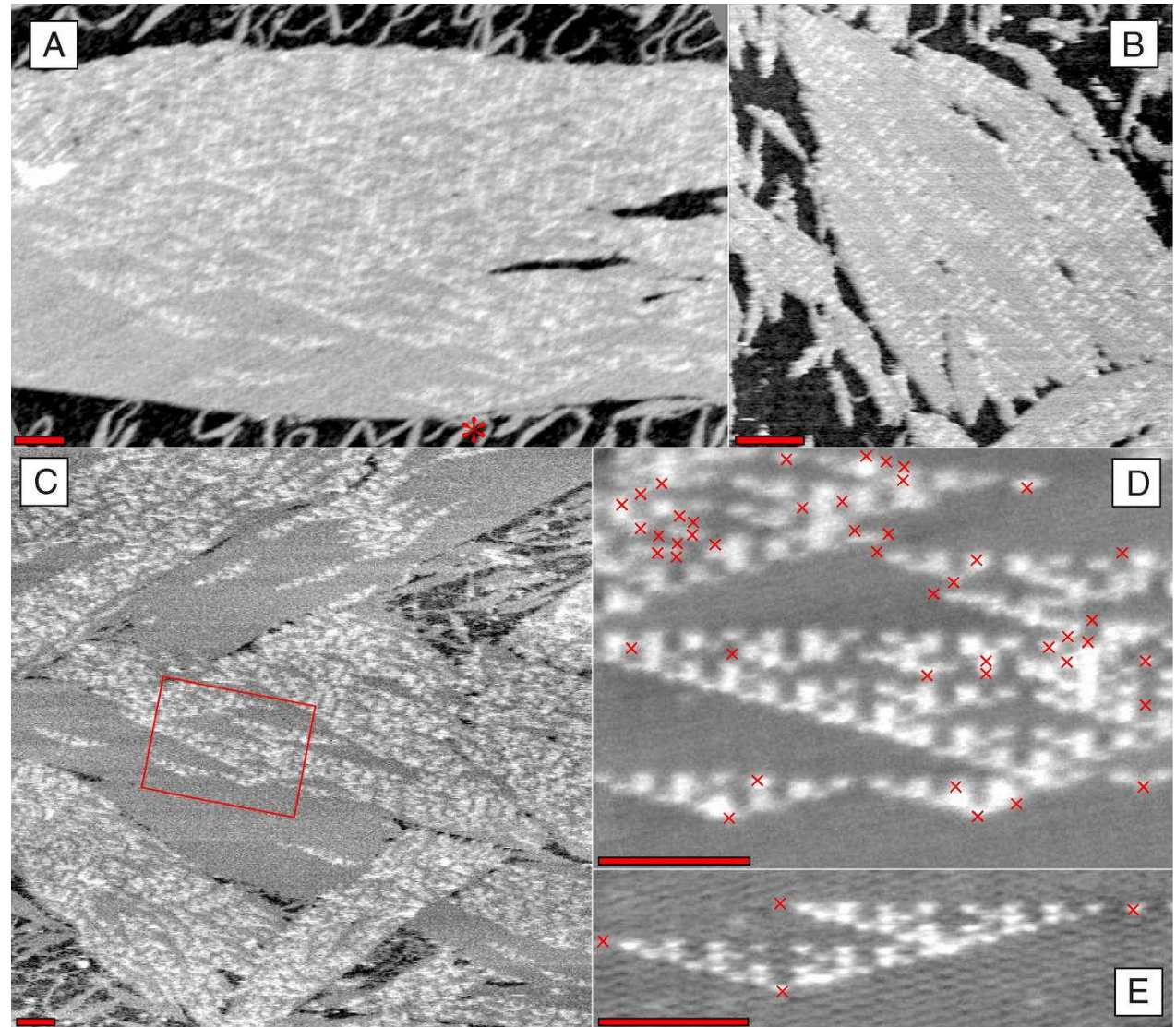


Qef's Website  
wikipedia



Algorithmic Self-Assembly of DNA Sierpinski Triangles (2004)

Rothmund, Papadakis, Winfree; PLoS Biology









© Peter Ruoff, Stavanger

*Conway's game of life*  
2-dim cellulaire automaat



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## ■ Afsluiting

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Alan Mathison Turing,  
FRS OBE, 1912 – 1954

*computability*

wát kunnen we berekenen?

