Tegels

van patronen tot berekeningen

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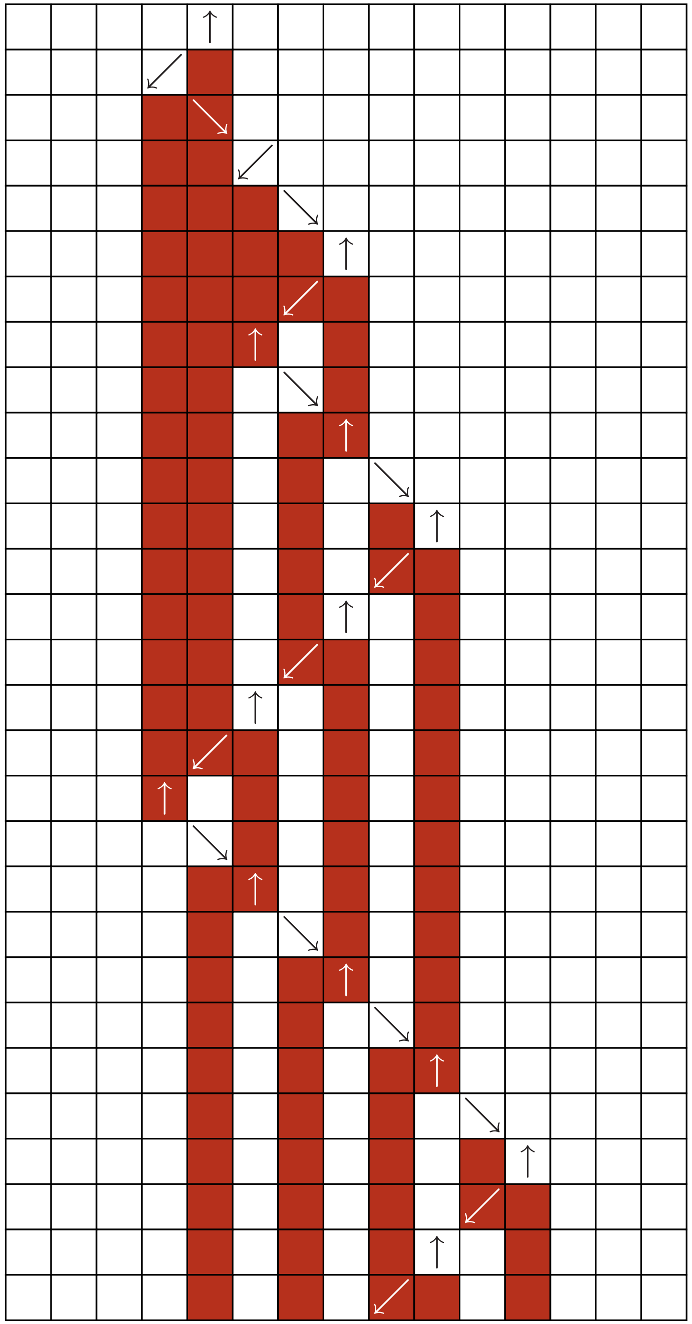
Universiteit Leiden, Informatica
www.liacs.nl/~hoogeboo/praatjes/tegels/



wiskunde: regelmaat, patronen

informatica: berekeningen

modellen:

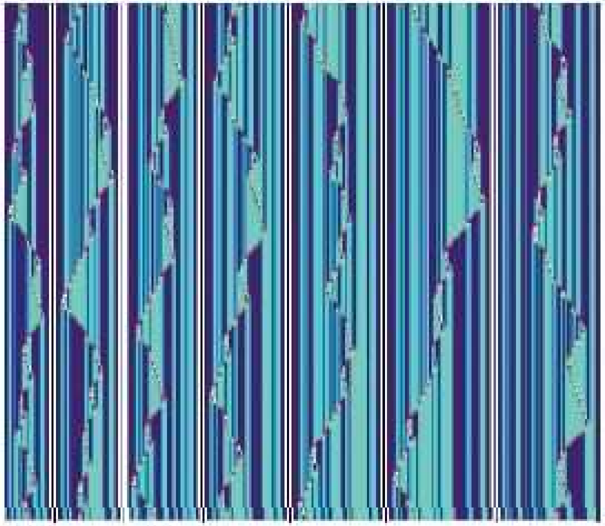
- Wang tiles
- Cellular automata
- Turing machines

Student wint prijs met bewijs

Het was een probleem voor de fijnproevers onder de informatici: wat is de kleinste universele Turing-machine? Een student gaf antwoord, in 44 pagina's.

Door Ionica Smeets

ROTTERDAM, 30 OKT. Alex Smith, een twintigjarige student informatica en elektronica uit Birmingham, heeft een openstaand probleem uit de informatica opgelost. Hij won daarmee vorige week 25.000 dollar. Smith bewees dat een zeer eenvoudige machine elke mogelijke berekening kan uitvoe-



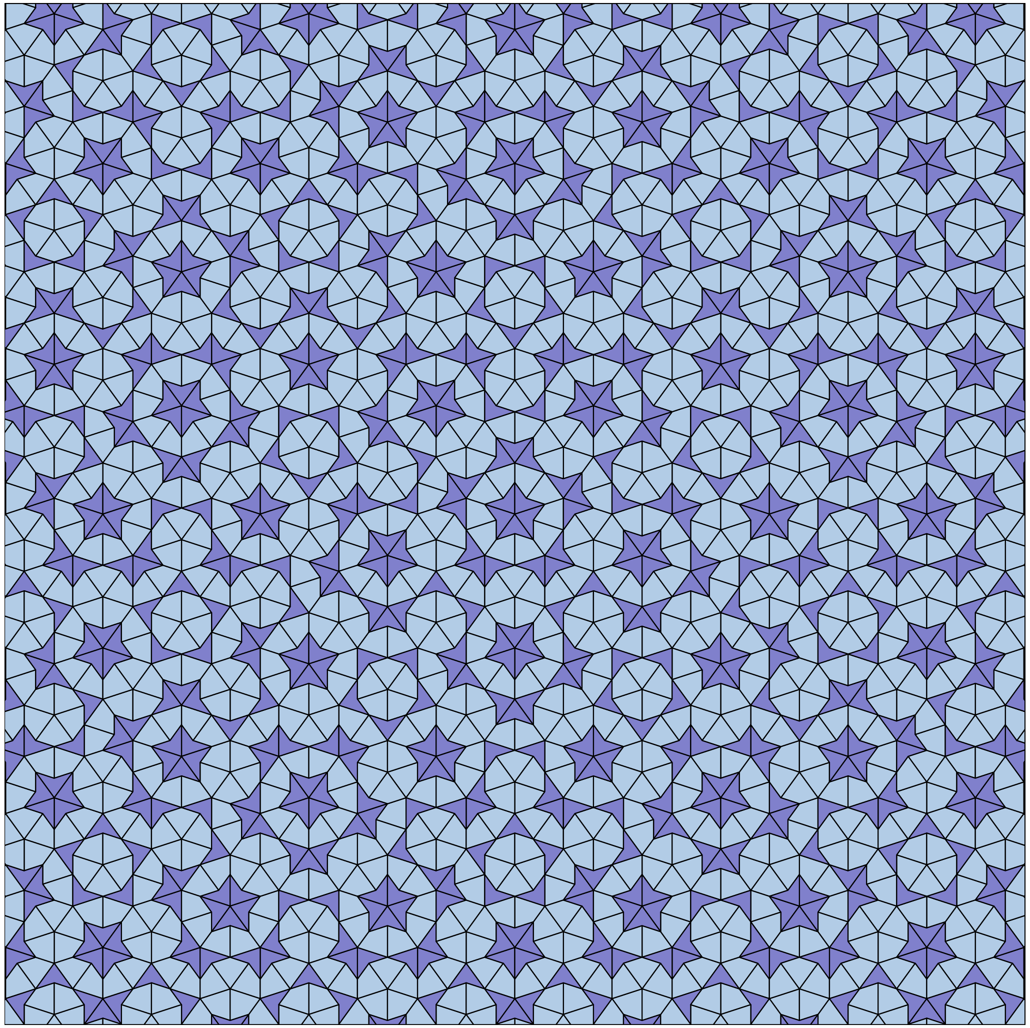
Schematische weergave van complexe berekening met een simpele Turing-machine. (Foto WI)

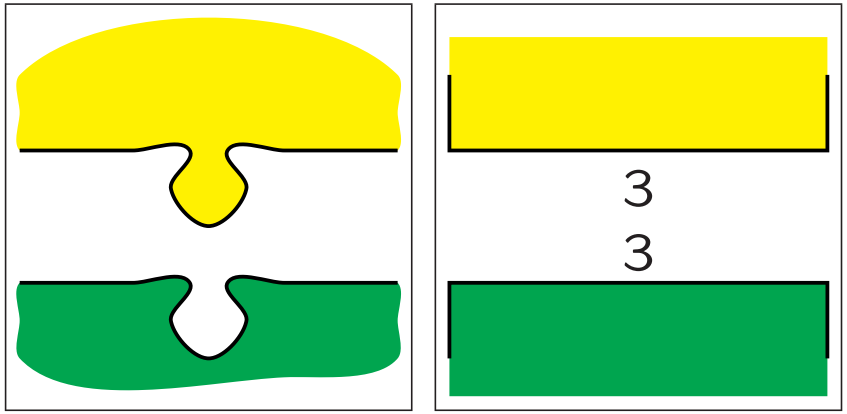
bepalen samen met de symbolen op de strook de stappen die de machine maakt.

Zo'n machine, schreef Wolfram in 2002, zou de kleinst mogelijk universele Turing-machine zijn. En in een ruim veertig pagina's tellend bewijs toont student Smith nu, tot zijn eigen verrassing, aan dat die bewering inderdaad juist is. In eerste instantie geloofde Smith juist dat hij kon bewijzen dat de machine niet universeel is, zo zei hij in een telefoongesprek met Wolfram.

