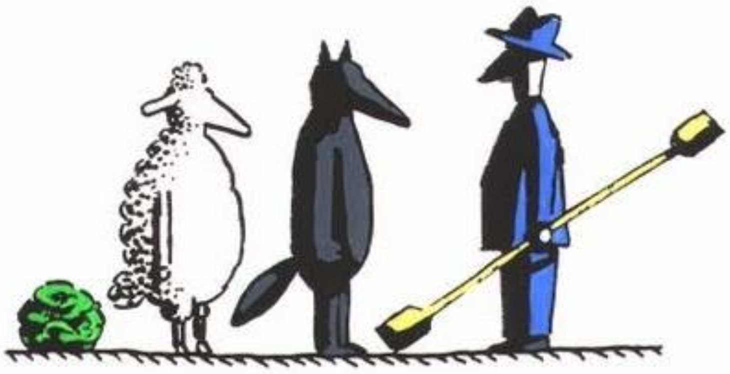


kool, geit en wolf

algoritmiek, NP-compleet en een beetje latijn

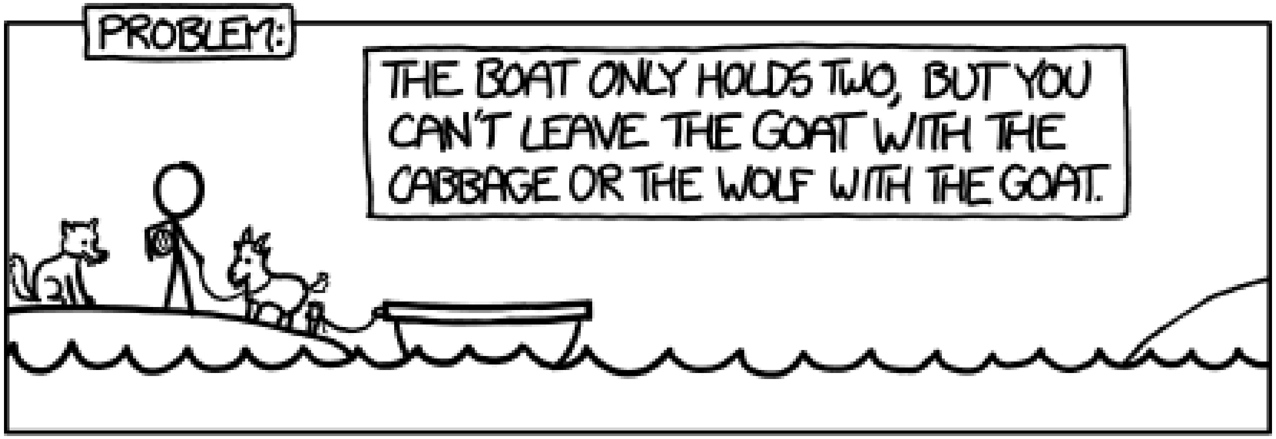
Leidsche Flesch Lunsch-lezing

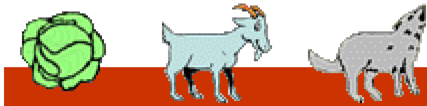
Hendrik Jan Hoogeboom

7.10’15

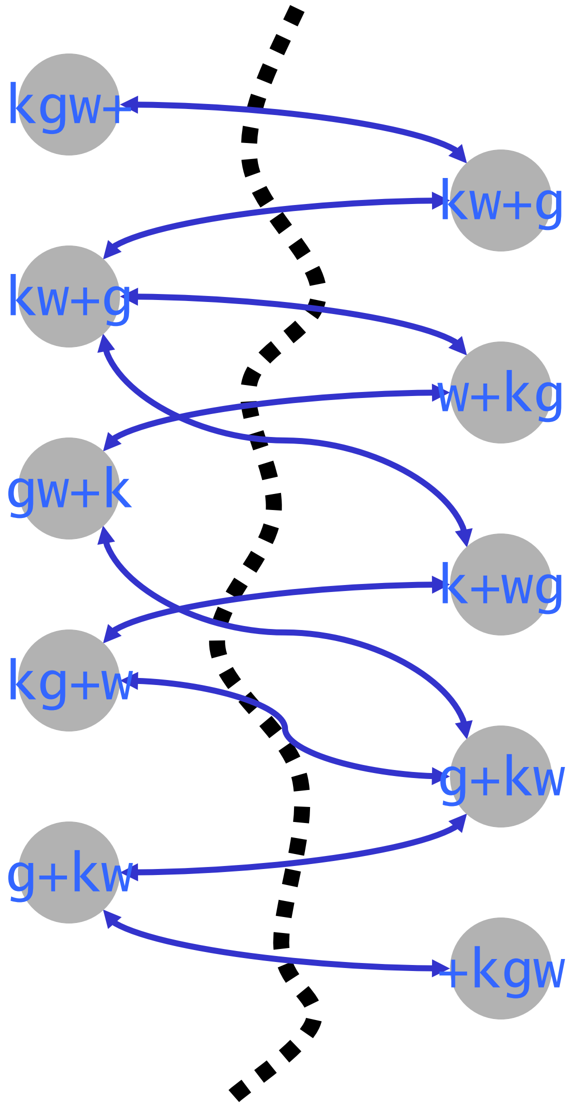
Algoritmiek: een puzzel

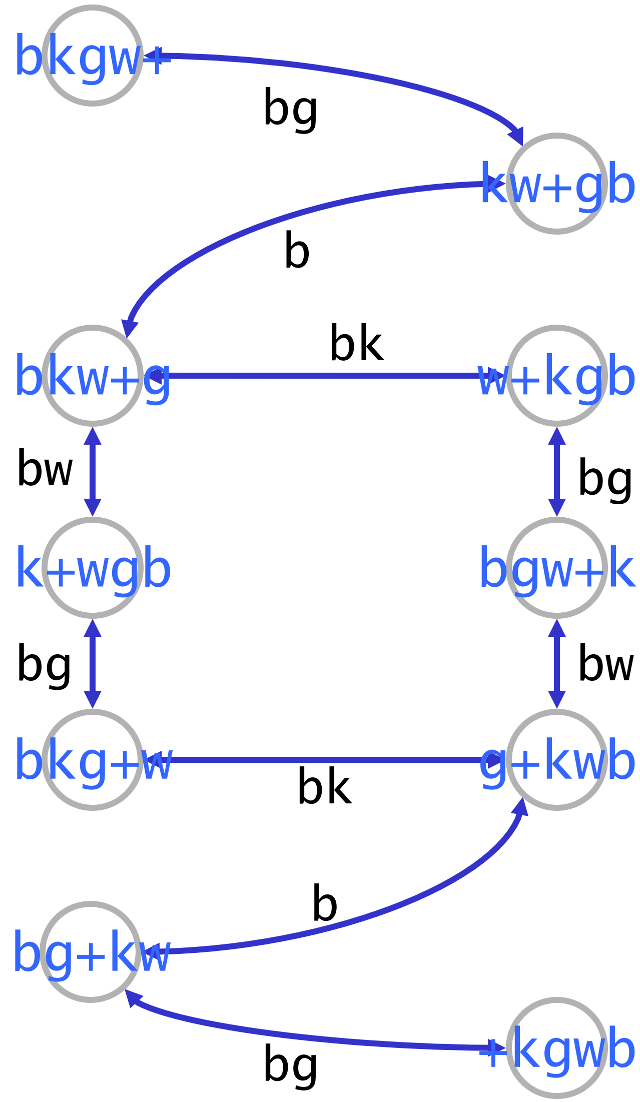
transportation problems

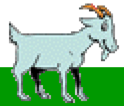
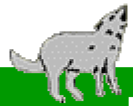




kool, geit, wolf





Propositiones ad Acuendos Juvenes

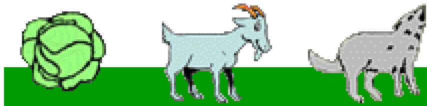
Alcuinus van York (York ~735 - Tours 804)

XVIII. PROPOSITIO DE HOMINE ET CAPRA ET LVPO.

Homo quidam debebat ultra fluuium transferre lupum, capram, et fasciculum cauli. Et non potuit aliam nauem inuenire, nisi quae duos tantum ex ipsis ferre ualebat. Praeceptum itaque ei fuerat, ut omnia haec ultra illaesa omnino transferret. Dicat, qui potest, quomodo eis illaesis transire potuit?

Solutio
Simili namque tenore ducerem prius capram et dimitterem foris lupum et caulum. Tum deinde uenirem, lupumque transferrem: lupoque foris misso capram naui receptam ultra reducerem; capramque foris missam caulum transueherem ultra; atque iterum remigassem, capramque assumptam ultra duxissem. Sicque faciendo facta erit remigatio salubris, absque uoragine lacerationis.

http://en.wikipedia.org/wiki/Propositiones_ad_acuendos_juvenes



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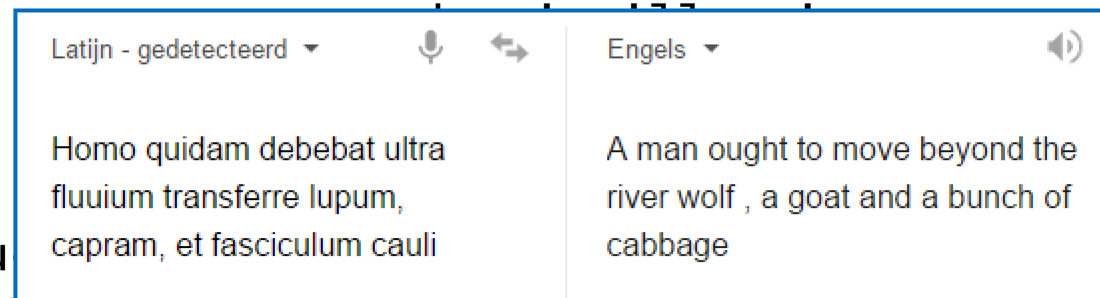