*Turingmachine*

*Enigma*

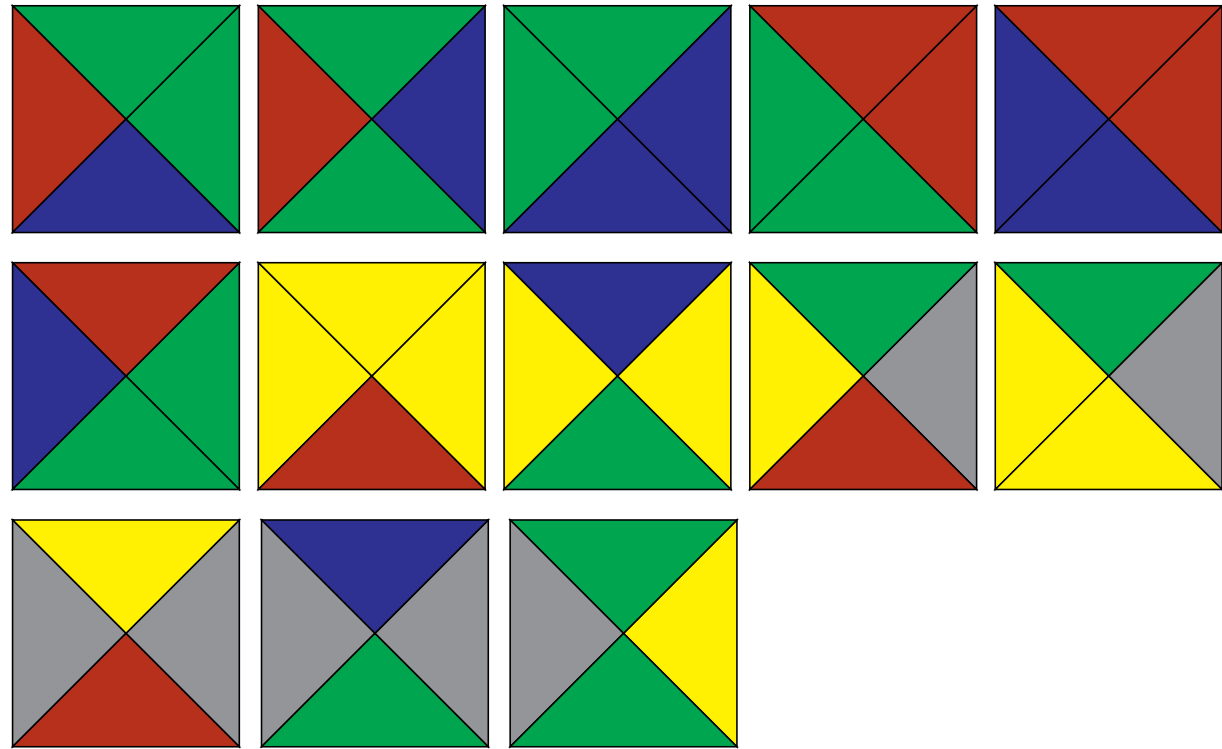
*breaking the code*

*artificial intelligence*

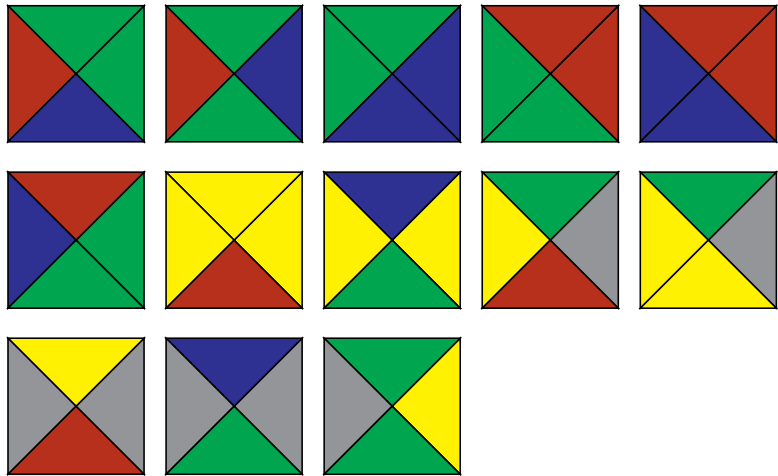
Turing test

*morfogenese*

biologische patroonvorming



Wang tiles, 1961



patroon zonder regelmaat (lastig)

Karel Culik II, 1996

invoer: verzameling tegels

gevraagd: bestaat er een  
passende betegeling van het vlak  
(van een rechthoek) ?

*er is géén algoritme dat dit  
probleem oplost*

Berger 1966

(echt niet!)