Aan de informaticapraktijk verandert dit bewijs niet zoveel, denkt Peter van Emde Boas, lector in de mathematische informatica aan het Institute for Logic, Language en Computation van de Universiteit van Amsterdam. „Mooi dat die jongen dit probleem heeft

■ Spelregels

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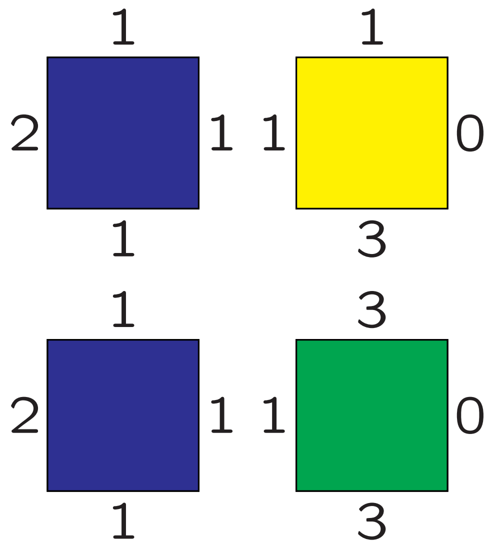
3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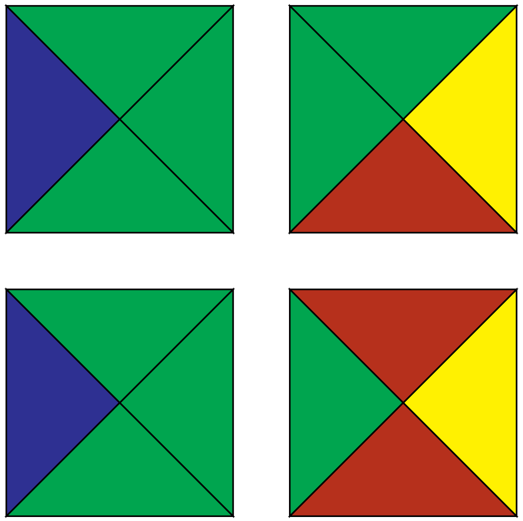
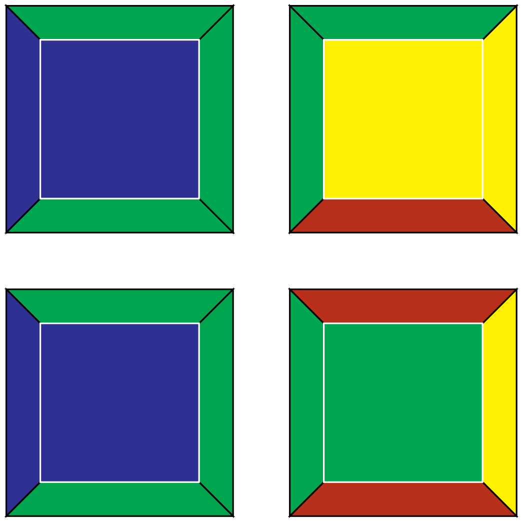
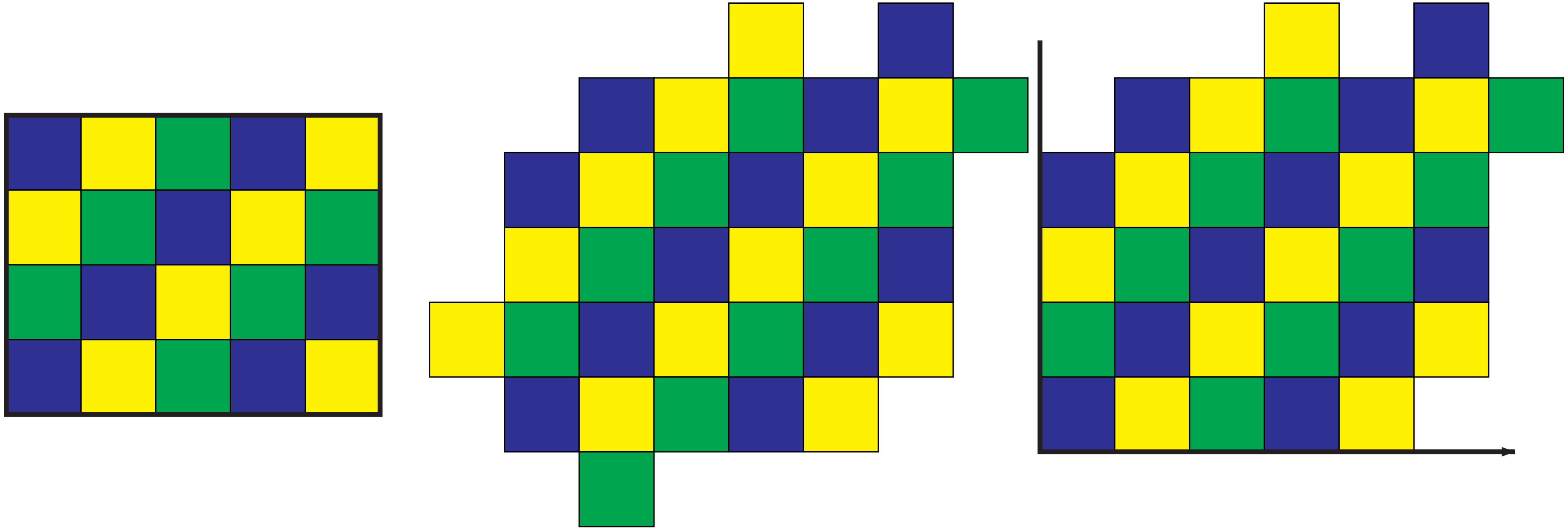
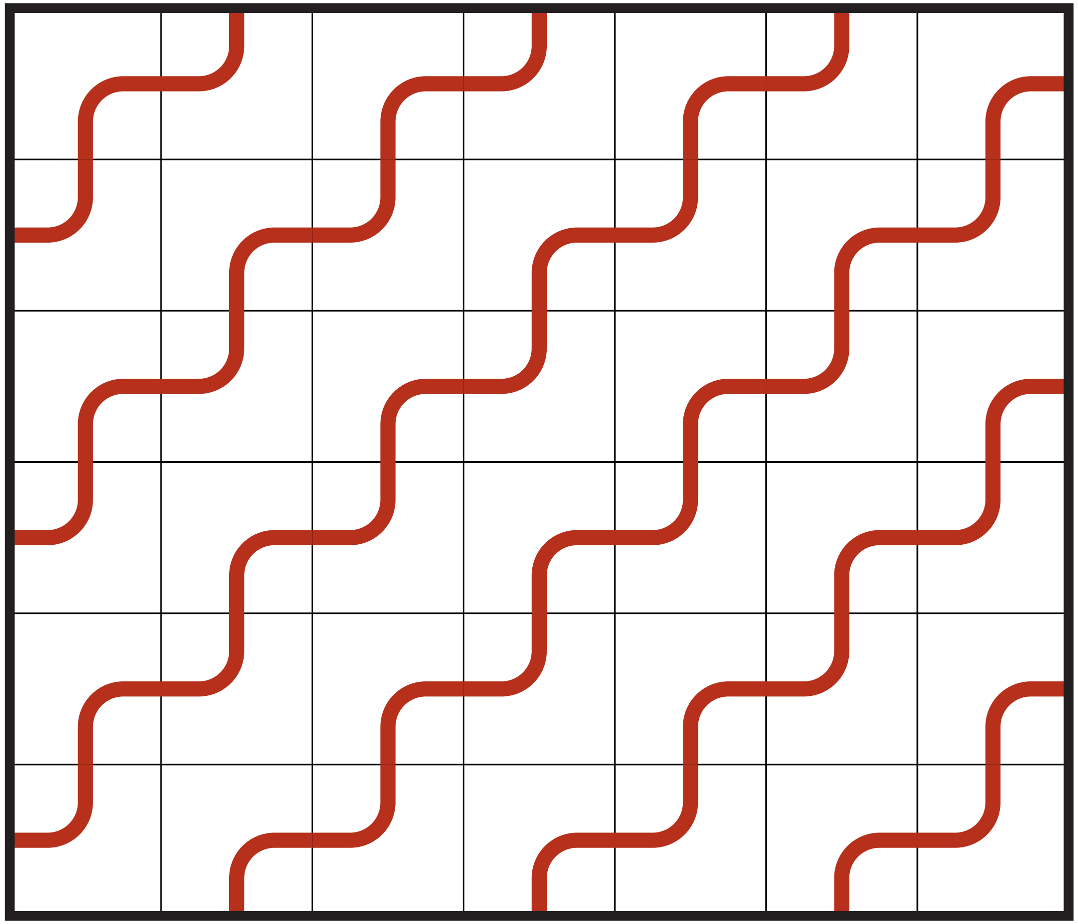


