Foundations of Computer Science

Fundamentele Informatica 1

Hendrik Jan Hoogeboom
Jeannette de Graaf

Bachelor Informatica (& specialisaties)
Universiteit Leiden

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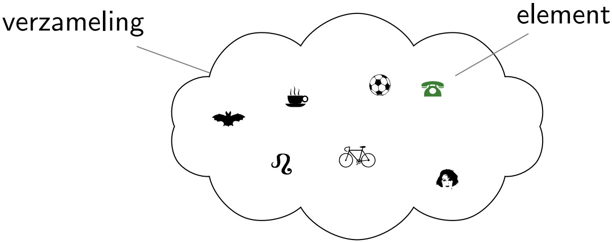
Universiteit Leiden
Leiden Institute of Advanced Computer Science

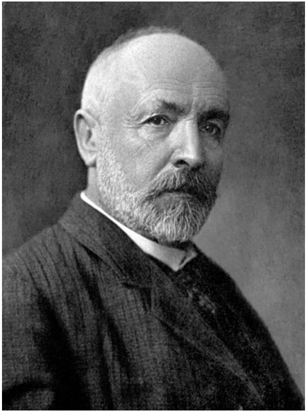
Hoofdstuk 1

Verzamelingen

verzameling



"Unter einer 'Menge' verstehen wir jede Zusammenfassung M von bestimmten wohlunterschiedenen Objekten m unserer Anschauung oder unseres Denkens (welche die 'Elemente' von M genannt werden) zu einem Ganzen. "



St Petersburg 1845 – Halle 1918

wikipedia

Über eine Eigenschaft des Imbegriffes aller reellen algebraischen Zahlen. Crelles Journal für Mathematik, 77 (258–263) 1874.


KOYKOYPIKOY.

" *a set may be viewed as any well-defined collection of objects* "

een verzameling wordt bepaald door haar elementen

opsommen

$$\{ 0, 2, 4, 6 \}$$
$$\{ 0, 2, 4, \ldots, 116, 118 \}$$
$$\{ 0, 1, 4, 9, \ldots \}$$

eigenschap P $\{ x \mid P(x) \}$

$$\{ x \mid x \text{ is een kwadraat} \}$$

" alle elementen waarvoor . . . "

$$\{ x \mid x = y^2 \text{ voor een geheel getal } y \}$$
$$\{ y^2 \mid y \text{ is geheel} \}$$

een verzameling wordt bepaald door haar elementen

$x \in A$ "x is element van A" "x zit in A"

$$1 \in \{1,2\} \quad 3 \notin \{1,2\}$$

gelijkheid

$$\{1,2\} = \{2,1\} = \{1,2,1\}$$

$$A = B$$
$$x \in A \text{ desda } x \in B$$

"dan en slechts dan als" $\Longleftrightarrow$

deelverzameling, *inclusie* $\subseteq$ "is bevat in"

$$\{3,5,9\} \subseteq \{1,3,5,7,9,11\}$$

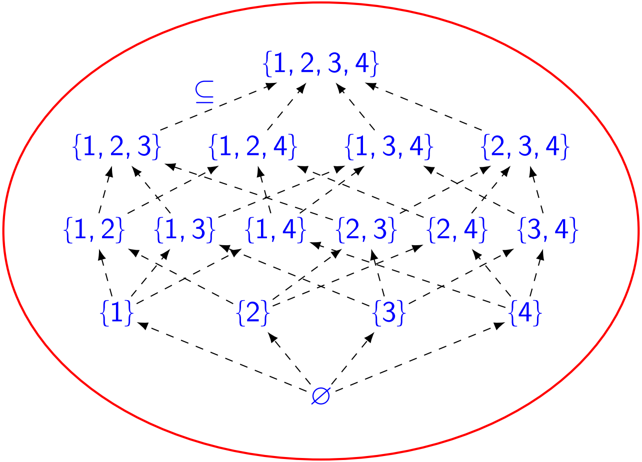
$$\{2,3,5,7\} \not\subseteq \{1,3,5,7,9,11\}$$

$$A \subseteq B$$
$$x \in A \text{ dan } x \in B$$
$$\Rightarrow$$

$$A = B \text{ desda } A \subseteq B \text{ en } B \subseteq A$$

echte deelverzameling $A \subset B$ $A \subseteq B$ én $A \neq B$

$$\{3,5,9\} \subset \{1,3,5,7,9,11\}$$


$\{1,2,3,4\}$
$\subseteq$
$\{1,2,3\}$ $\{1,2,4\}$ $\{1,3,4\}$ $\{2,3,4\}$
$\{1,2\}$ $\{1,3\}$ $\{1,4\}$ $\{2,3\}$ $\{2,4\}$ $\{3,4\}$
$\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$
$\emptyset$

Thm. 1.1

Voor verzamelingen A, B en C geldt

1. $A \subseteq A$ *reflexief*
2. als $A \subseteq B$ en $B \subseteq A$ dan $A = B$ *anti-symmetrisch*
3. als $A \subseteq B$ en $B \subseteq C$ dan $A \subseteq C$ *transitief*

Voor getallen x, y en z geldt

1. $x \leqslant x$ *reflexief*
2. als $x \leqslant y$ en $y \leqslant x$ dan $x = y$ *anti-symmetrisch*
3. als $x \leqslant y$ en $y \leqslant z$ dan $x \leqslant z$ *transitief*

partiele ordening

$\mathbb{N}$ *natuurlijke* getallen $\mathbb{N}^+$

$\{ 0, 1, 2, 3, \ldots \}$

$\mathbb{Z}$ *gehele* getallen *integers*

$\{ \ldots, -2, -1, 0, 1, 2, \ldots \}$,

$\mathbb{Q}$ *rationale* getallen

breuken p/q, maar $2/4 = 1/2$

$\mathbb{R}$ *reëele* getallen

$\frac{-1}{\sqrt{2}}$, e en π.