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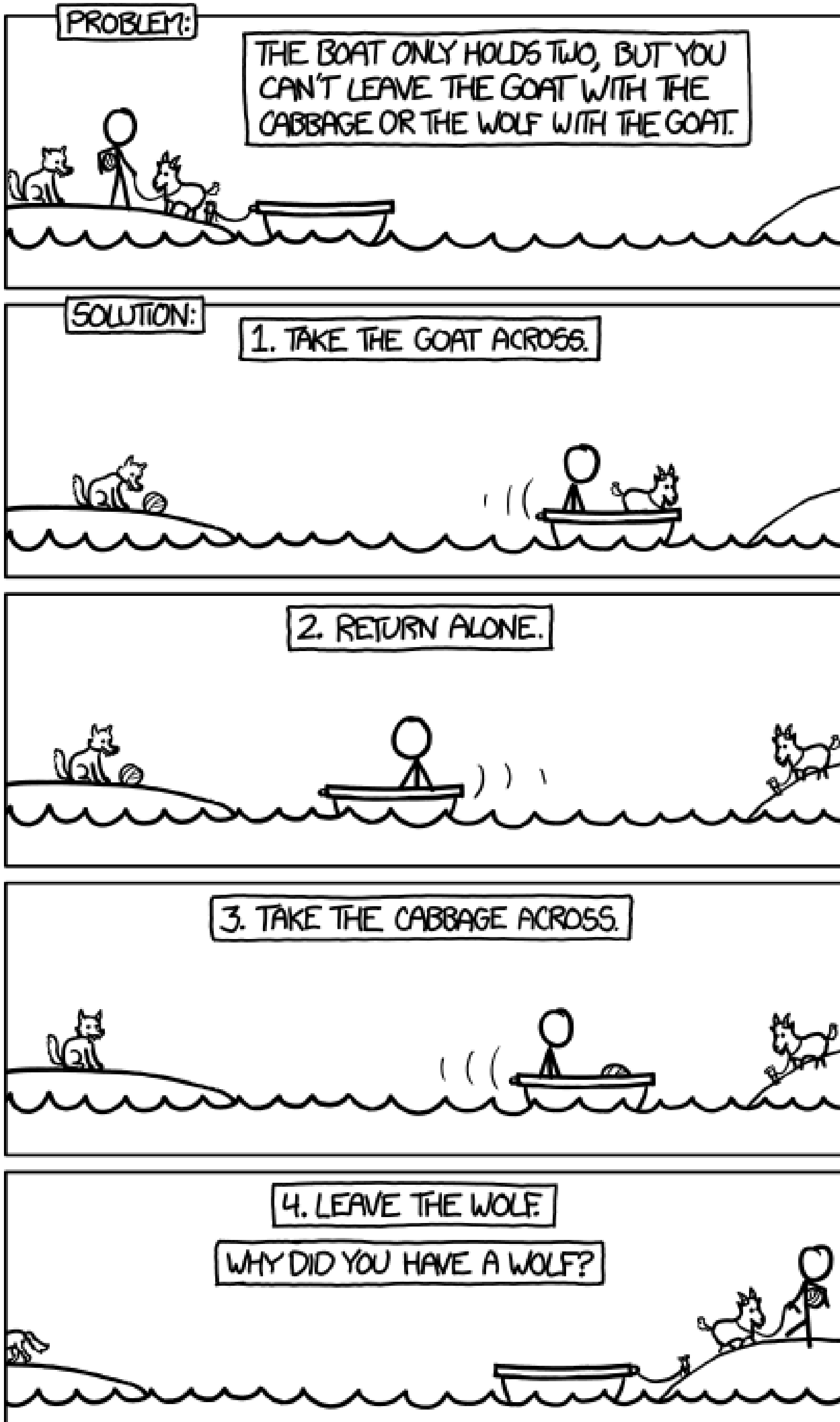
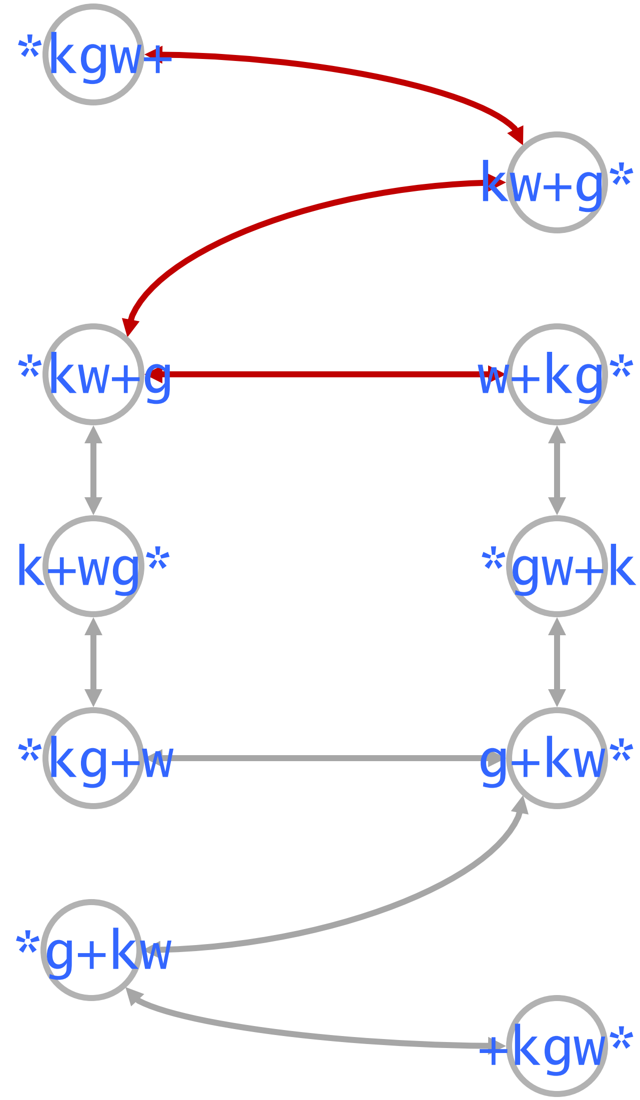
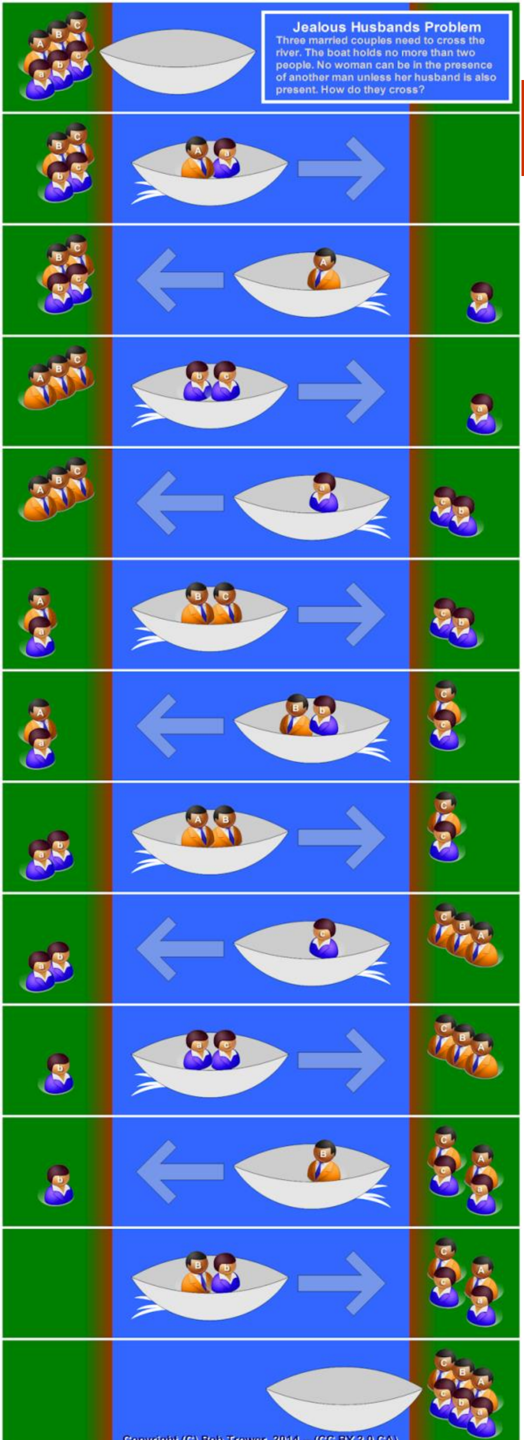
Logic Boat





http://xkcd.com/1134/



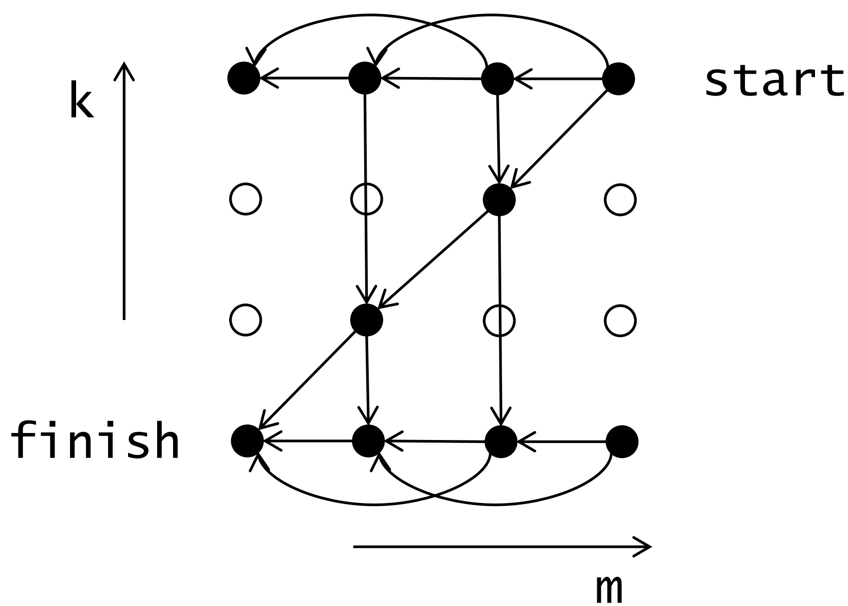
jaloerse echtgenoten

trip number	left bank	travel	right bank
(start)	Aa Bb Cc		
1	Ab Cc	Aa →	
2	Ab Cc	← A	a
3	A B C	bc →	a
4	A B C	← a	b c
5	Aa	BC →	b c
6	Aa	← Bb	Cc
7	a b	AB →	Cc
8	a b	← c	A B C
9	b	a c →	A B C
10	b	← B	Aa Cc
11		Bb →	Aa Cc
(finish)			Aa Bb Cc

missionarissen en kannibalen

3 missionarissen, 3 kannibalen, 2 per boot.
als de missionarissen in de meerderheid zijn bekeren ze de kannibalen; hoe komen deze veilig naar de overkant?