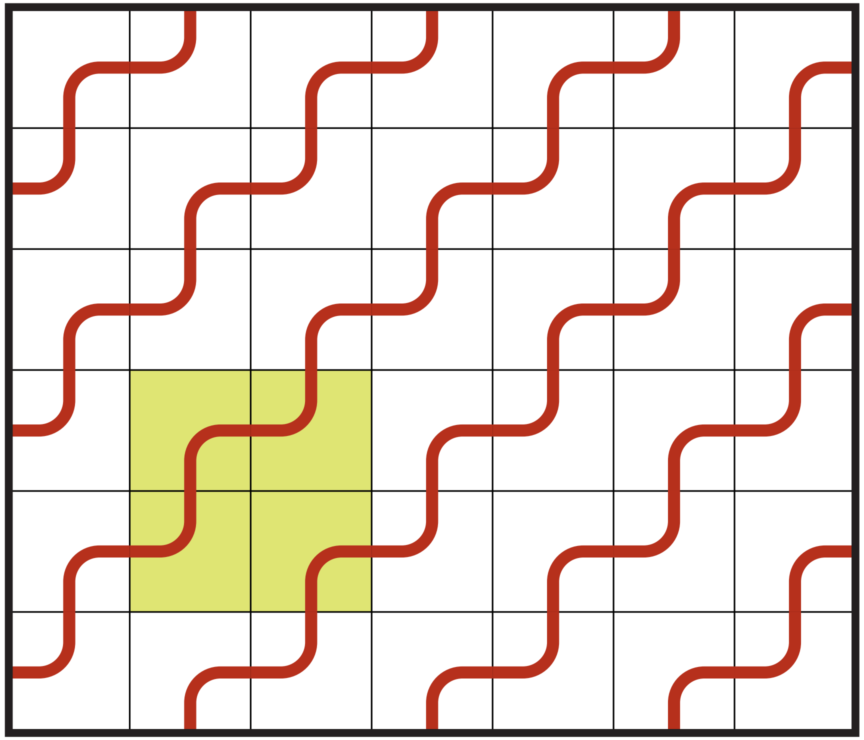


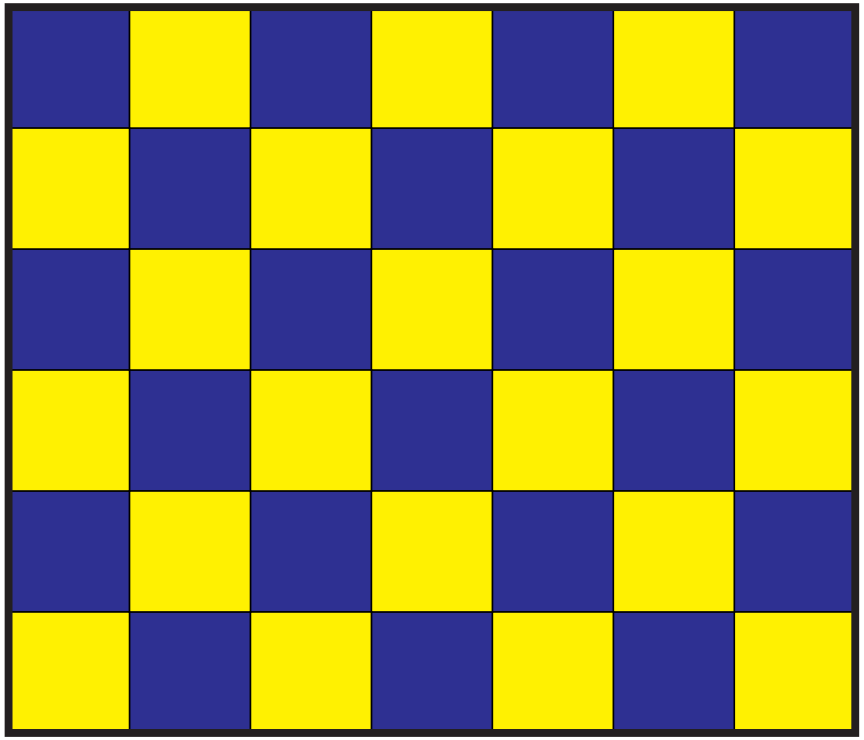


- rechthoek
- (half) vlak
- kwadrant

■ Patronen







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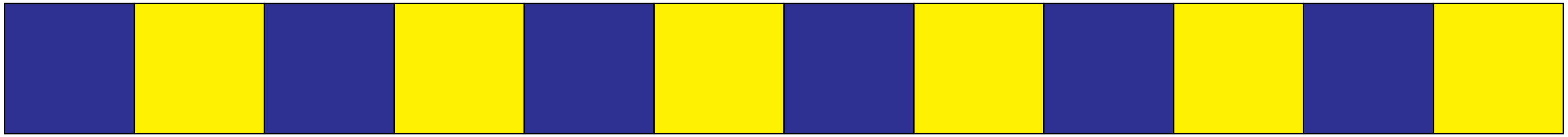
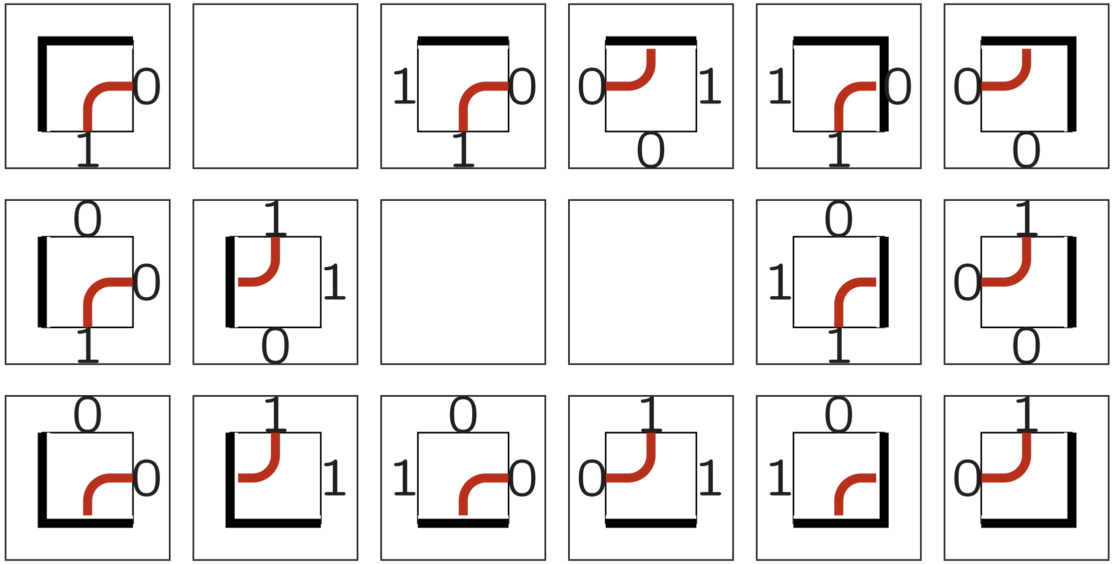
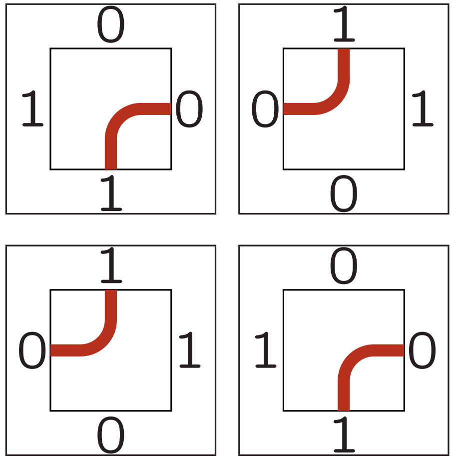
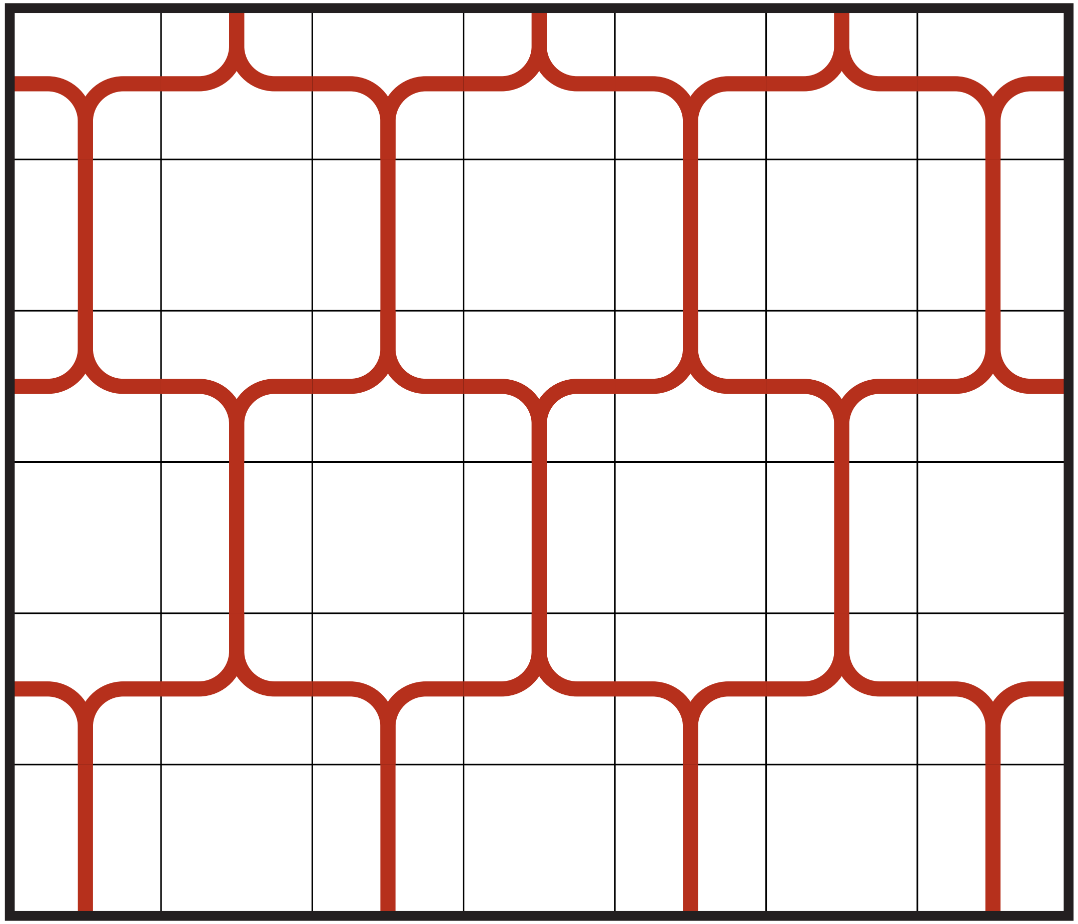