$$\{\ x \mid x \text{ is even }\ \}$$

$$\{0, 2, 4, 6, \dots\} \quad \text{vs} \quad \{\dots, -4, -2, 0, 2, 4, 6, \dots\}$$

universum U

lege verzameling $\{\ \}$ $\varnothing$ $\emptyset$

Thm 1.2

voor elke verzameling A geldt $\varnothing \subseteq A \subseteq U$

Boolese operaties

	logica		verzamelingen	
conjunctie	en	$\wedge$	$\cap$	doorsnede
disjunctie	of	$\vee$	$\cup$	vereniging
negatie	niet	$\neg$	^{c}	complement

$$A = \{1,2,3,4\} \qquad B = \{1,3,5,7\} \qquad U = \{1,2,3,4,5,6,7,8\}$$

en $\quad A \cap B = \{\, x \in U \mid (x \in A) \wedge (x \in B) \,\}$ $\quad$ *doorsnede*

$$A \cap B = \{\, 1, 3 \,\}$$

of $\quad A \cup B = \{\, x \in U \mid (x \in A) \vee (x \in B) \,\}$ $\quad$ *vereniging*

$$A \cup B = \{\, 1, 2, 3, 4, 5, 7 \,\}$$

Thm 1.3

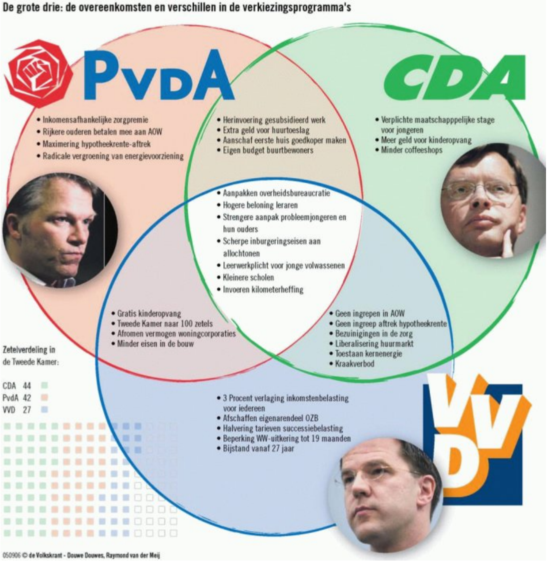
Voor elk tweetal verzamelingen A en B geldt $A \cap B \subseteq A \subseteq A \cup B$

$$A \cap B \subseteq B \subseteq A \cup B$$

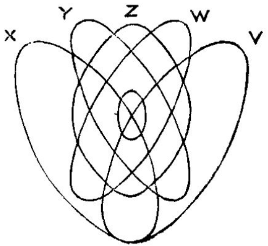
niet $\quad A^c = \{\, x \in U \mid \neg(x \in A) \,\} = \{x \in U \mid x \notin A \,\}$ $\quad$ *complement*

$$A^c = \{\, 5, 6, 7, 8 \,\}$$

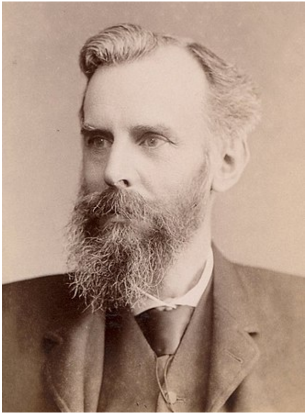

COCORICOOOOOO



050906 (C) de Volkskrant – Douwe Douwes, Raymond van der Meij



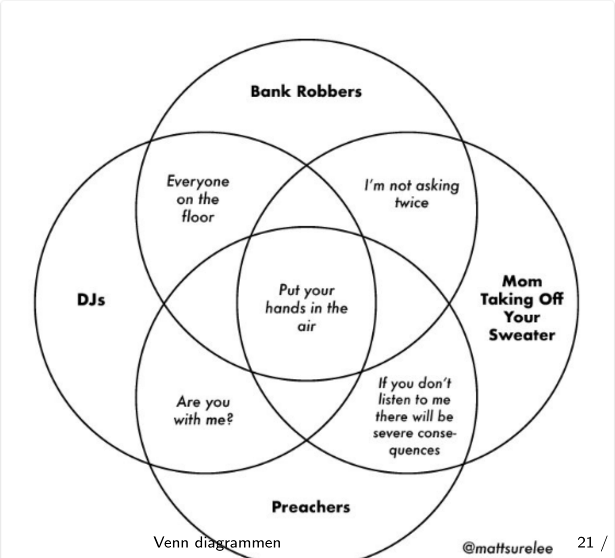
On the Diagrammatic and Mechanical Representation of Propositions and Reasonings. Dublin Philosophical Magazine and Journal of Science 9, 1—18, 1880.

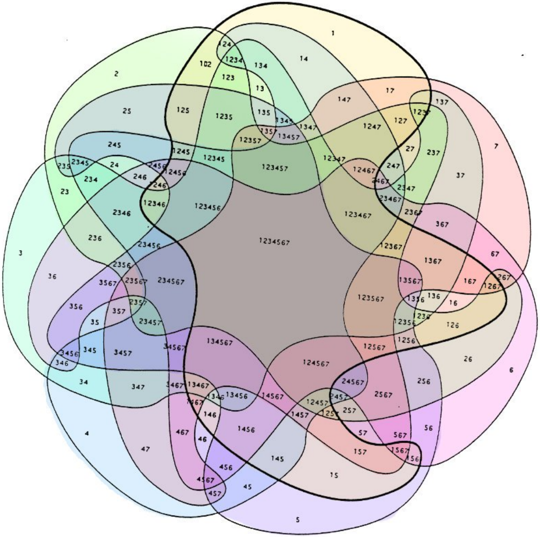


Hull 1834 – Cambridge 1923

wikipedia

I added a layer to that "Put your hands in the air" Venn diagram going around.