(k,m)
k=0 of k≥m
k=3 of k≤m (andere oever!)



Cannibals and missionaries, Ruby Lim
APL '92 Proc. int. conf. APL, 135-142 (1992)

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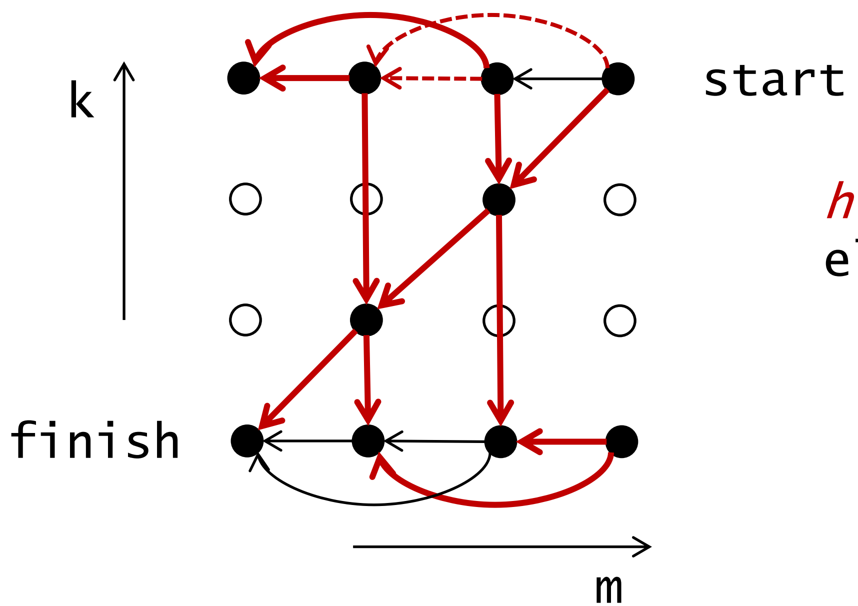
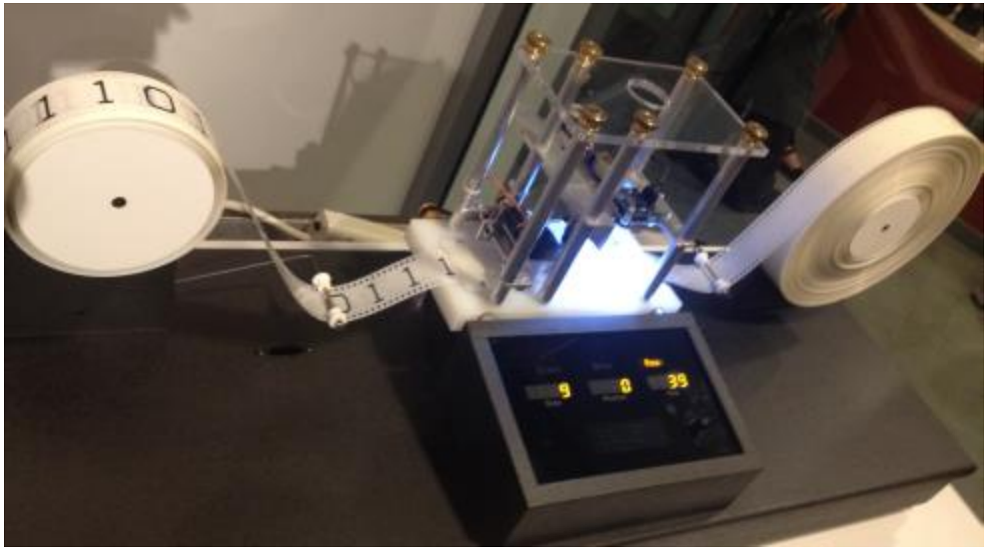
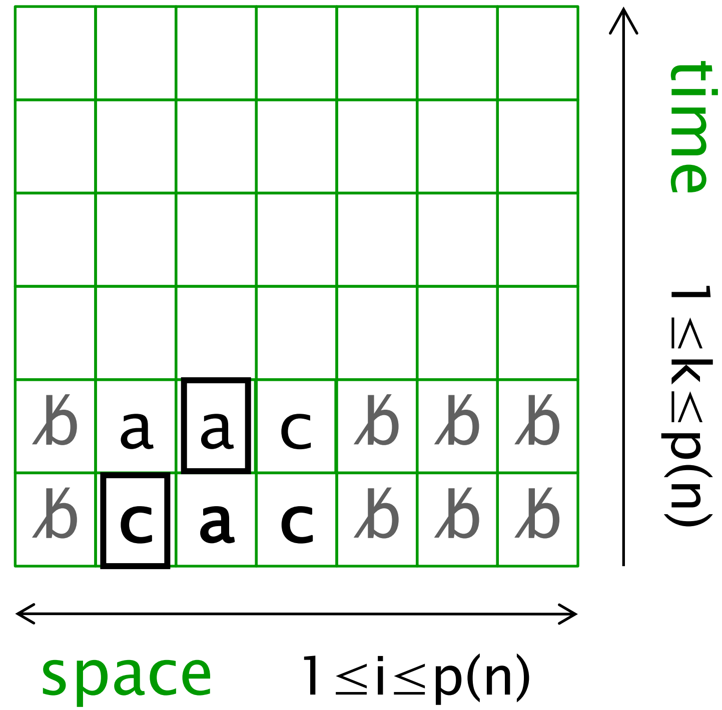


heen en weer!
elf stappen

Complexiteit: NP compleet

Turing machine





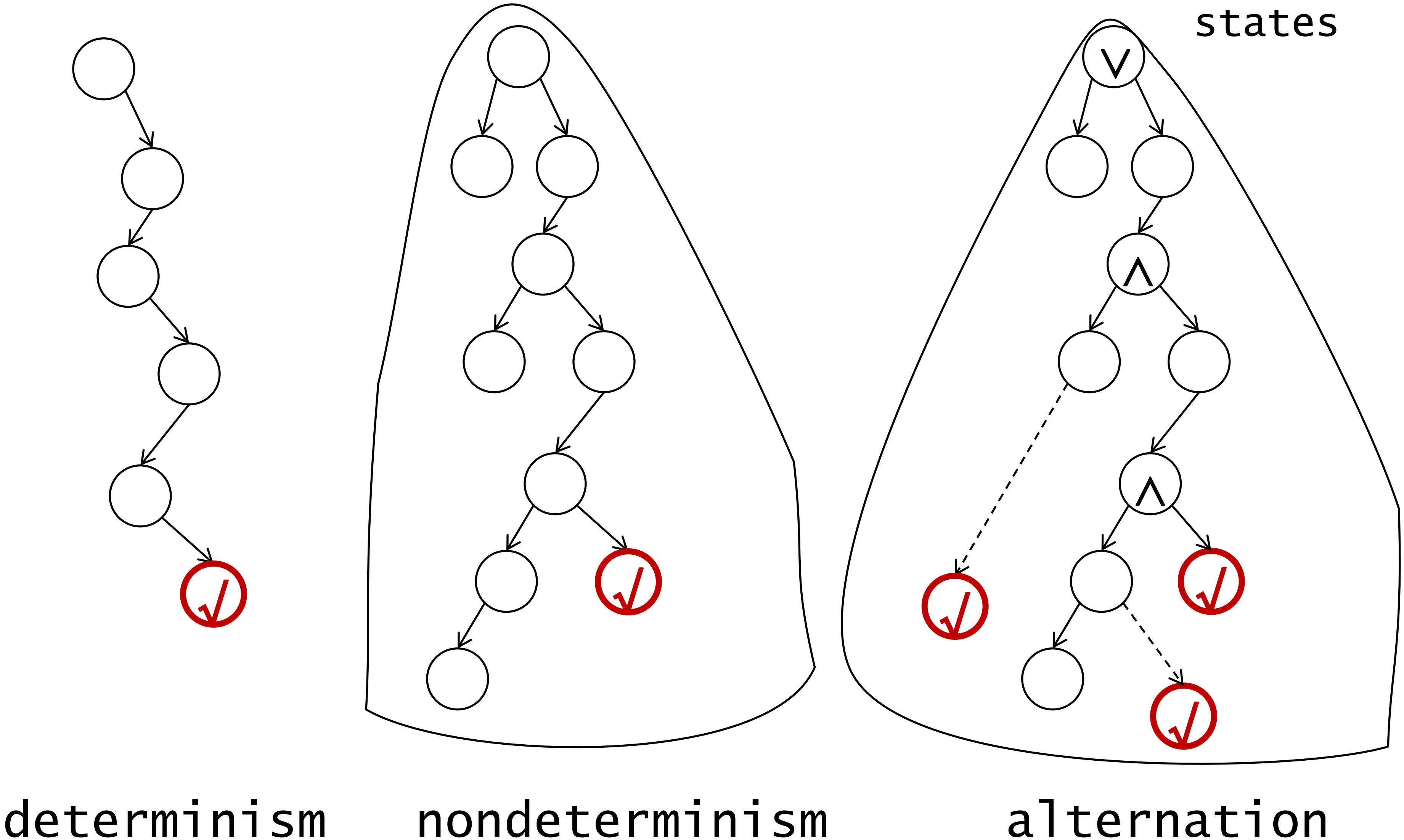
(p,c,q,a,R) instructie

- p toestand
- c letter gelezen
- q nieuwe toestand
- a letter geschreven
- R richting verplaatst

A.M. Turing (1936). On Computable Numbers, with an Application to the Entscheidungs problem.
Proc London Math Soc (1937)
Emil Post. Finite Combinatory Processes—Formulation 1, J Symbolic Logic (1936)

computation tree

existential and *universal* states



dimensions

existential and *universal* states
computation = tree

	log.	*polynomial*		*exp.*
	space	time	space	time
determinism	L	P	PSPACE	EXPTIME
nondeterminism	NL	NP	NPSPACE	NEXPTIME
alternation	AL	AP	APSPACE	AEXPTIME

AL AP APSPACE AEXPTIME

P ⊆ NP ⊆ PSPACE ⊆ EXPTIME ⊆ NEXPTIME ⊆ EXPSPACE

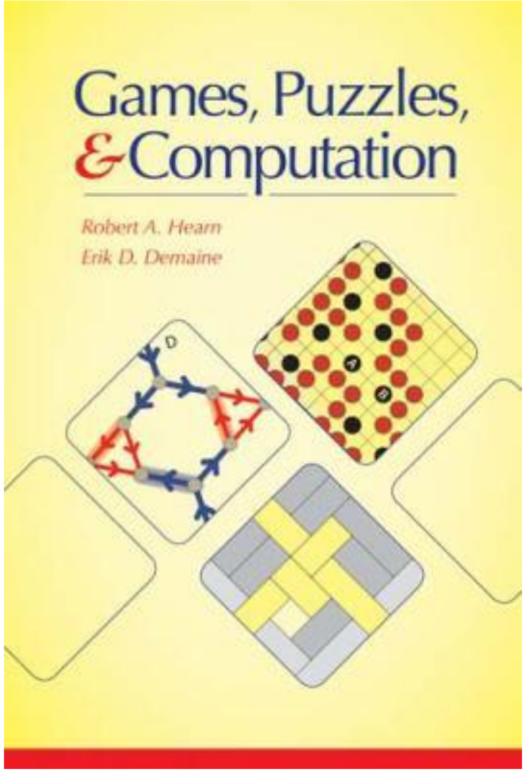
NPSPACE NEXPSPACE

Chandra, Kozen, Stockmeyer. 'Alternation', J ACM 1981.