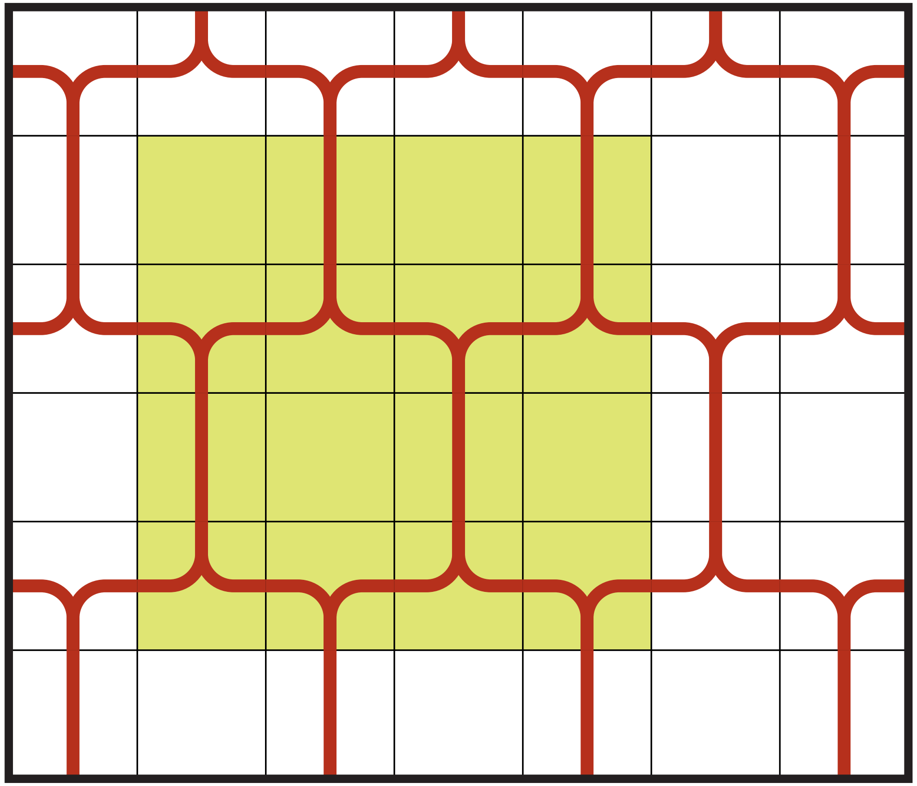


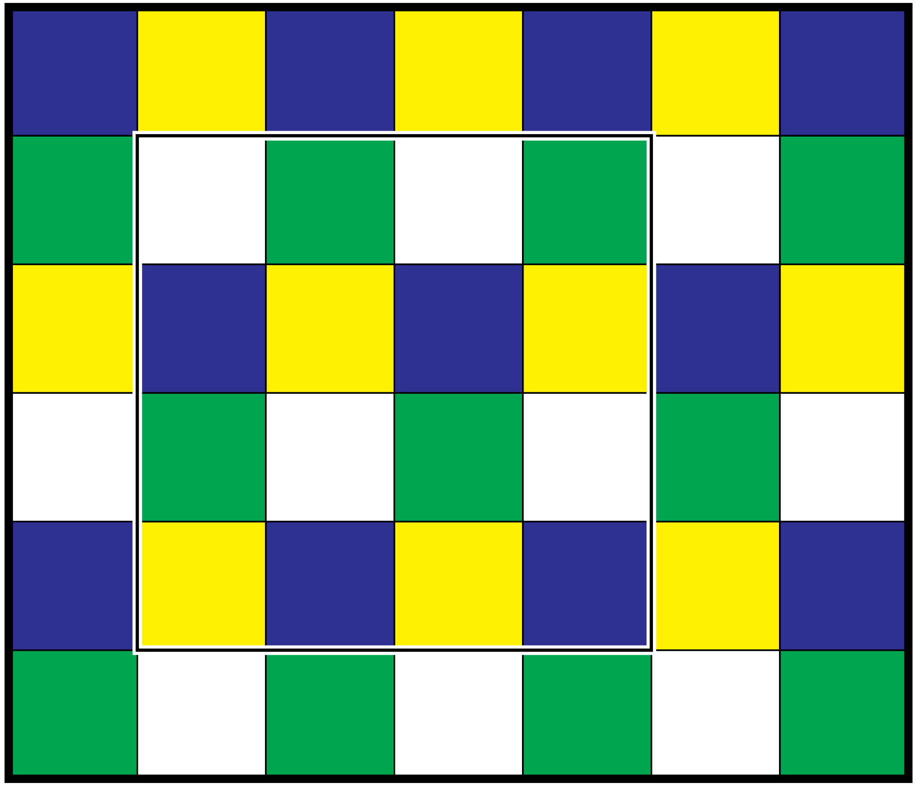


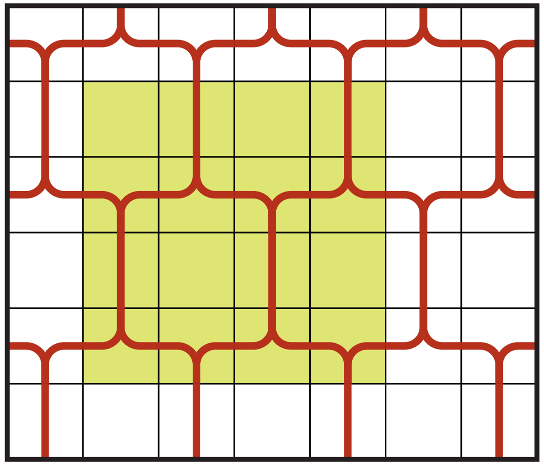
twee tegels

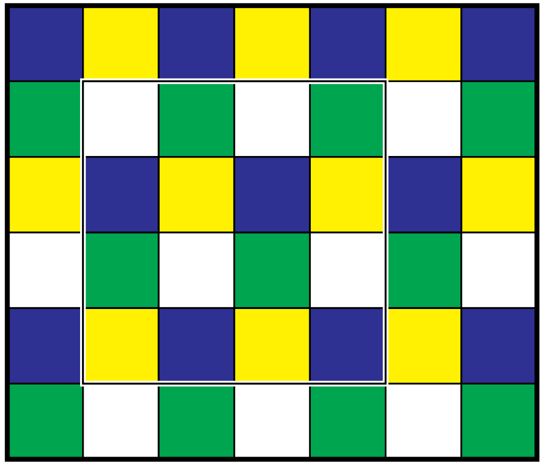
en wat randjes

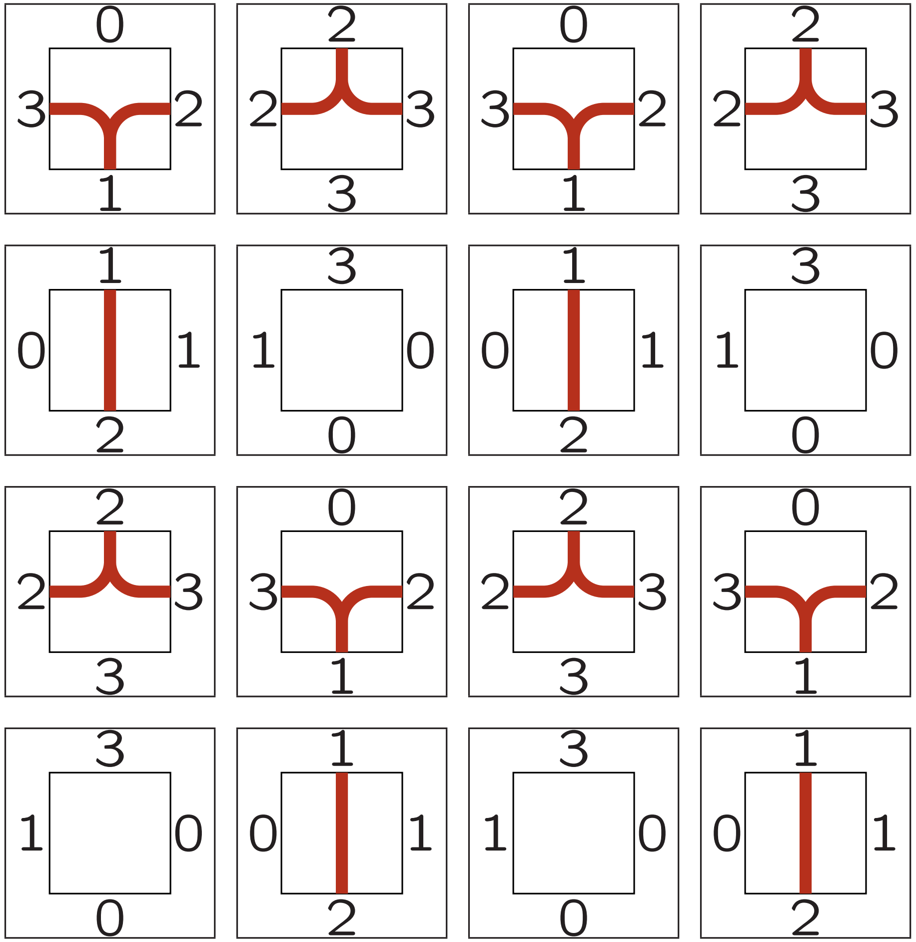