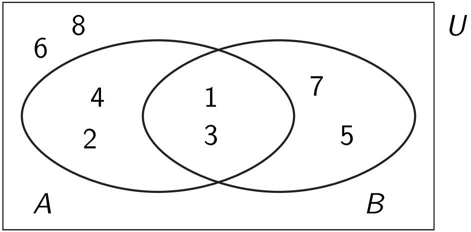


twitter.com/tweetsauce/ (bron) (interactief)

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 2, 3, 4\} \qquad B = \{1, 3, 5, 7\}$$

'klein' 'oneven'

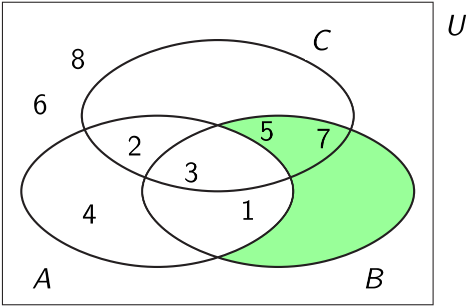


$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 2, 3, 4\} \qquad B = \{1, 3, 5, 7\}$$

'klein' 'oneven'

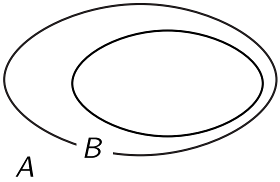
$$C = \{2, 3, 5, 7\}$$ 'priem'

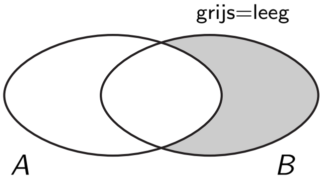


"alle grote oneven getallen zijn priem"

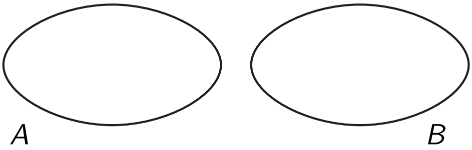
Euler vs. Venn

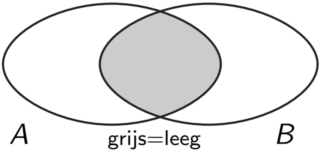
deelverzameling





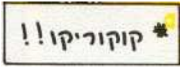
disjunct





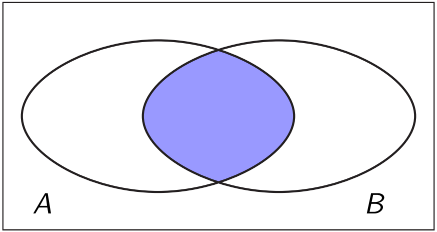
Overzicht

1. Verzamelingen
   - Definities
   - Venn diagrammen
   - **Boolese operaties**
   - Verzamelingenalgebra
   - Inclusie en exclusie
   - Collecties
   - Postscriptum



en

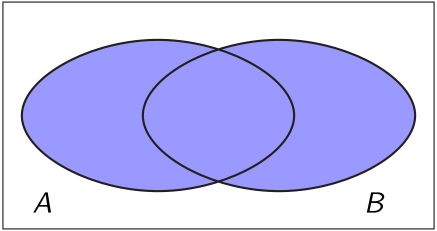
$$A \cap B = \{\, x \in U \mid (x \in A) \wedge (x \in B) \,\}$$



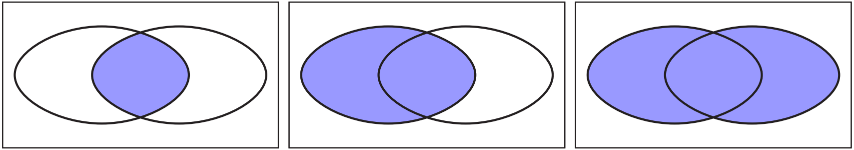
i∩ntersection

disjunct $A \cap B = \varnothing$ *disjoint* of

$$A \cup B = \{ x \in U \mid (x \in A) \vee (x \in B) \}$$



∪nion



Thm 1.3

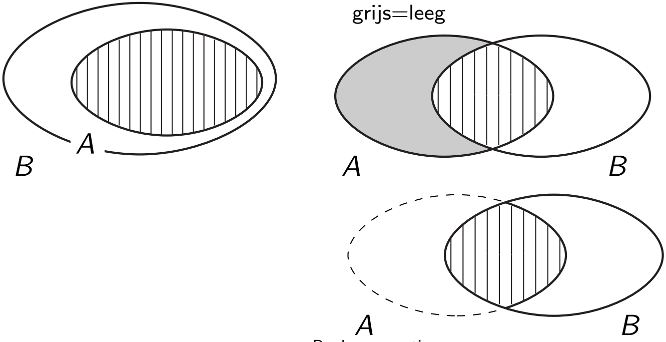
Voor elk tweetal verzamelingen A en B geldt $A \cap B \subseteq A \subseteq A \cup B$
$A \cap B \subseteq B \subseteq A \cup B$

Thm 1.4

de volgende beweringen zijn equivalent

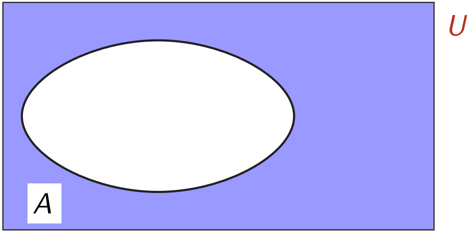
$$A \subseteq B, \quad A \cap B = A, \text{ en } \quad A \cup B = B$$

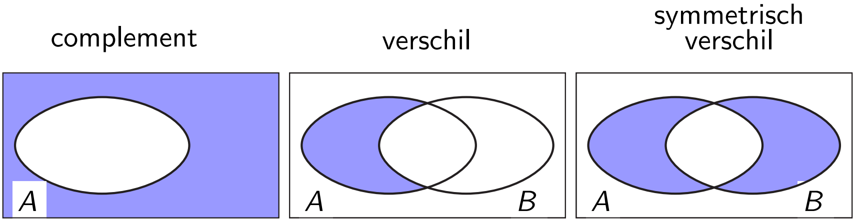
$$A \subseteq B \quad \text{desda} \quad A \cap B = A$$