Games, Puzzles, & Computation

Robert A. Hearn
Erik D. Demaine

(2009, AKPeters)

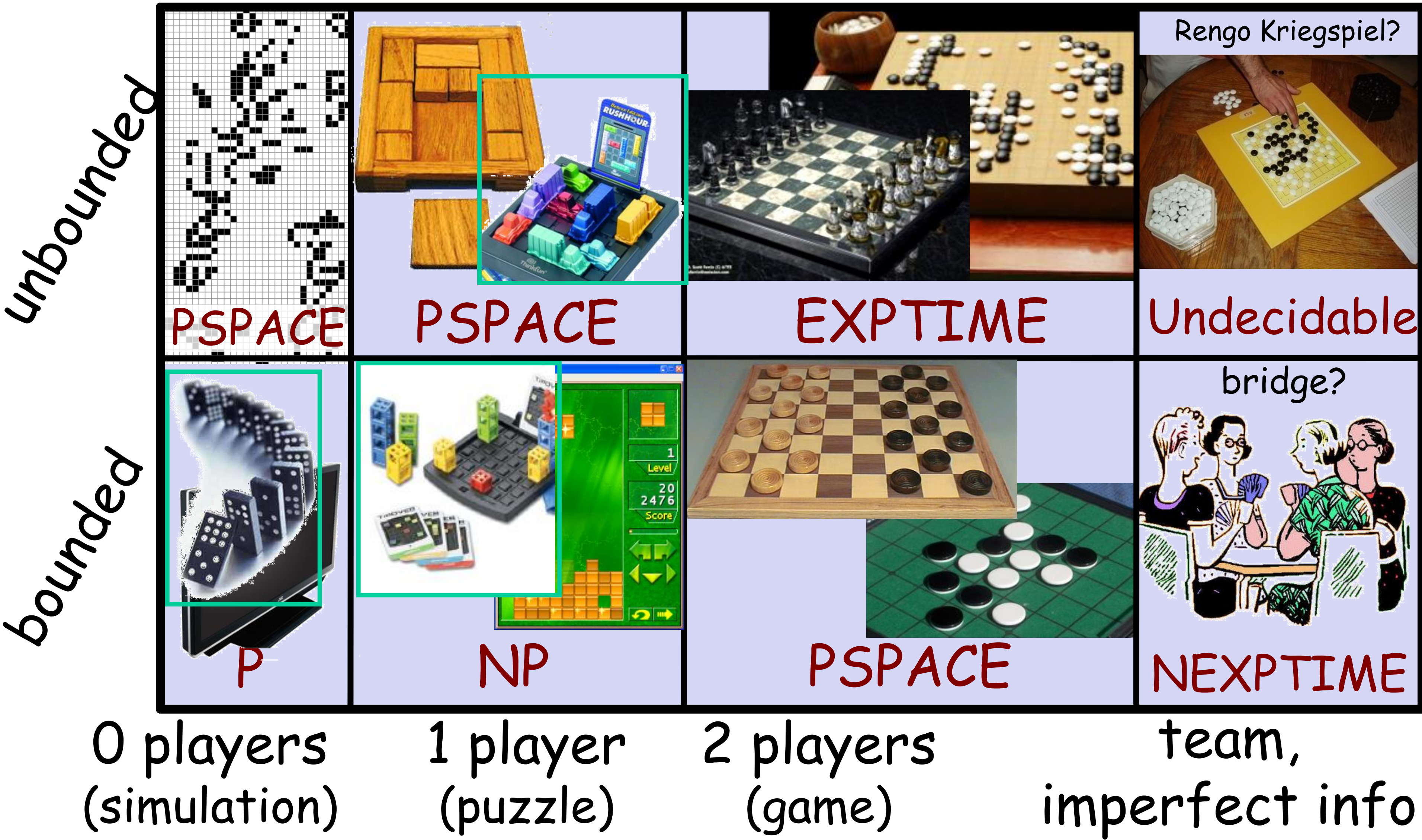


E. Demaine and R.A. Hearn. Constraint Logic: A Uniform Framework for Modeling Computation as Games. In: Proceedings of the 23rd Annual IEEE Conference on Computational Complexity, June 2008. http://www.dartmouth.edu/~rah/constraint-logic.pdf

R.A. Hearn. Games, Puzzles, and Computation PhD thesis, MIT, 2006. http://www.dartmouth.edu/~rah/

Complexity of Games & Puzzles

[Demaine, Hearn & many others]



game categories and their natural complexities

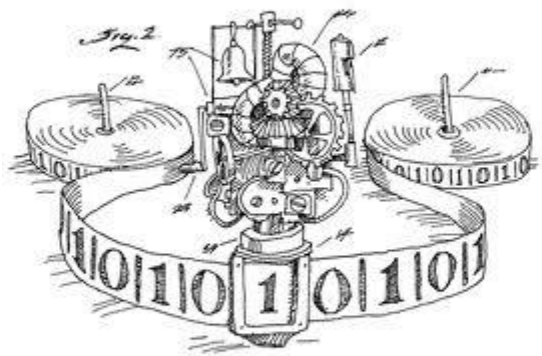
(polynomial)
TM resources

Rush Hour
River Crossing

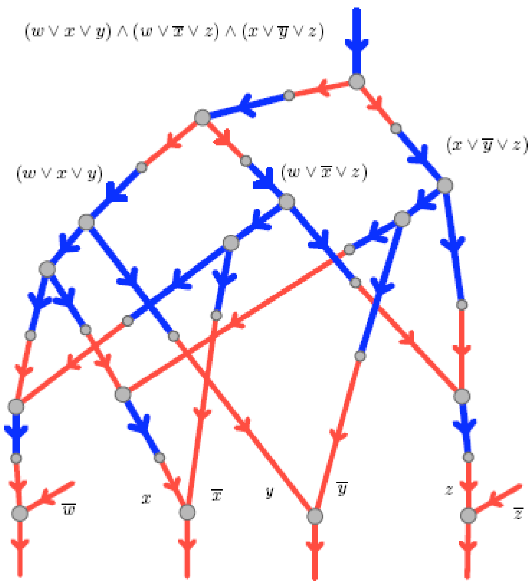
unbounded **SPACE**	**PSPACE**	**PSPACE** NPSPACE	**EXPTIME** APSPACE	undecid
bounded **TIME**	**P**	**NP**	**PSPACE** AP	NEXPTIME
#	**zero** *simulation* ***determ.***	**one** *puzzle* ***nondeterm.***	**two** *game* ***alternat.***	team *imperfect informat.*

Toppling Dominoes *TipOver*

NL ⊆ P ⊆ NP ⊆ PSPACE ⊆ EXPTIME ⊆ NEXPTIME
NPSPACE

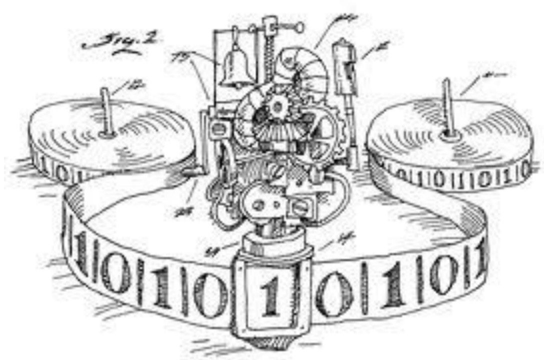


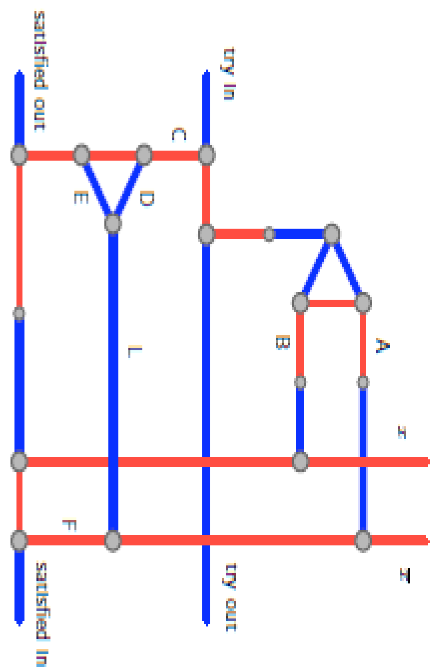
$(w \vee x \vee y) \wedge (w \vee \overline{x} \vee z) \wedge (x \vee \overline{y} \vee z)$

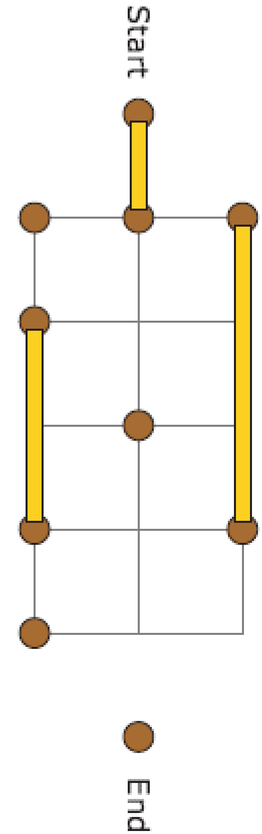


NP	part I constraint logic 'graph games'	part II games in particular
3SAT	Bounded NCL	TipOver

PSPACE & Plank Puzzle







PSPACE

QBF

part I
constraint logic
'graph games'