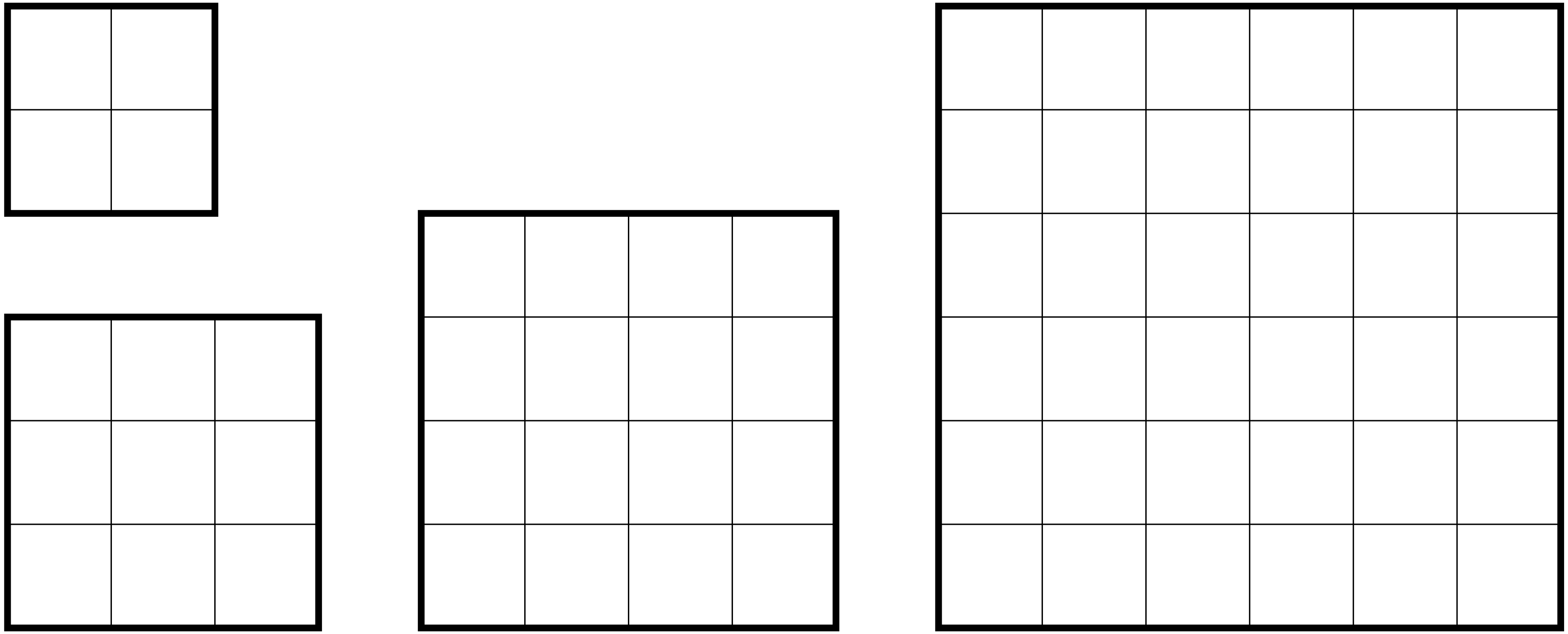


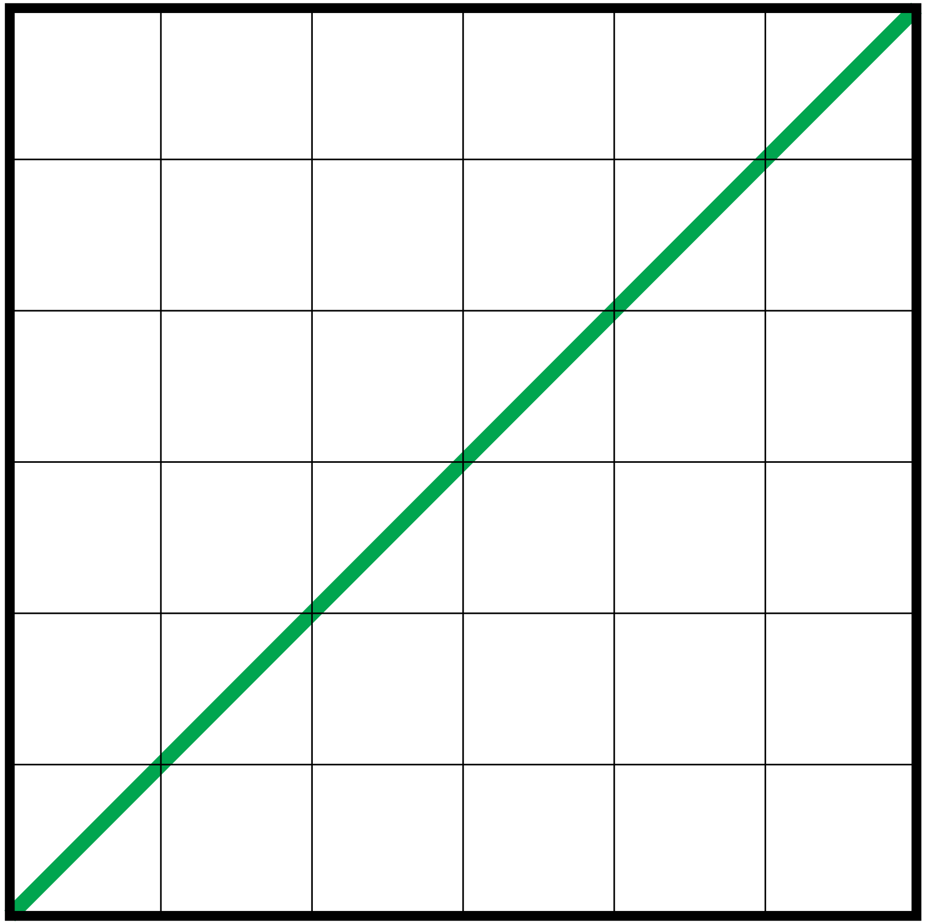


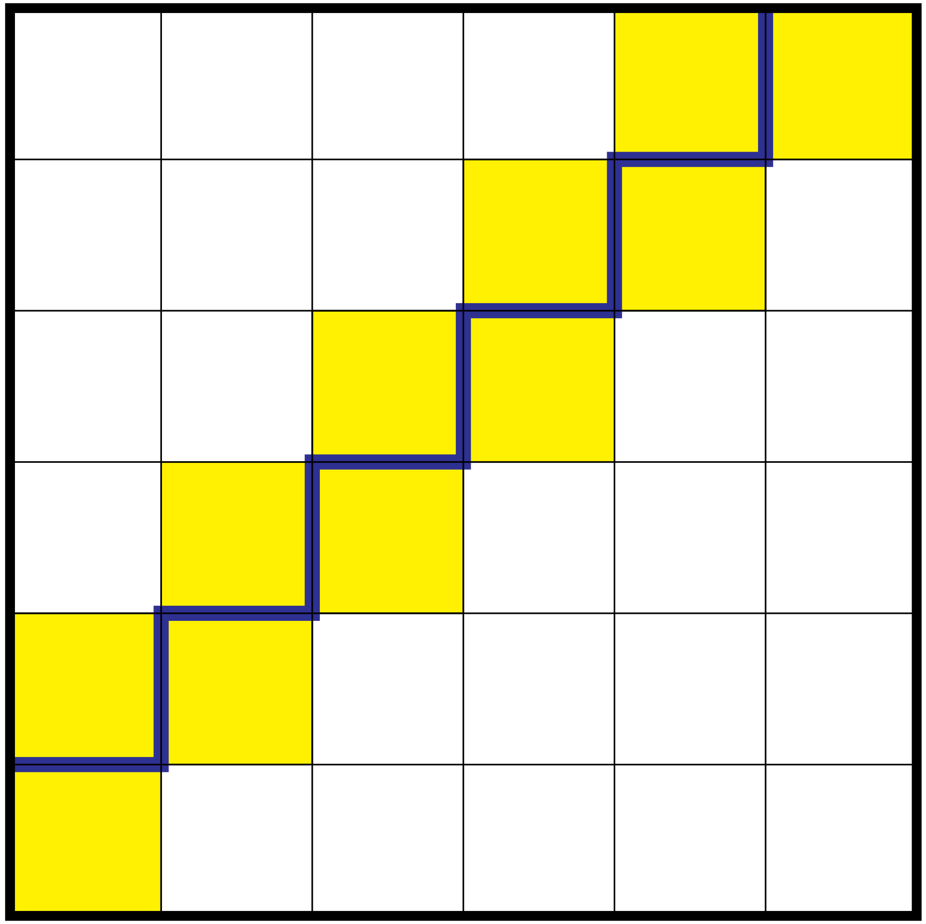
vier tegels

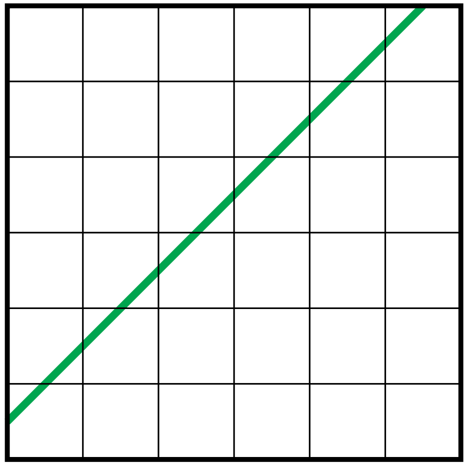
(en eventueel randjes)