niet

$$A^c = \{ x \in U \mid \neg(x \in A) \} = \{x \in U \mid x \notin A \}$$





$A^c = U \setminus A$

$A \setminus B$ $\quad A - B$

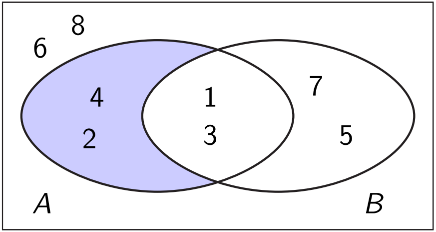
$A \cap B^c$

$A \oplus B$

$(A \setminus B) \cup (B \setminus A)$

$$U = \{1,2,3,4,5,6,7,8\}$$
$$A = \{1,2,3,4\} \qquad B = \{1,3,5,7\}$$

klein oneven



$B^c = \{2,4,6,8\}$ even

$A \cap B^c = \{2,4\}$ klein en even

$= A \setminus B$

$A^c = \{5,6,7,8\}$ groot

$A^c \cup B = \{1,3,5,6,7,8\}$

groot of oneven

$$A \cap B^c = (A^c \cup B)^c$$

universum: *strings* (woorden)   rijtjes symbolen

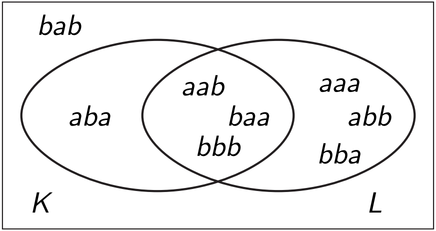
verzameling: "taal"

$$K = \{\, x \in \{a, b\}^* \mid x \text{ heeft een even aantal } a\text{'s} \,\}$$

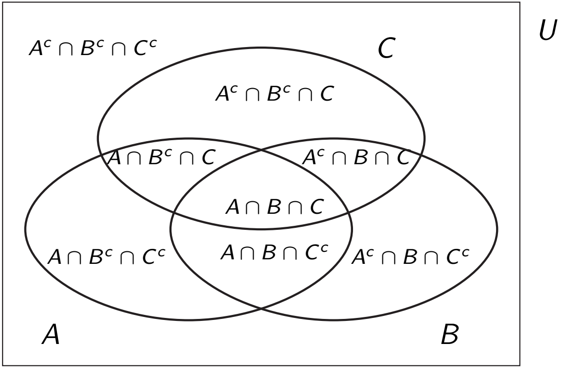
$$\{\lambda, b, aa, bb, aab, aba, baa, bbb, \ldots\}$$

$$L = \{\, x \in \{a, b\}^* \mid x \text{ heeft twee gelijke letters achter elkaar}\}$$

$$\{aa, bb, aaa, aab, abb, baa, bba, bbb, \ldots\}$$

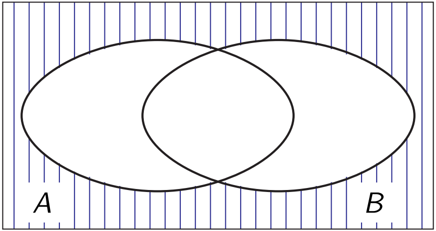


$aab, baa, bbb \in K \cap L$ $aba \in K \setminus L$ $aaa, abb, bba \in L \setminus K$ $bab \in (K \cup L)^c$

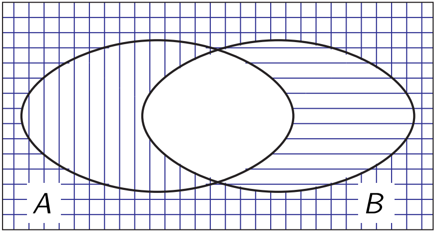

$A^c \cap B^c \cap C^c$
C
U
$A^c \cap B^c \cap C$
$A \cap B^c \cap C$
$A^c \cap B \cap C$
$A \cap B \cap C$
$A \cap B^c \cap C^c$
$A \cap B \cap C^c$
$A^c \cap B \cap C^c$
A
B

"niet (klein of oneven) = groot en even"

$$(A \cup B)^c = A^c \cap B^c$$



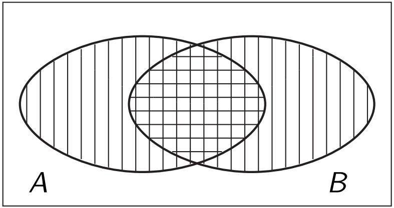
||| $(A \cup B)^c$



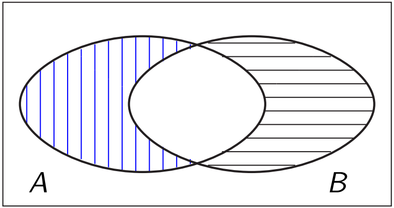
$\equiv$ A^c ||| B^c # $A^c \cap B^c$

symmetrisch verschil

$$A \oplus B = (A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$



$|||$ $(A \cup B)^c$ $\equiv$ $(A \cap B)^c$



$|||$ $A \setminus B$ $|||$ $B \setminus A$

$$(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$

①

definities

als $x \in$ (links) …

… dan $x \in$ (rechts)

wat doe je?

overtuig de lezer

beargumenteer

twee voorbeelden

②

Venn diagrammen

links en rechts

wat doe je?

construeer gebieden

arceren

betekenis streepjes

welk gebied

③

verzamelingenalgebra

vaste vorm

stap voor stap

herschrijf $\cdot \setminus \cdot$

benoem de regel

lastig

$$(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$

we bewijzen de gelijkheid door twee inclusies ...