NCL

part II
games in particular

plank puzzle
(river crossing)

3SAT

literal
(negated) variable

clause

$(w \vee x \vee y) \wedge (w \vee \neg x \vee z) \wedge \ldots \wedge (x \vee \neg y \vee z)$

3 conjunctive normalform

3SAT
given: given formula ϕ in 3CNF
question: is ϕ satisfiable?
(can we find a variable assignment making formula true)

Cook’71/Levin’73 3SAT is NP-complete

specify computation

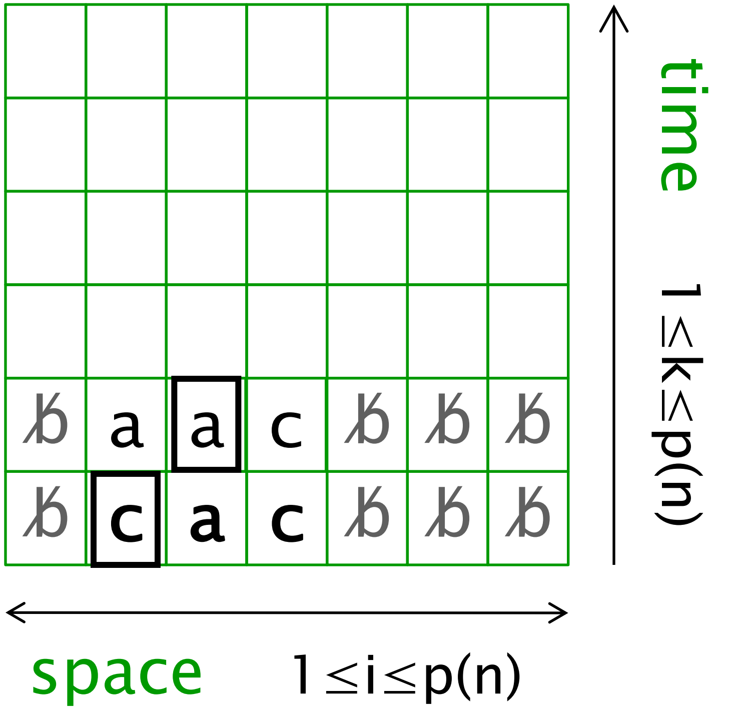
at step k …

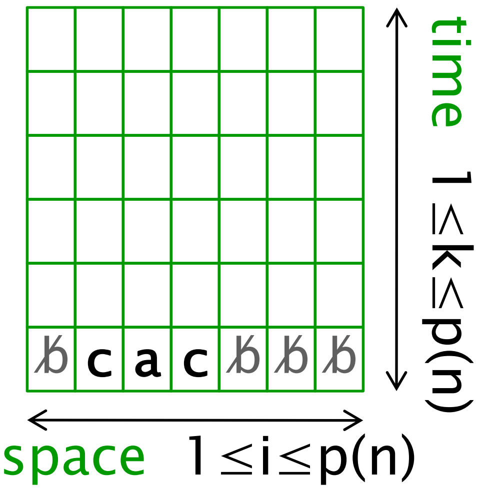
T_{iak} cell i contains a

H_{ik} head at position i

Q_{qk} state q

variabelen





specify computation
at step k …

T_{iak}	cell i contains a
H_{ik}	head at position i
Q_{qk}	state q

(p,a,p,a,0) for each p,a

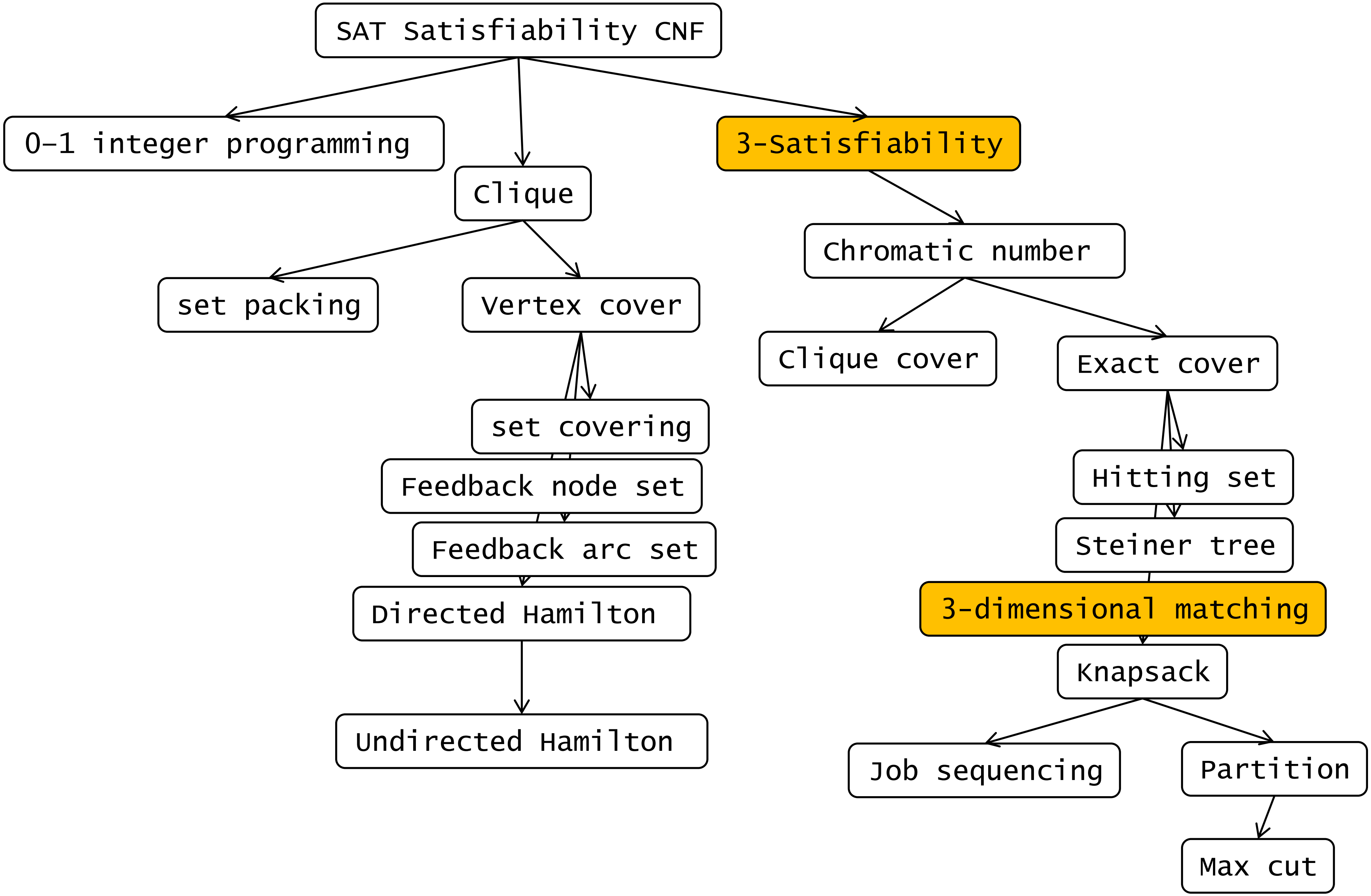
conjunction of

$T_{ix[i]0}$	initial tape $x[i]=x_i$ or $x[i]=\not{b}$
Q_{q00}	initial state
H_{00}	initial position
$T_{iak} \rightarrow \neg T_{ibk}$	single symbol $a \neq b$
$Q_{pk} \rightarrow \neg Q_{qk}$	single state $p \neq q$
$H_{ik} \rightarrow \neg H_{jk}$	single head $i \neq j$
$T_{iak} \wedge T_{ib.k+1} \rightarrow H_{ik}$	changed only if written $a \neq b$

$$H_{ik} \wedge Q_{pk} \wedge T_{iak} \rightarrow \bigvee_{(p,a,q,b,d)} H_{i+d.k+1} \wedge Q_{q.k+1} \wedge T_{ib.k+1}$$

$Q_{h.p(n)}$	accept

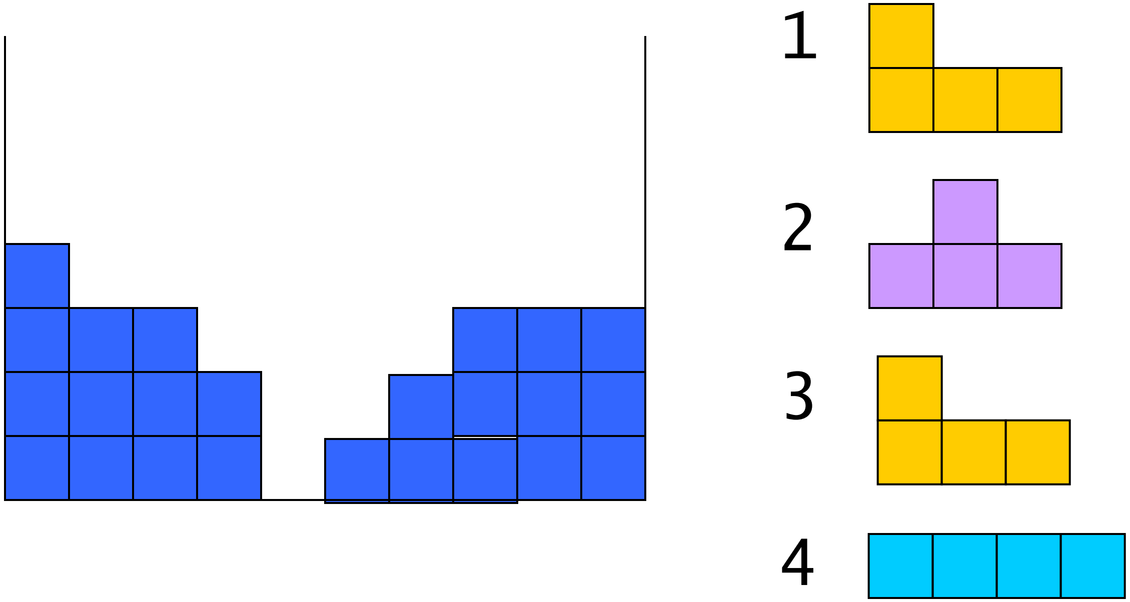
Karp's 21 NP-complete problems



Richard M. Karp (1972) Reducibility Among Combinatorial Problems

Tetris is NP complete

"Given an initial game board and a sequence of pieces, can the board be cleared?"



Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell.
Tetris is Hard, Even to Approximate. Selected Papers from COCOON 2003.
Int. J. of Comput Geom and Appl 2004.

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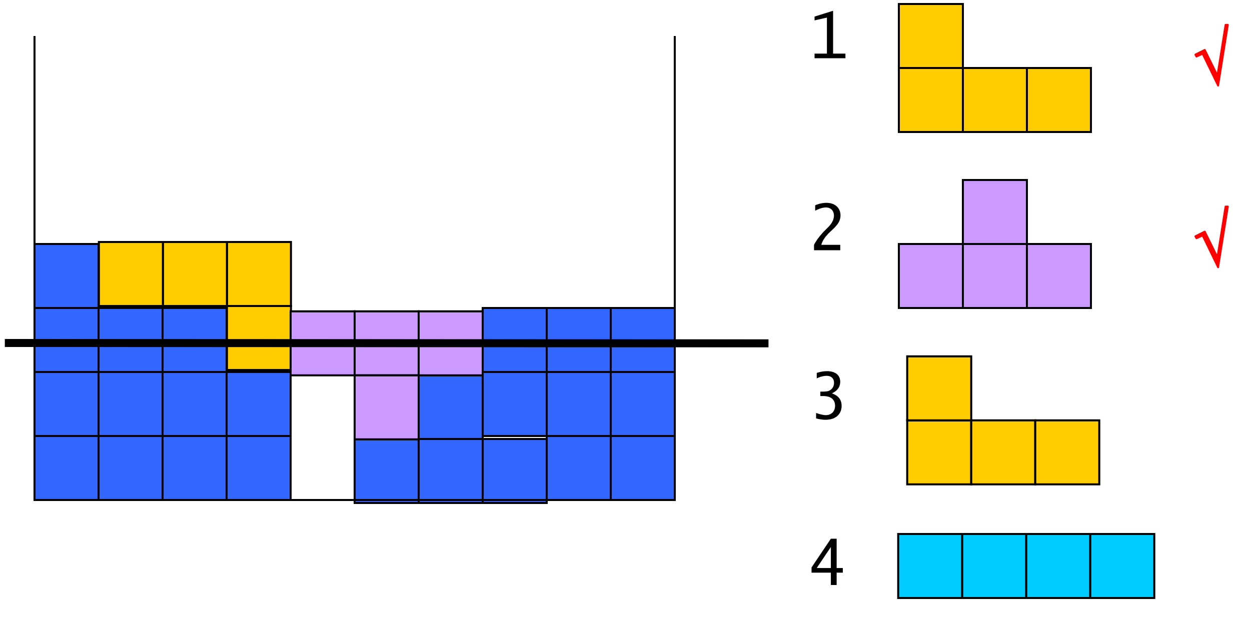

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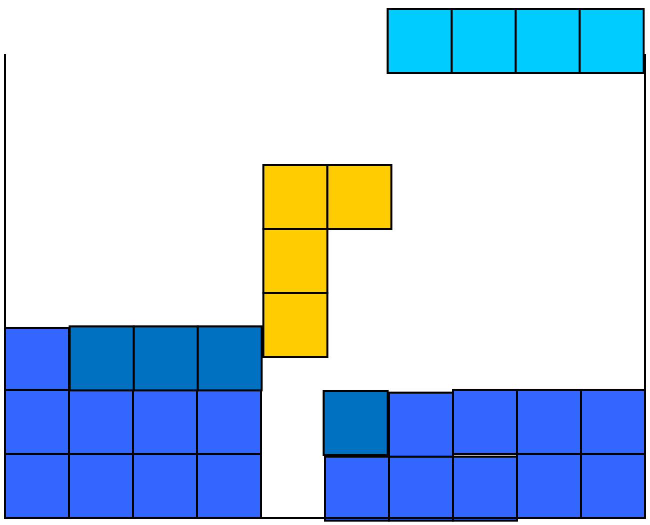
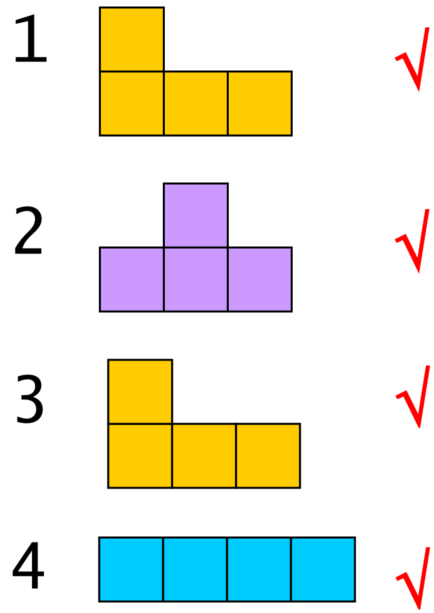


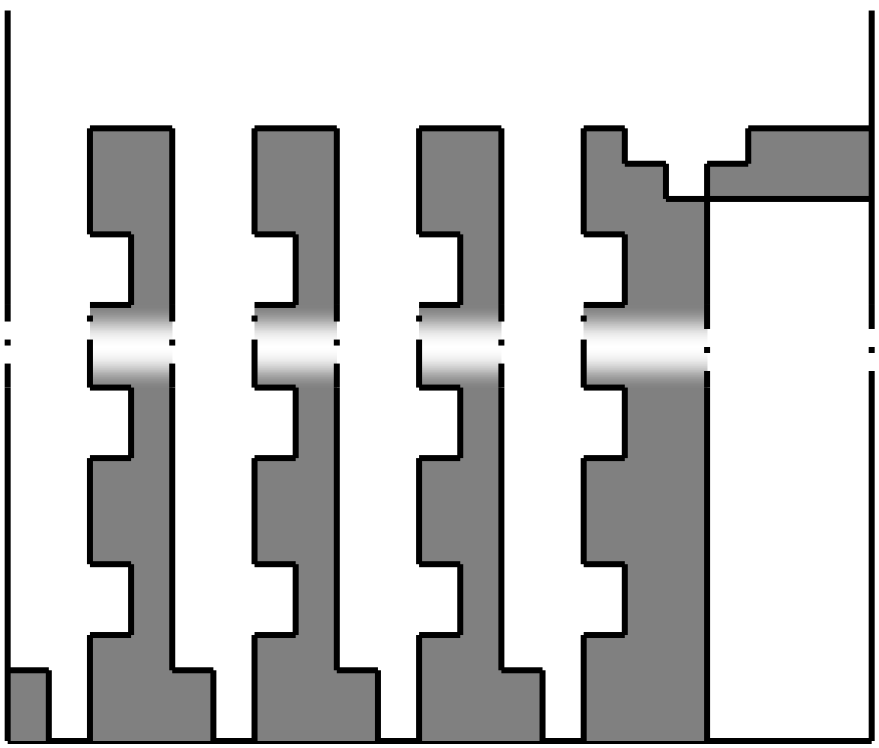


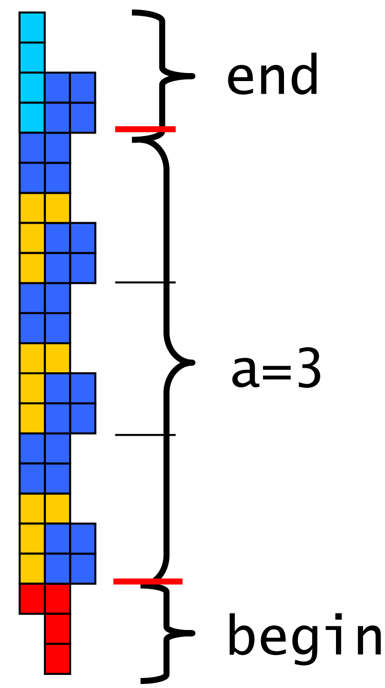
yes!

Tetris is NP complete

"Given an initial game board and a sequence of pieces, can the board be cleared?"

reduction from 3-partitioning problem
(can we divide set of numbers into triples?)





'generalized' river crossing

D drivers
P passengers
c capaciteit boot
m max verplaatsingen
verboden combinaties:
F_R rechteroever
F_L linkeroever
F_B boot

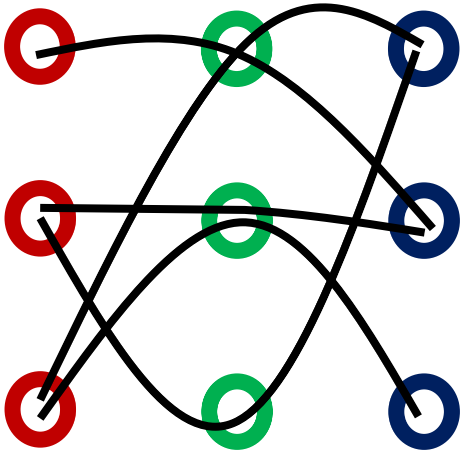
D = { Man }, P = { Wolf, Goat, Cabbage },
F_L = F_R = { {Wolf, Goat}, {Goat, Cabbage},
{Wolf, Goat, Cabbage} },
F_B = ∅,
c = 2, m = 100.

verboden: want anders polynomiaal, nl. graaf-wandelen

Thm. [Ito etal.] Polynomiaal als $F_B = \emptyset$, en $m=\infty$.

Ito, Langerman, Yoshida. Generalized River Crossing Problems, Th. Comp. Sys. 2015

3 dim matching



gegeven:
U,V,W, met |U|=|V|=|W|=k
en $M \subseteq U \times V \times W$

vraag:
bestaat er een P⊆M met |P|=k
die alle elementen één keer bevat ?

{a,b,c} x {p,q,r} x {x,y,z}

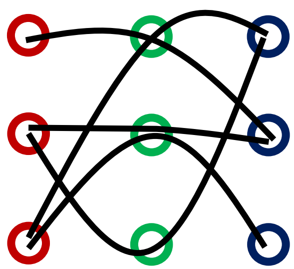
M = { (a,p,y), (b,q,y), (b,r,x), (c,p,x), (c,q,z) }

3D matching

U,V,W, met

|U|=|V|=|W|=k en

M ⊆ U x V x W



river crossing

D=U∪{s}, P=V∪W, c=3, m=3k-1,

$F_R = \emptyset$, $F_L = \emptyset$,

F_B^C =

{ {u,v,w} | (u,v,w) ∈ M }

∪ { {s,u_{2i-1},u_{2i}} | i≤k/2 }

complement!