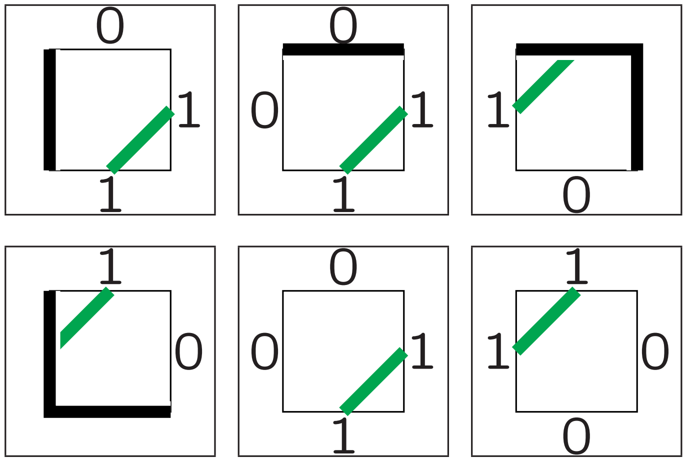
Vorm

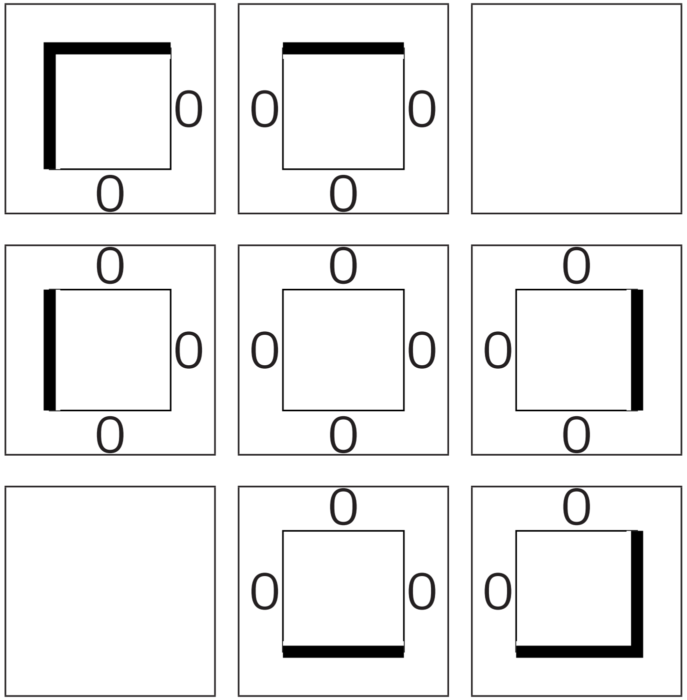


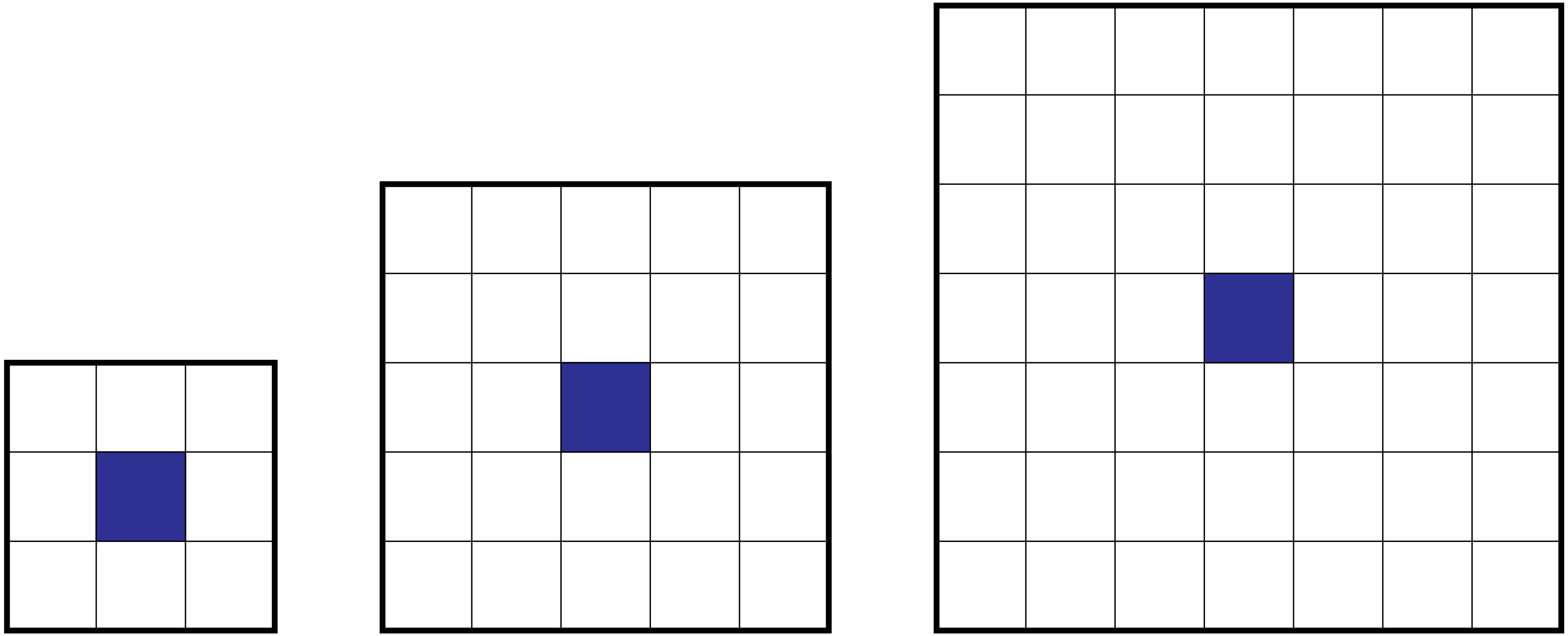


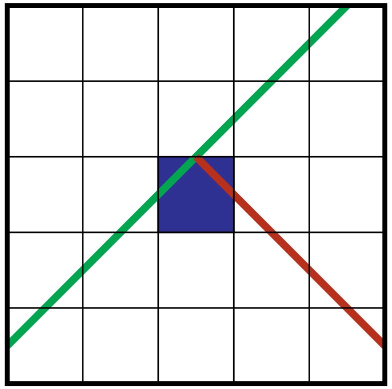


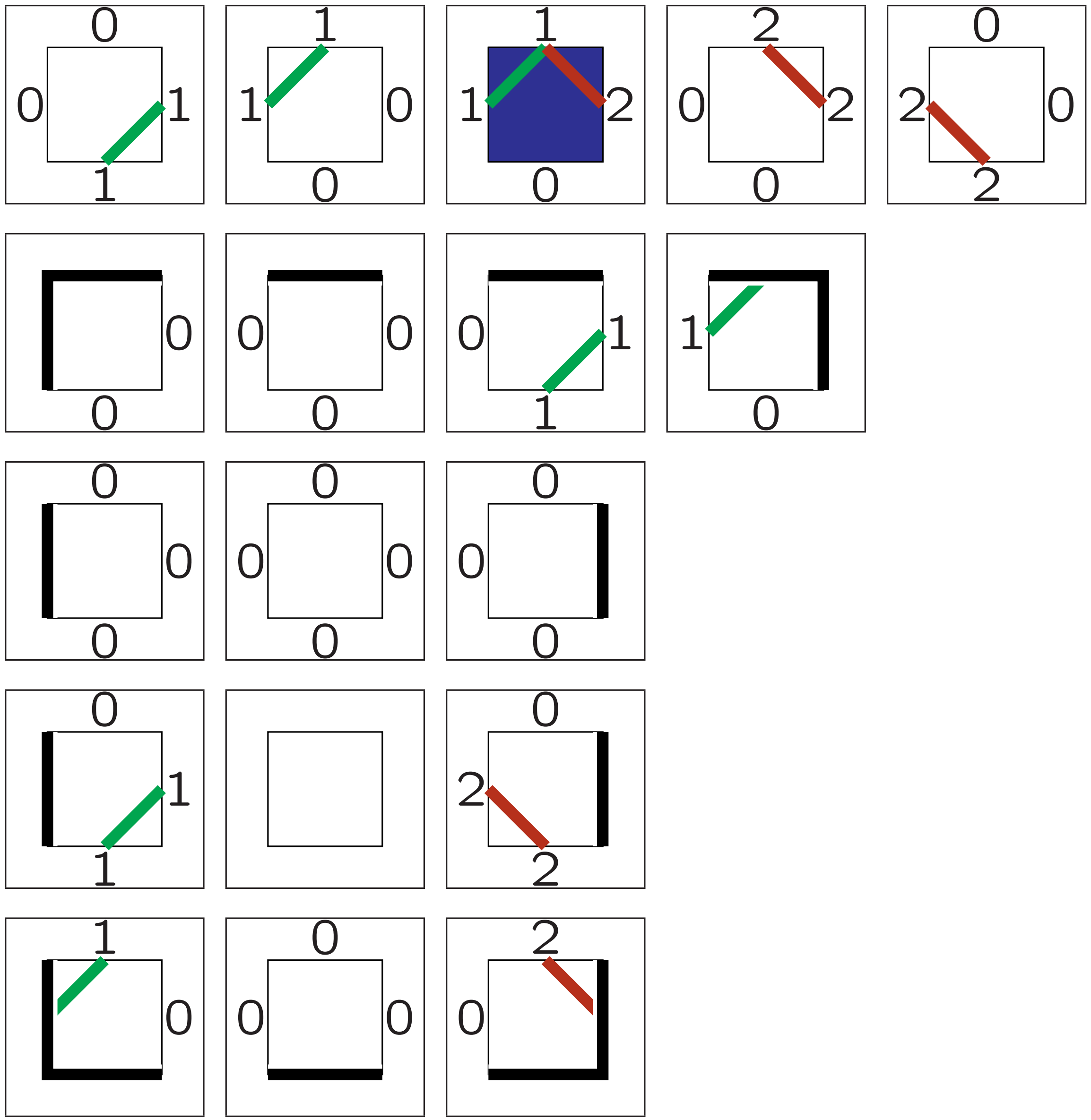








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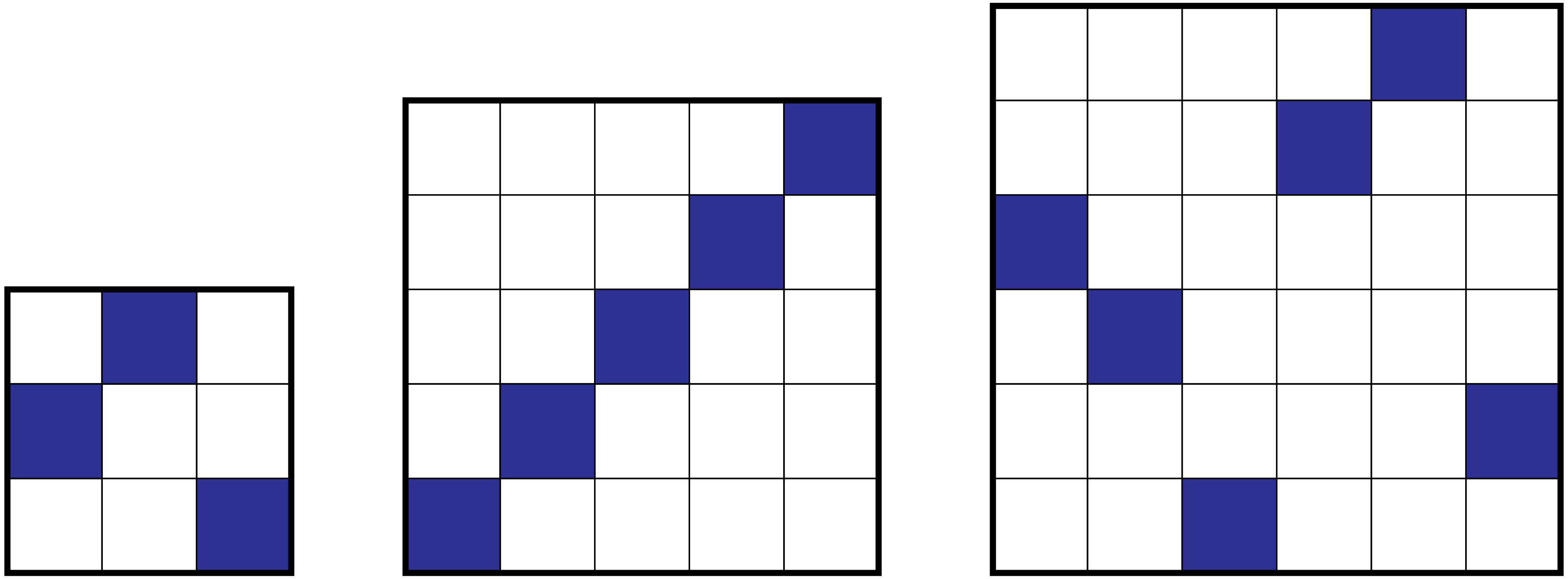