($\subseteq$) neem $x \in (A \cup B) \setminus (A \cap B)$

laten zien dat $x \in (A \setminus B) \cup (B \setminus A)$

dan (1) $x \in A \cup B$ maar (2) $x \notin A \cap B$

dus $x \in A$ of $x \in B$ (1)

neem $x \in A$ — later doen we $x \in B$

dan $x \notin B$, want anders $x \in A \cap B$ (2)

dus $x \in A \setminus B$, dus in $(A \setminus B) \cup (B \setminus A)$

neem $x \in B$, dan volgt eenzelfde redenering

($\supseteq$) neem $x \in (A \setminus B) \cup (B \setminus A)$ — en redeneer verder

$$A \subseteq B \quad \text{desda} \quad A \cap B = A$$
$$\Longleftrightarrow$$

als $A \subseteq B$ dan $A \cap B = A$ ✓

$A \cap B \subseteq A$

altijd → $A \subseteq A \cap B$

als $x \in A$ --- $A \subseteq B$ ---▸ dan $x \in A \cap B$

neem $x \in A$ ⟶ dus $x \in A \cap B$

gegeven $A \subseteq B$

dus $x \in B$

als $A \cap B = A$ dan $A \subseteq B$ ✓

als $x \in A$ --- $A \cap B = A$ ---▸ dan $x \in B$

neem $x \in A$

gegeven $A = A \cap B$

dus $x \in A \cap B$

dus $x \in B$

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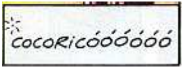


$$23 + 11 + 17 + 9 = (23 + 17) + (11 + 9) = 40 + 20 = 60$$

binaire operator $\star$

commutatief $x \star y = y \star x$ voor alle ...

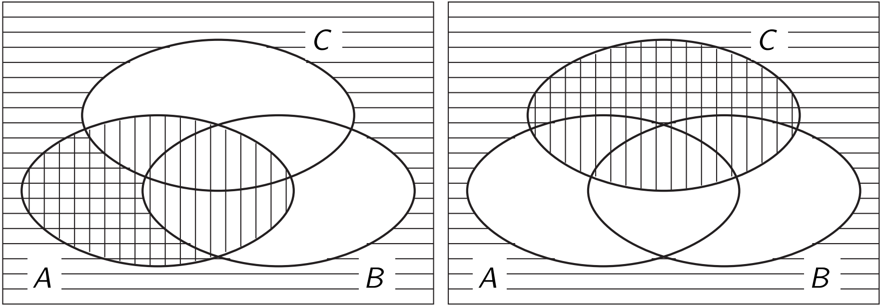
associatief $x \star (y \star z) = (x \star y) \star z$ voor alle ...

commutatief	$A \cup B = B \cup A$	$A \cap B = B \cap A$
associatief	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$

verschil $\setminus$ niet commutatief, niet associatief

$A \downarrow B = (A \cup B)^c$ NOR commutatief

$A \downarrow (B \downarrow C) \stackrel{?}{=} (A \downarrow B) \downarrow C$ associatief?



||| A ≡ $B \downarrow C$
ongearceerd

≡ $A \downarrow B$ ||| C
ongearceerd

$$(B \cup C) \setminus A \neq (A \cup B) \setminus C$$

$$1 \cdot x = x \quad 0 \cdot x = 0 \qquad 0 + x = x$$

binaire operator $\star$

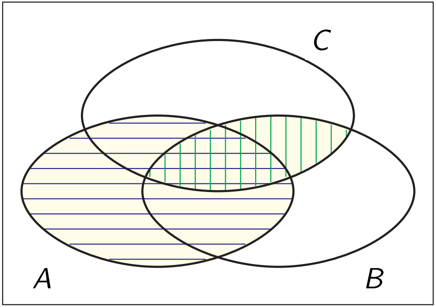
een $1 \star x = x$ voor alle ...

nul $0 \star x = 0$ voor alle ...

een	$A \cup \varnothing = A$	$A \cap U = A$
nul	$A \cup U = U$	$A \cap \varnothing = \varnothing$

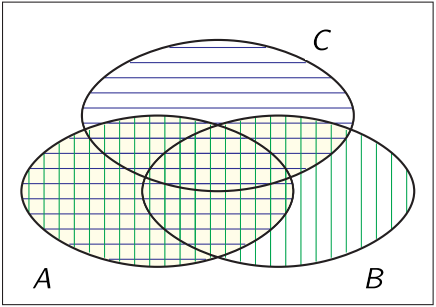
$$3 \cdot (2 + 7) = (3 \cdot 2) + (3 \cdot 7)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$



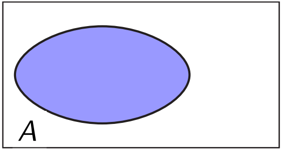
$\equiv A$ $\quad ||| \; B \cap C$

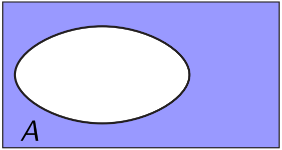
alles gestreept



$\equiv A \cup C$ $\quad ||| \; A \cup B$

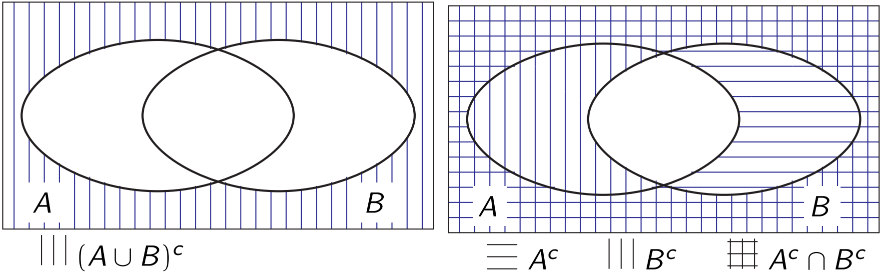
dubbel gestreept





$$A \cup A^c = U \quad A^{cc} = A \quad A \cap A^c = \varnothing$$

$$(A \cup B)^c = A^c \cap B^c$$

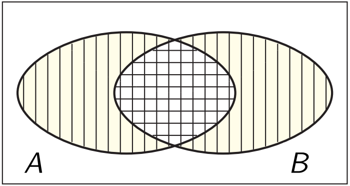


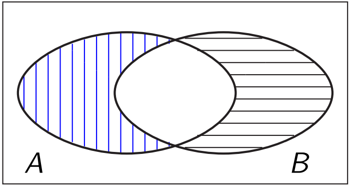
idempotent	$A \cup A = A$	$A \cap A = A$
associatief	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
commutatief	$A \cup B = B \cup A$	$A \cap B = B \cap A$
distributief	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	
		$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
identiteit	$A \cup \varnothing = A$	$A \cap U = A$
	$A \cup U = U$	$A \cap \varnothing = \varnothing$
dubbel compl.	$A^{cc} = A$	
complement	$A \cup A^c = U$	$A \cap A^c = \varnothing$
	$\varnothing^c = U$	$U^c = \varnothing$
De Morgan	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$