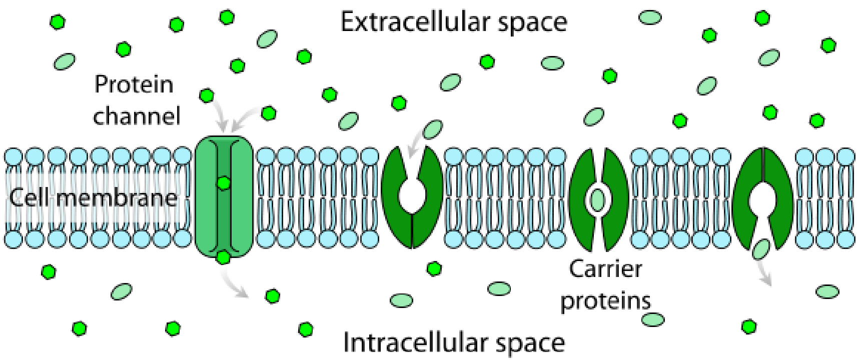
oplossing ⟺ oplossing

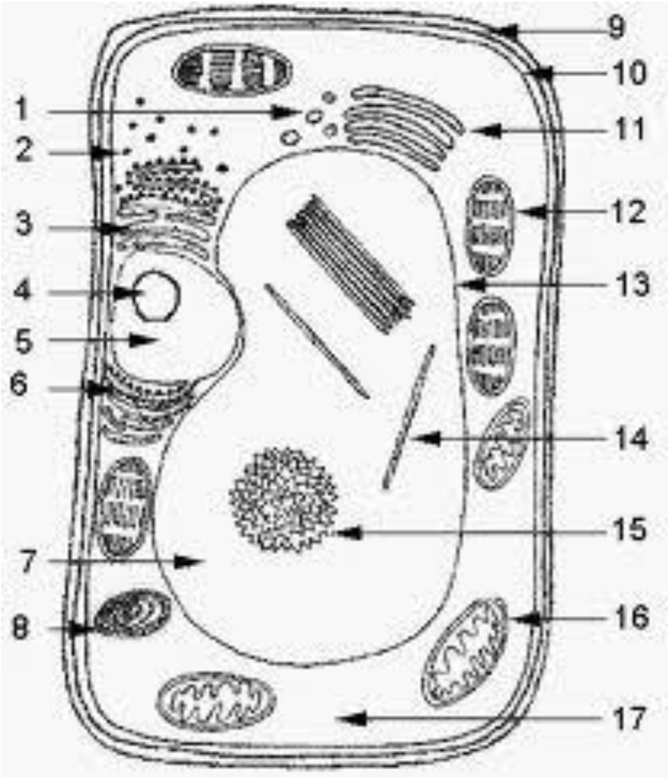
P =
{ (a,p,y),
(b,r,x),
(c,q,z) }

{a,p,y} →
← a
{b,r,x} →
← b
{s,a,b} →

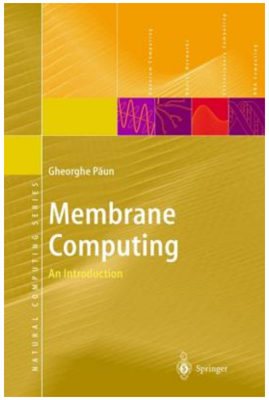
Natural Computing: een model

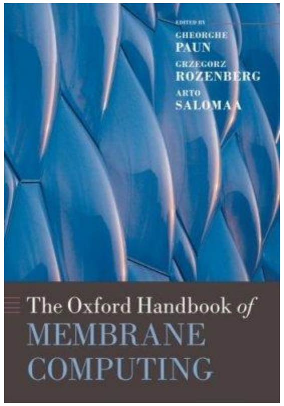
nature inspires

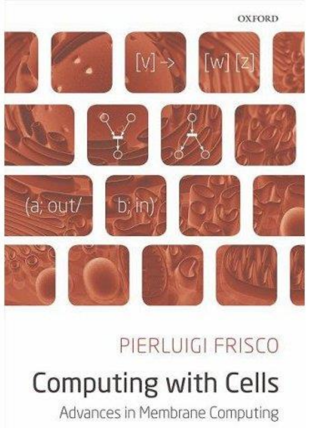




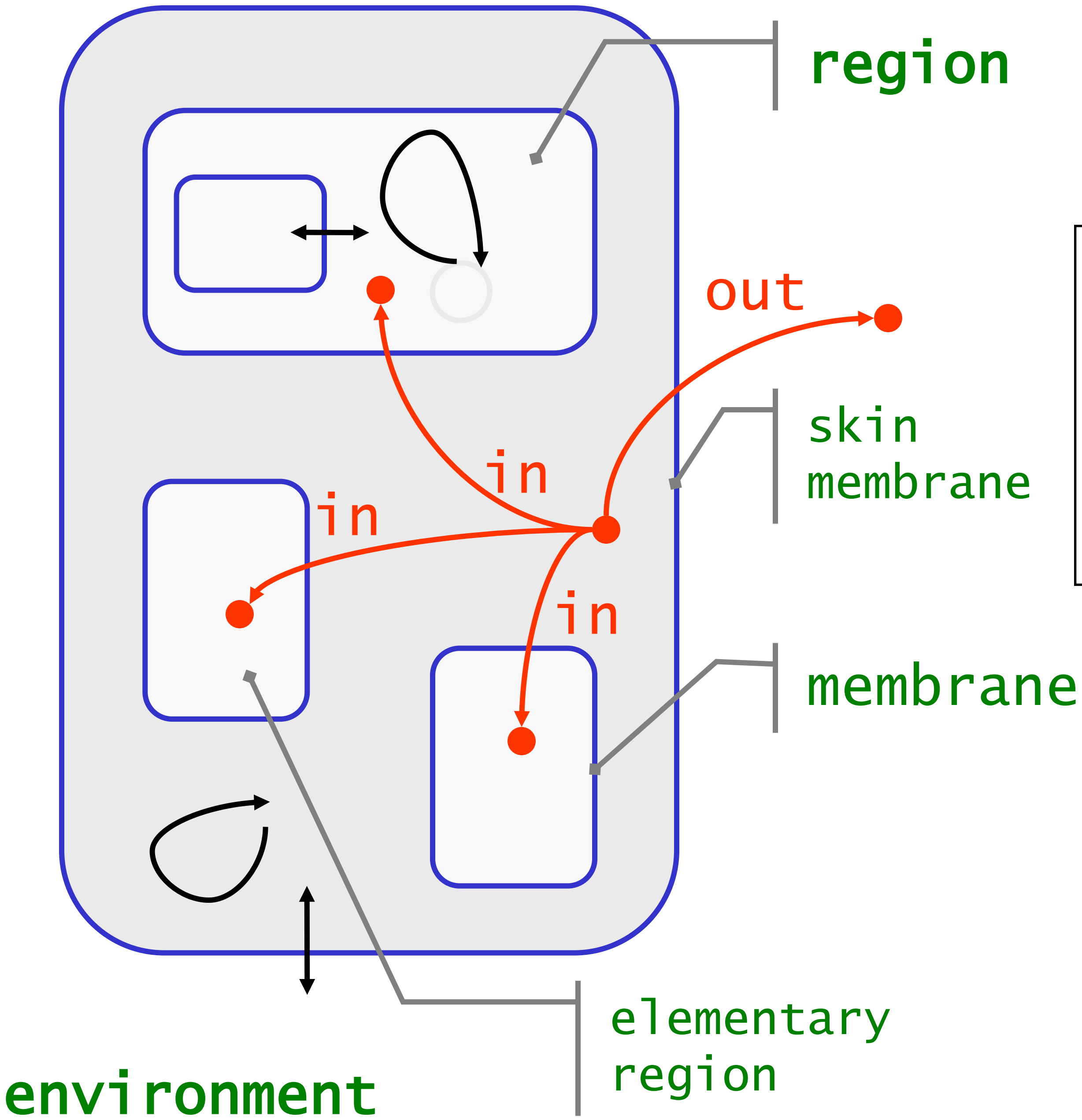
nested compartments
              - information
membranes - communication