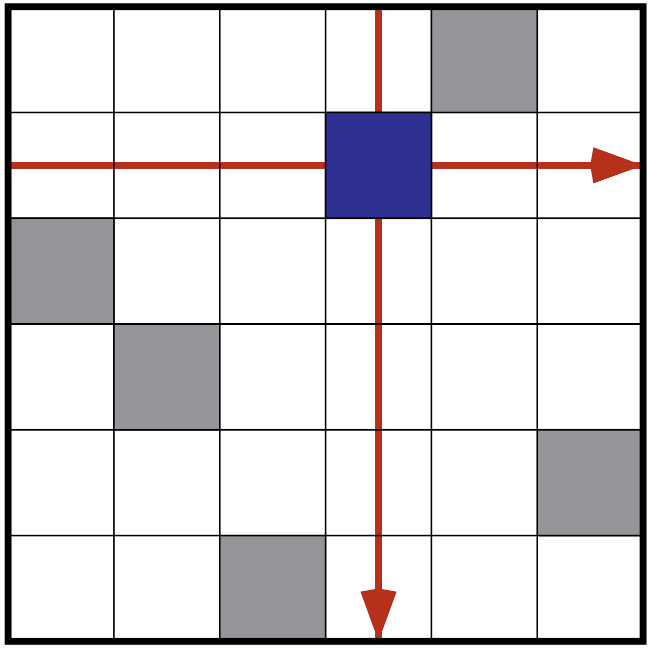

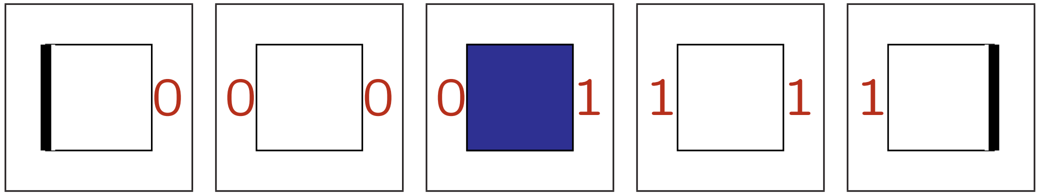
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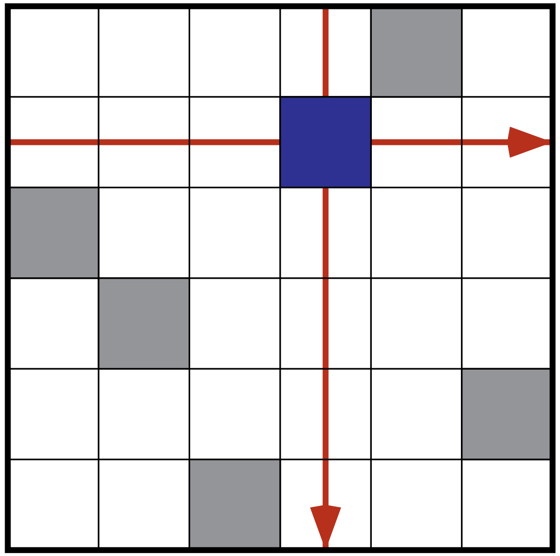

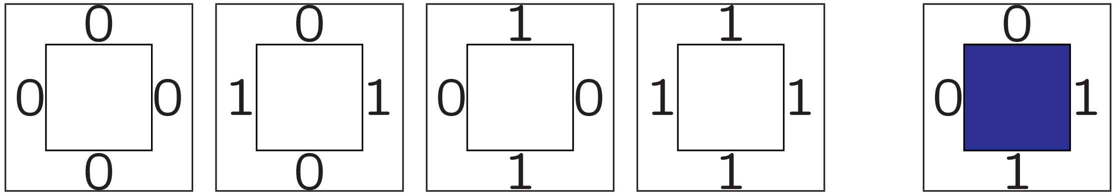

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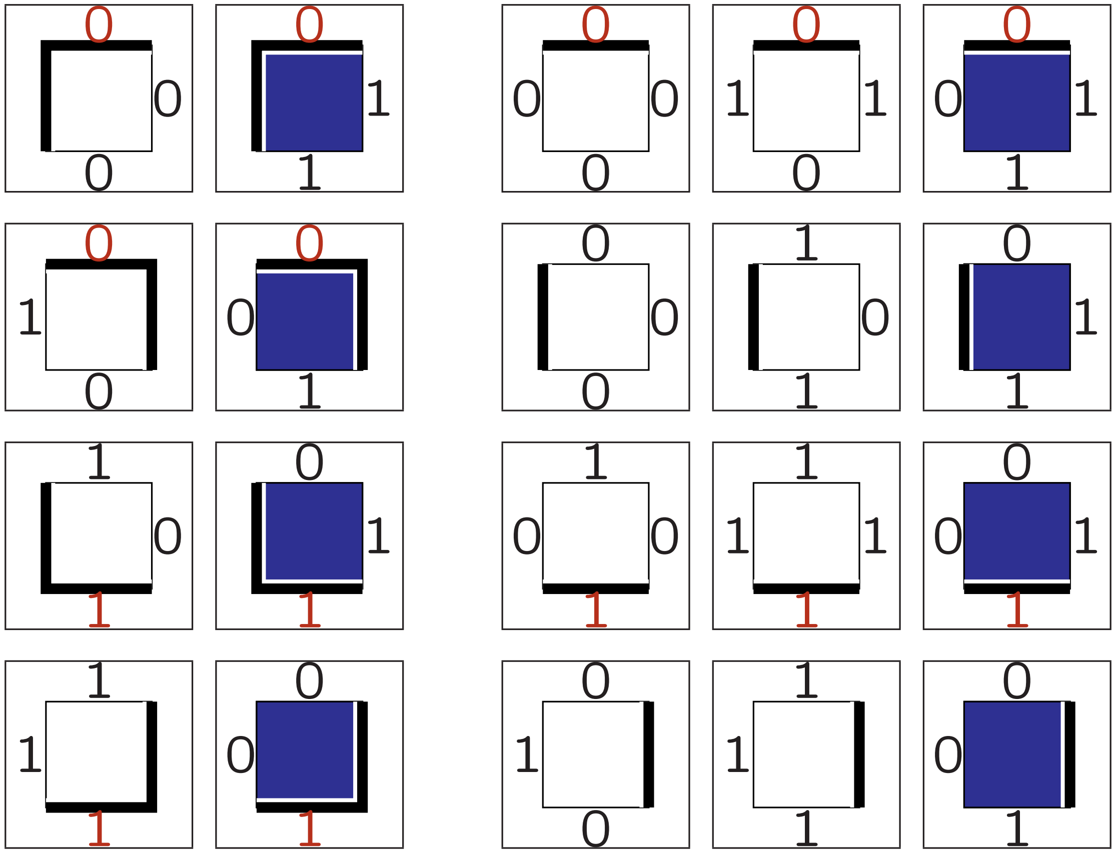
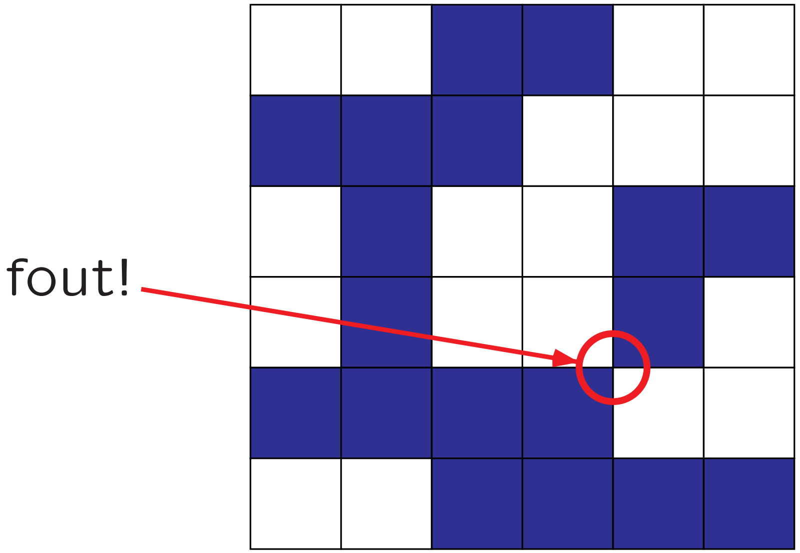
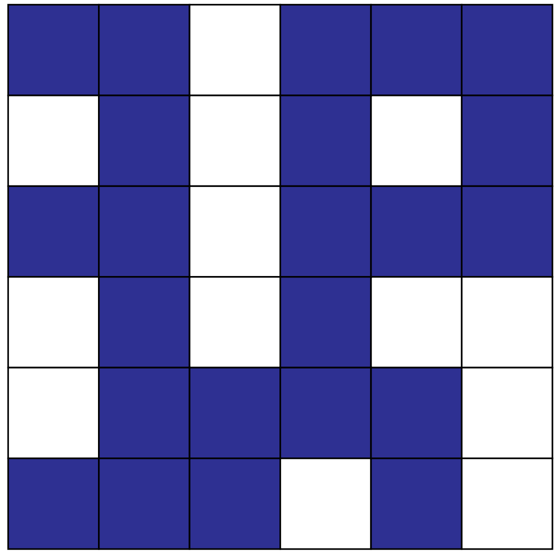
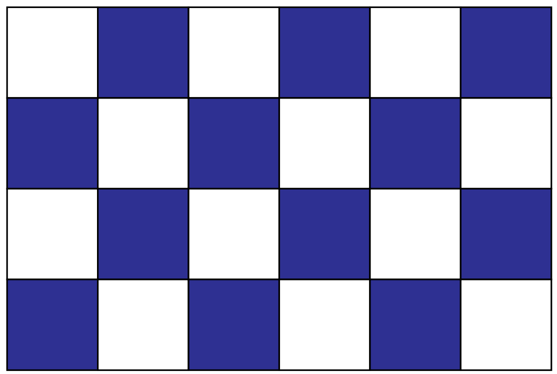