Tabel: De axioma's van de *verzamelingenalgebra*.

symmetrisch verschil

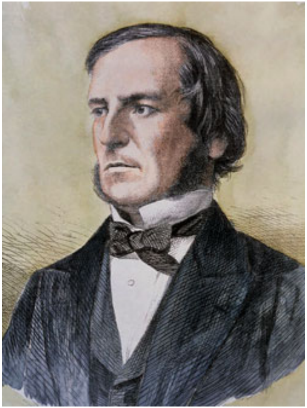
$$(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$





$(A \cup B) \setminus (A \cap B) =$	*omschrijven*
$(A \cup B) \cap (A \cap B)^c =$	distributief
$(A \cap (A \cap B)^c) \cup (B \cap (A \cap B)^c) =$	DeMorgan×2
$(A \cap (A^c \cup B^c)) \cup (B \cap (A^c \cup B^c)) =$	distributief×2
$((A \cap A^c) \cup (A \cap B^c)) \cup ((B \cap A^c) \cup (B \cap B^c)) =$	complement×2
$(\varnothing \cup (A \cap B^c)) \cup ((B \cap A^c) \cup \varnothing) =$	nul-element×2
$(A \cap B^c) \cup (B \cap A^c)$	omschrijven×2
$(A \setminus B) \cup (B \setminus A)$	

Boole's work founded the discipline of algebraic logic. It is often, but mistakenly, credited as being the source of what we know today as Boolean algebra. (wikipedia)



Lincoln 1815 – Cork 1864

wikipedia

An Investigation of *the Laws of Thought* on Which are Founded the Mathematical Theories of Logic and Probabilities, 1854

absorptie

$A \cup (A \cap B) = A$ en $A \cap (A \cup B) = A$

$$
\begin{aligned}
A \cup (A \cap B) &= \text{(identiteit)} \\
(A \cap U) \cup (A \cap B) &= \text{(distributiviteit)}\uparrow \\
A \cap (U \cup B) &= \text{(identiteit)} \\
A \cap (B \cup U) &= \text{(commutativiteit)}* \\
A \cap U &= \text{(identiteit)} \\
A &
\end{aligned}
$$

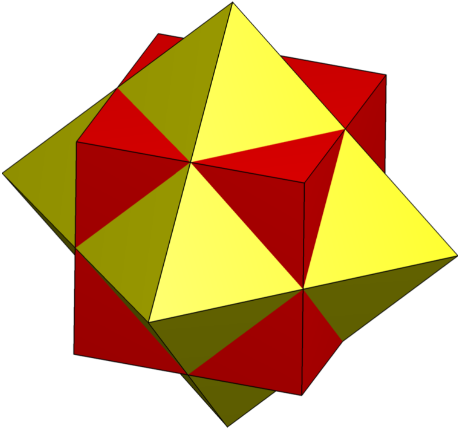
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ distributief

$A \cup U = U$ identiteit

idempotentie

$A \cap A =$	nulelement
$(A \cap A) \cup \varnothing =$	complement
$(A \cap A) \cup (A \cap A^c) =$	distributief
$A \cap (A \cup A^c) =$	complement
$A \cap U =$	eenelement
A	

☒Ch.15: Boolean Algebra



wikipedia compound of a cube . . .
(c) Robert Webb www.software3d.com

distributief	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	
		$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
identiteit	$A \cup \varnothing = A$	$A \cap U = A$
	$A \cup U = U$	$A \cap \varnothing = \varnothing$
complement	$A \cup A^c = U$	$A \cap A^c = \varnothing$
De Morgan	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$

$$\varphi \overset{\textit{duaal}}{\longleftrightarrow} \varphi^* \qquad \begin{cases} \cup \longleftrightarrow \cap \\ \varnothing \longleftrightarrow U \end{cases}$$

$$\varphi = \psi \quad \text{desda} \quad \varphi^* = \psi^*$$

Basic Identities of Boolean Algebra

Let X be a boolean variable and 0,1 constants

1. $X + 0 = X$ -- Zero Axiom
2. $X \cdot 1 = X$ -- Unit Axiom
3. $X + 1 = 1$ -- Unit Property
4. $X \cdot 0 = 0$ -- Zero Property

5. $X + X = X$ -- Idempotence
6. $X \cdot X = X$ -- Idempotence
7. $X + X' = 1$ -- Complement
8. $X \cdot X' = 0$ -- Complement
9. $(X')' = X$ -- Involution

1,4.Zero	$X + 0 = X$	$X \cdot 0 = 0$
3,2.Unit	$X + 1 = 1$	$X \cdot 1 = X$
5.Idempotence	$X + X = X$	$X \cdot X = X$
7.Complement	$X + X' = 1$	$X \cdot X' = 0$
9.Involution	$(X')' = X$	
10.Commutat.	$X + Y = Y + X$	$X \cdot Y = Y \cdot X$
12.Associative	$X + (Y + Z) = (X + Y) + Z$	$X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$
14.Distributive	$X \cdot (Y + Z) = X \cdot Y + X \cdot Z$	
		$X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$
16.DeMorgan's	$(X + Y)' = X' \cdot Y'$	$(X \cdot Y)' = X' + Y'$

Tabel: Boolean Algebra Properties FoDSD

Karnaugh

0	1	1	1
0	0	0	1
0	0	0	1
1	1	1	1

$$A \cap B \cup B \cap C^c \cap D^c \cup A^c \cap B \cap C \cup C^c \cap D = B \cup C^c \cap D$$ ☹

$$(A \cap B) \cup (B \cap C^c \cap D^c) \cup (A^c \cap B \cap C) \cup (C^c \cap D) = B \cup (C^c \cap D)$$