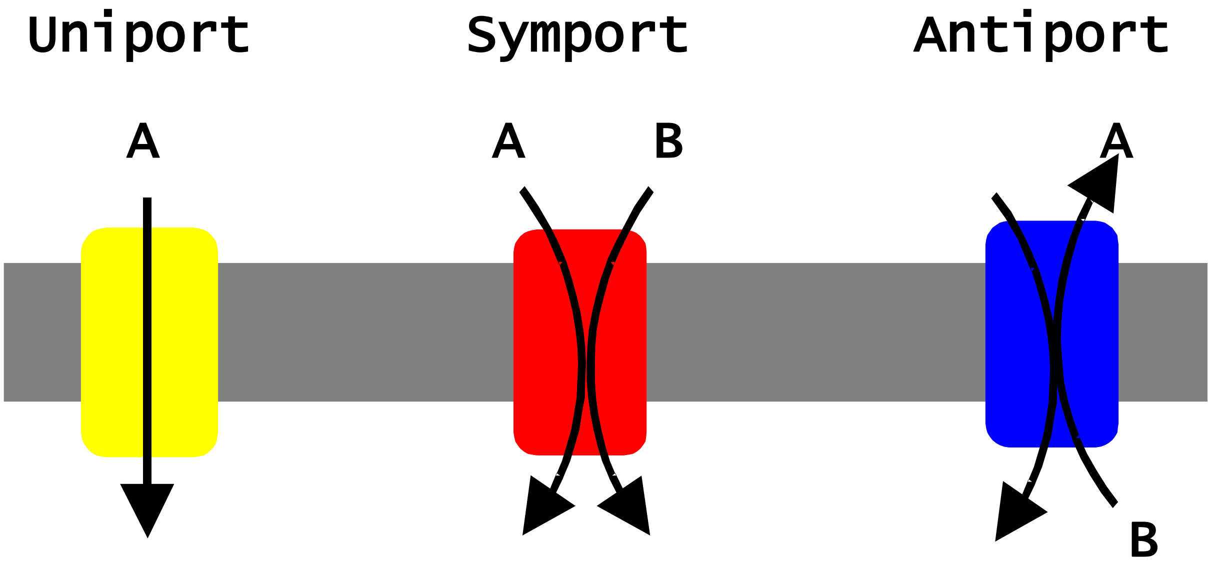


Structure



keuze:
type objecten,
reacties,
communicatie,
output

simpele systemen

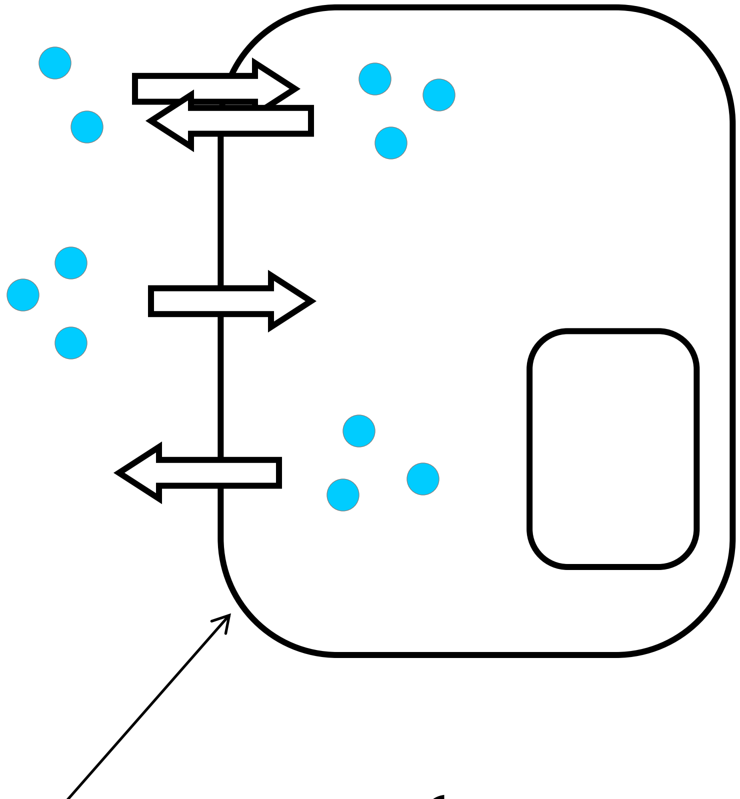


- alléén samen verplaatsen

- hoeveel membranen?
- hoeveel tegelijk?
- antiport nodig?

P systems with symport/antiport

Păun & Păun



regels per membraan

Contents

- objects
  - multiset symbols
  - infinite supply in environment

Rules

$(a_1 \ldots a_k \leftrightarrow b_1 \ldots b_\ell)$ antiport

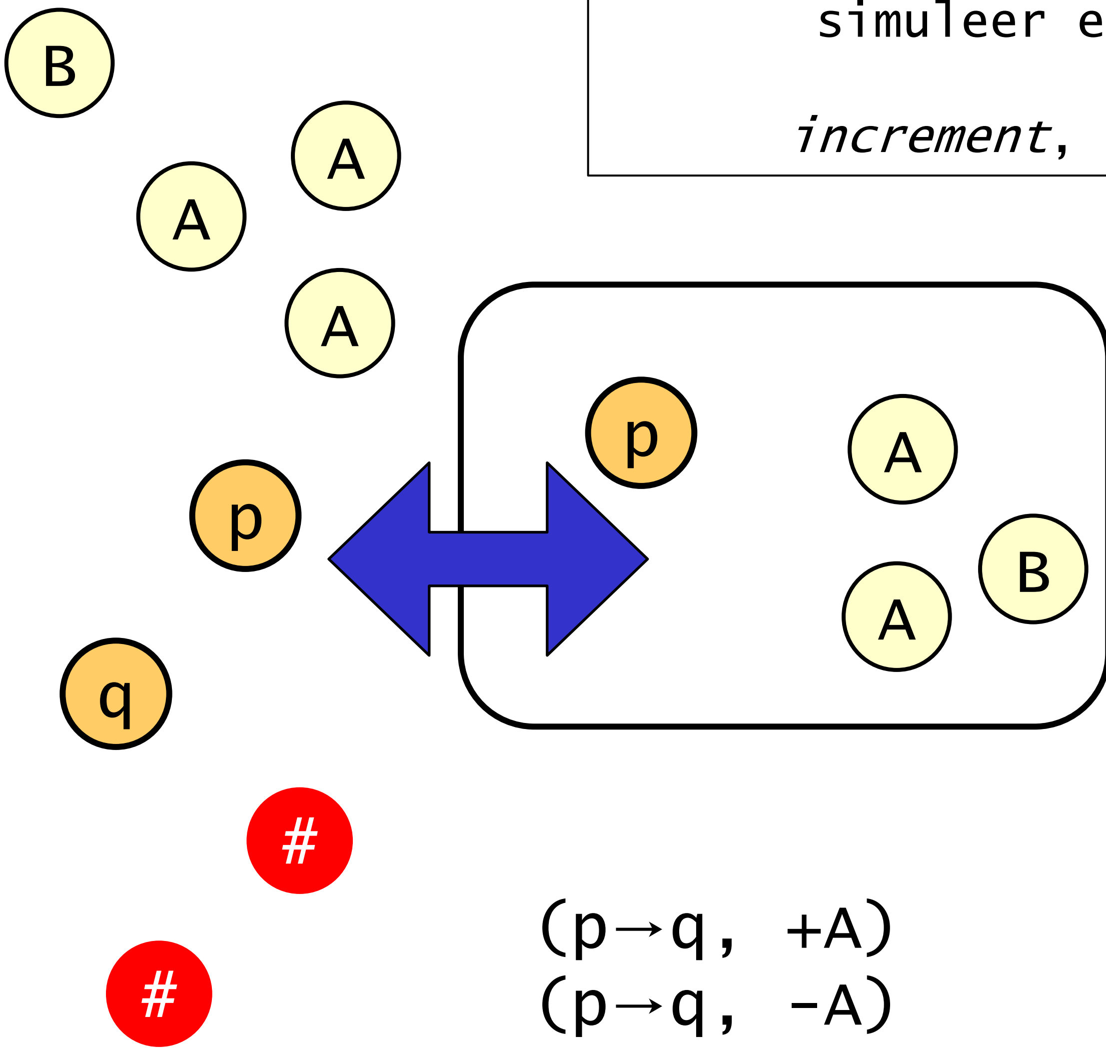
$(a_1 \ldots a_k \rightarrow)$ symport (in)

$(\leftarrow a_1 \ldots a_k)$ (out)

single membrane will do

sadly enough

simuleer een **counter automaton**
(=register machine)
increment, *decrement*, *zero test*



toestand p
counter A=2
B=1

$(p \rightarrow q, +A)$
$(p \rightarrow q, -A)$

counter aut

$(\ qA \leftrightarrow p\ )$
$(\ q \leftrightarrow pA\ )$

antiport

zero test: programming trick

infinite = blocking

conflicting counters A & A'

A A'

always present #

(p→q, A=0)

(A'r ↔ p) +A'
(r' ↔ r) 'wait'
(q ↔ r'A') -A'

(# ↔ AA')
(# ↔ #)

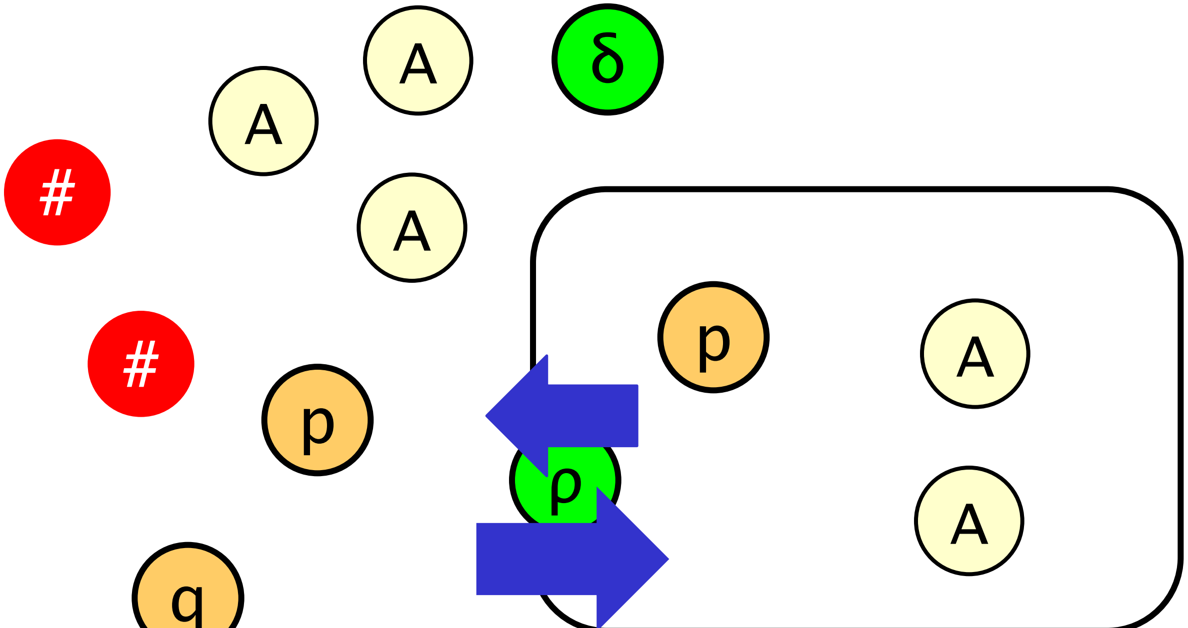
counter aut 'simulation' antiport

verboden combinatie!

max parallellism: forced move

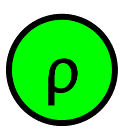
antiport to symport

add ‘drivers’



$\rho = (qA \leftrightarrow p)$

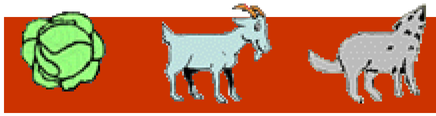
speciaal voor deze regel



$(\rho qA \rightarrow \ )$

$(\ \leftarrow \rho p)$

drie persoons bootje!



conclusie

drie persoons-bootje + verboden combinatie

we zijn weer terug!

Remark. The construction in the proof of Theorem 1 above reminds us of the classical wolf, cabbage, and goat problem. attributed to [AY99, Propositio XVIII][1]. The carrier v crossing the membrane echoes the little boat crossing the river, whereas the carrier ∂ models the conflicting presence of goat with either wolf or cabbage on the banks of the river (here the membrane).

[1] Propositio de Homine et Capra et Lupo. Homo quidam debebat ultra flavium transferre lupum, capram, et fasciculum cauli. Et non potuit aliam navem invenire, nisi

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klaar

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