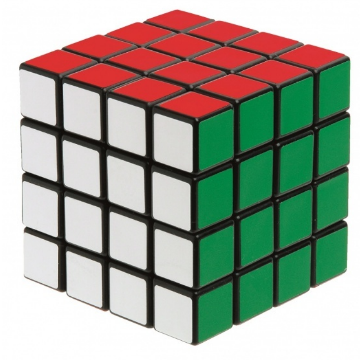
AB + BC'D' + A'BC + C'D =	absorption
AB + BC'D' + A'BC + (C'D + BC'D) =	commutative
AB + (BC'D' + BC'D) + A'BC + C'D =	distributive
AB + BC'(D'+D) + A'BC + C'D =	complement+unit
AB + BC' + A'BC + C'D =	complement+distributive
(ABC+ABC') + (ABC'+A'BC') + A'BC + C'D =	commutative
ABC + A'BC + (ABC' + ABC') + A'BC' + C'D =	idempotence
ABC + A'BC + ABC' + A'BC' + C'D =	distributive
(A+A')B(C+C') + C'D =	complement+unit
B + C'D	

Overzicht

1. Verzamelingen
   - Definities
   - Venn diagrammen
   - Boolese operaties
   - Verzamelingenalgebra
   - **Inclusie en exclusie**
   - Collecties
   - Postscriptum

hoeveel kubusjes zichtbaar

$4 \times 4 \times 4 \qquad 4^3$

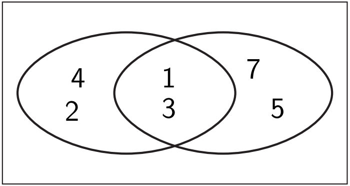


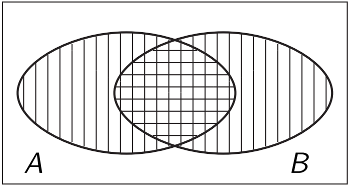
$3 \cdot 4^2 - 3 \cdot 4 + 1$

Jumbo Rubik's cube

Schaum

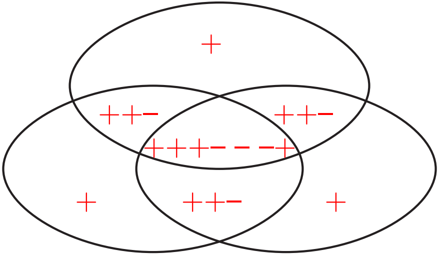
A eindig   *aantal elementen* $n(A)$ $|A|$ $\#A$





Lem.1.9

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Cor.1.10, Thm.5.8

Voor eindige verzamelingen A, B en C geldt dat
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$.

Lem.1.9 *

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A \cup (B \cup C)| \overset{*}{=} |A| + |\underbrace{B \cup C}_{①}| - |\underbrace{A \cap (B \cup C)}_{②}|$$

① $|B \cup C| \overset{*}{=} |B| + |C| - |B \cap C|$

② $A \cap (B \cup C) \overset{\text{distr}}{=} (A \cap B) \cup (A \cap C)$

$$-|(A \cap B) \cup (A \cap C)| \overset{*}{=} -|A \cap B| - |A \cap C| + |\underbrace{(A \cap B) \cap (A \cap C)}_{A \cap B \cap C}|$$

Cor.1.10, Thm.5.8

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|.$

Cor.1.10, Thm.5.8

Voor eindige verzamelingen A, B en C geldt dat
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

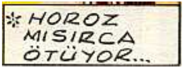
$U = \{1, 2, \ldots, 1000\}$ getallen deelbaar door 2,3 of 5

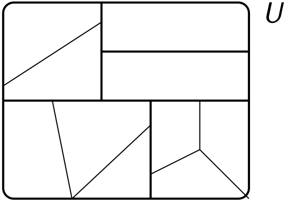
$$A_2 \cup A_3 \cup A_5$$

$$A_n = \{\, x \in U \mid x \text{ is deelbaar door } n \,\}$$
$$|A_n| = 1000 \div n \text{ (zonder rest)}$$

kgv!

$$\begin{gathered} |\; A_2 \cup A_3 \cup A_5 \;| = \\ (1000 \div 2) + (1000 \div 3) + (1000 \div 5) \\ -(1000 \div 6) - (1000 \div 10) - (1000 \div 15) \\ +(1000 \div 30) = \\ 500 + 333 + 200 - 166 - 100 - 66 + 33 = 734 \end{gathered}$$


* HOROZ
MISIRCA
ÖTÜYOR...



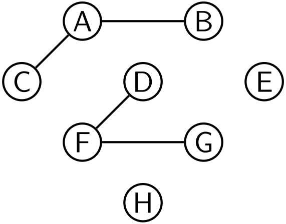
$\mathcal{A} = \{ A_1, A_2, \ldots, A_n \}$ $\quad$ $\mathcal{A} = \{ A_i \}_{i \in I}$

$A_i \subseteq U,\ A_i \neq \varnothing$

$A_1 \cup A_2 \cup \cdots \cup A_n = U$

$\bigcup_{i=1}^{n} A_i$ $\quad$ $\bigcup_{i \in I} A_i = U$

$A_i \cap A_j = \varnothing$ als $i \neq j$



$\{ \{ A, B, C \}, \{ D, F, G \}, \{ E \}, \{ H \} \}$

$\mathbb{Z}$ restklassen $\{\ [0], [1], \ldots, [6]\ \}$

$[0] = \{\ldots, -14, -7, 0, 7, 14, \ldots\}$

$[1] = \{\ldots, -13, -6, 1, 8, 15, \ldots\}$

$\ldots$

$[6] = \{\ldots, -8, -1, 6, 13, 19, \ldots\}$

$\mathbb{N}^+$ $\{\ D_k\ \}_{k \in \mathbb{N}}$

$D_0 = \{\ 1, 3, 5, 7, \ldots\}$

$D_1 = \{\ 2{\cdot}1, 2{\cdot}3, 2{\cdot}5, \ldots\}$

$D_2 = \{\ 4{\cdot}1, 4{\cdot}3, 4{\cdot}5, \ldots\}$

$\ldots$

$D_k = \{\ 2^k{\cdot}m \mid m \text{ oneven}\ \}$

$(A \cup B)^c = A^c \cap B^c$ $\neg(p \vee q) = \neg p \wedge \neg q$

$(\ \bigcup_{i \in I} A_i\ )^c = \bigcap_{i \in I} A_i^c$ $\neg\exists i : P(i) = \forall i : \neg P(i)$

$x \in \bigcup_{i \in I} A_i$ desda *er is een* $i \in I$ met $x \in A_i$

$\{1, 2, 3, 4\}$

$\{1, 2, 3\}$ $\{1, 2, 4\}$ $\{1, 3, 4\}$ $\{2, 3, 4\}$

$\{1, 2\}$ $\{1, 3\}$ $\{1, 4\}$ $\{2, 3\}$ $\{2, 4\}$ $\{3, 4\}$

$\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$

$\varnothing$

definitie

$\mathcal{P}(A) = \{ X \mid X \subseteq A \}$ ($X \in \mathcal{P}(A)$ desda $X \subseteq A$)

$$\mathcal{P}(\{ a, b, c \}) = \{ \{ a, b, c \}, \{ a, b \}, \{ a, c \}, \{ b, c \}, \{ a \}, \{ b \}, \{ c \}, \varnothing \}$$

$\varnothing \subseteq A$ $\varnothing \in \mathcal{P}(A)$ $A \subseteq A$ $A \in \mathcal{P}(A)$

ook $\varnothing \subseteq \mathcal{P}(A)$

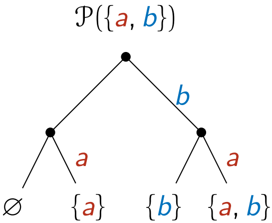
A eindig dan $|\mathcal{P}(A)| = 2^{|A|}$

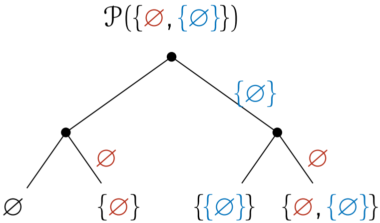
$2^0 = 1$ $\mathcal{P}(\varnothing) = \{ \varnothing \}$

$2^1 = 2$ $\mathcal{P}(\ \mathcal{P}(\varnothing)\ ) = \mathcal{P}(\ \{\varnothing\}\ ) = \{ \varnothing, \{ \varnothing \} \} = \{ \{ \ \}, \{ \varnothing \} \}$

$2^2 = 4$ $\mathcal{P}(\mathcal{P}(\mathcal{P}(\varnothing))) = \mathcal{P}(\{ \varnothing, \{\varnothing\} \}) = \{ \{ \ \}, \{ \varnothing \}, \{ \{\varnothing\} \}, \{ \varnothing, \{\varnothing\} \} \}$

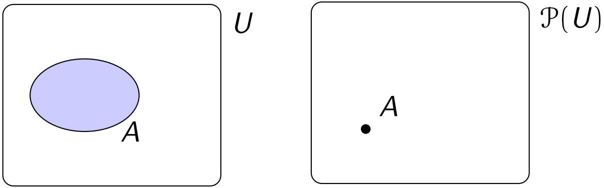
machtsverzameling





$\{\ \emptyset, \{a\}, \{b\}, \{a, b\}\ \}$

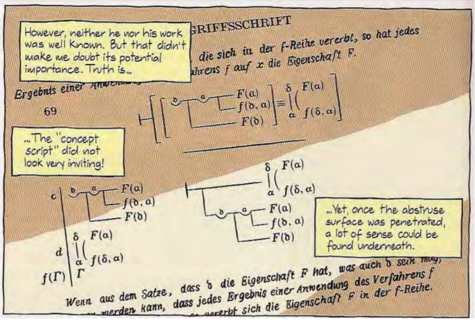
$\{\ \emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\ \}$

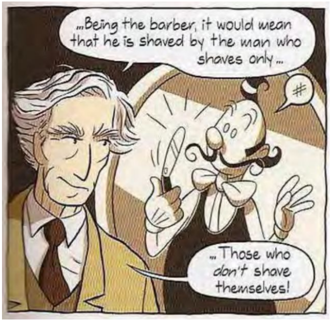


$A \subseteq U$ $A \in \mathcal{P}(U)$

$$\varnothing \notin \varnothing \qquad \varnothing \subseteq \varnothing$$
$$\varnothing \in \{\, \varnothing \,\} \qquad \varnothing \subseteq \{\, \varnothing \,\}$$


GIGGERIGI !



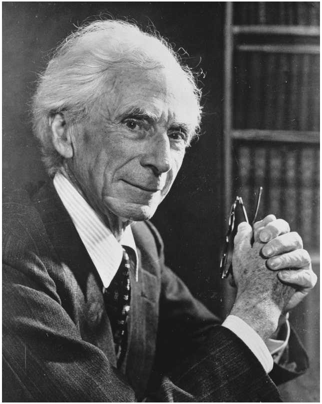


A. Doxiadis, C. Papadimitriou, A. Papadatos:
Logicomix, An epic search for truth, 2009.

$*54{\cdot}43.\quad \vdash :.\ \alpha, \beta \in 1 \,.\, \supset :\ \alpha \cap \beta = \Lambda \,.\, \equiv .\ \alpha \cup \beta \in 2$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$*.*

A.N. Whitehead, B. Russell, Principia Mathematica, 1910



Trellech 1872 –
Penrhyndeudraeth 1970

wikipedia nationaal archief

$$Z \stackrel{def}{=} \{ V \mid V \notin V \}$$

$$V \stackrel{?}{\in} Z \quad \text{desda} \quad V \notin V$$

$$Z \stackrel{?}{\in} Z \quad \text{desda} \quad Z \notin Z$$

Georg Cantor (1874) on-aftelbaar (*overaftelbaar*)

Entscheidungsproblem David Hilbert

Kurt Gödel (1931) on-volledigheid ‘deze stelling heeft geen bewijs’

Alonzo Church (1935) λ-calculus
Emil Post (1936) finite combinatory processes
Alan Turing (1936) Turing machine on-berekenbaar

END.