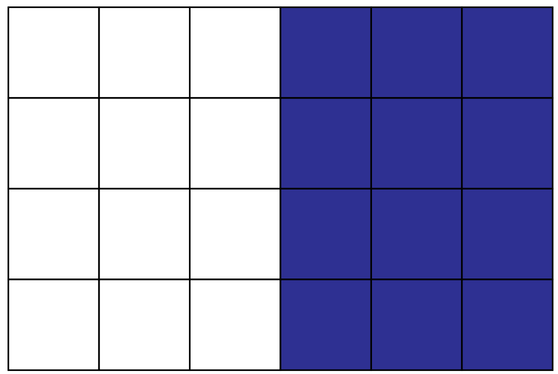
- alle blauwe tegels zijn verbonden (*moeilijk*)



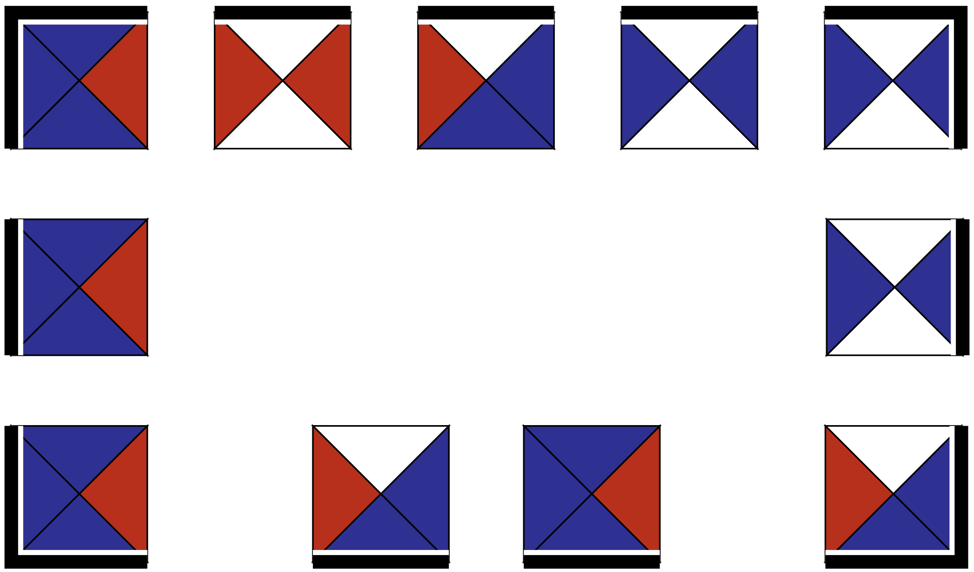


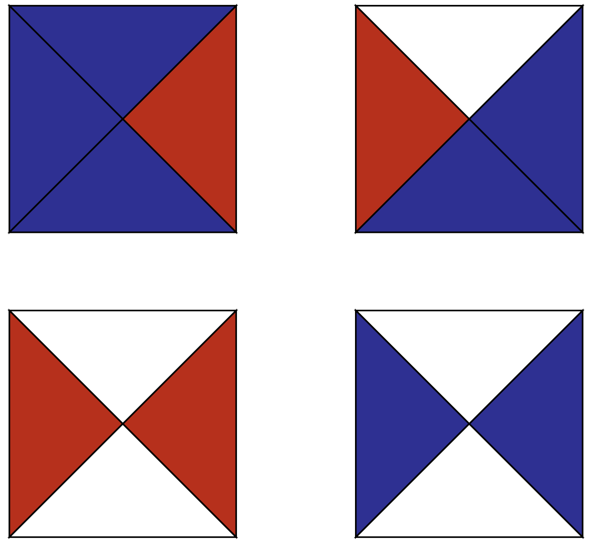
- evenveel van beide kleuren (*heel moeilijk*)



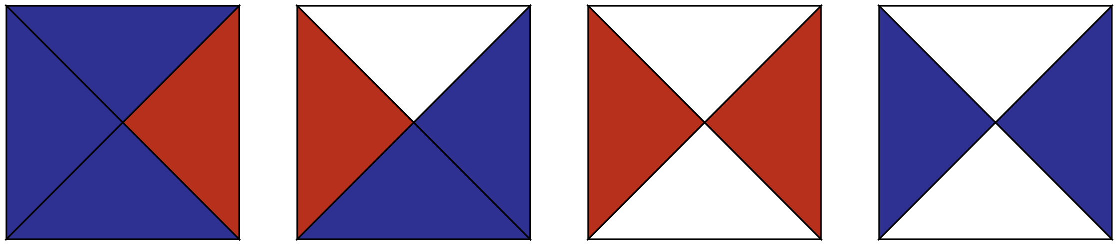


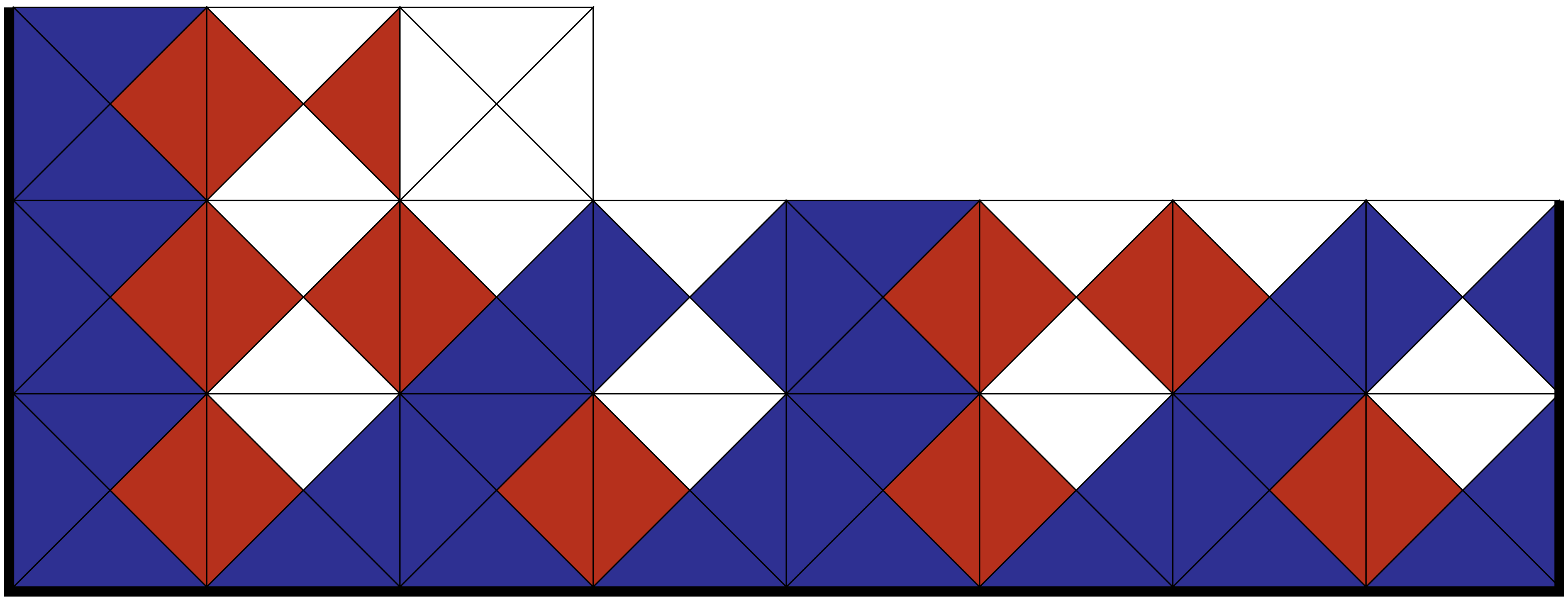
■ Rekenen

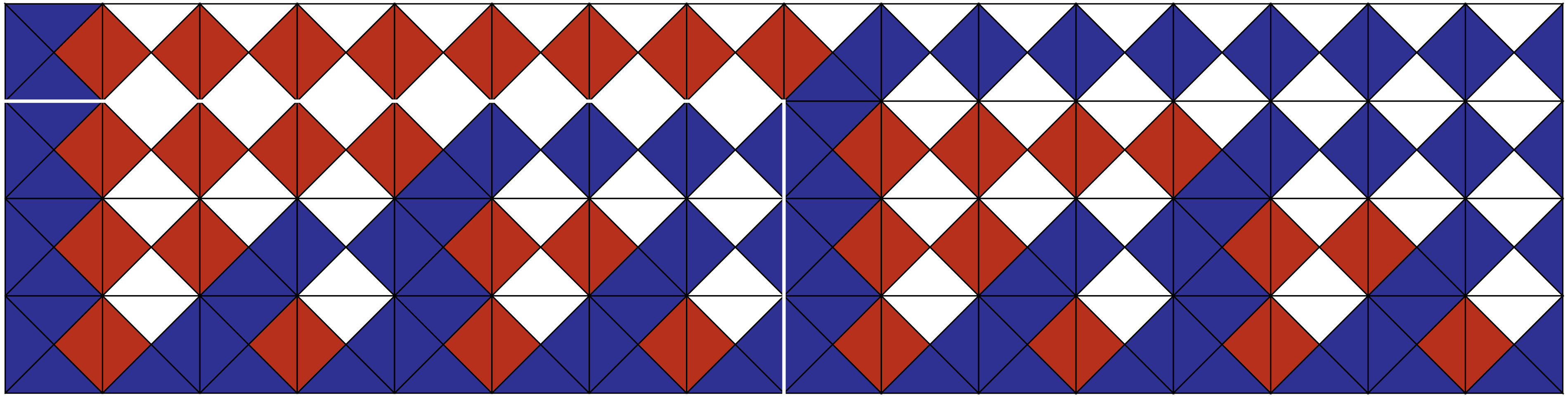


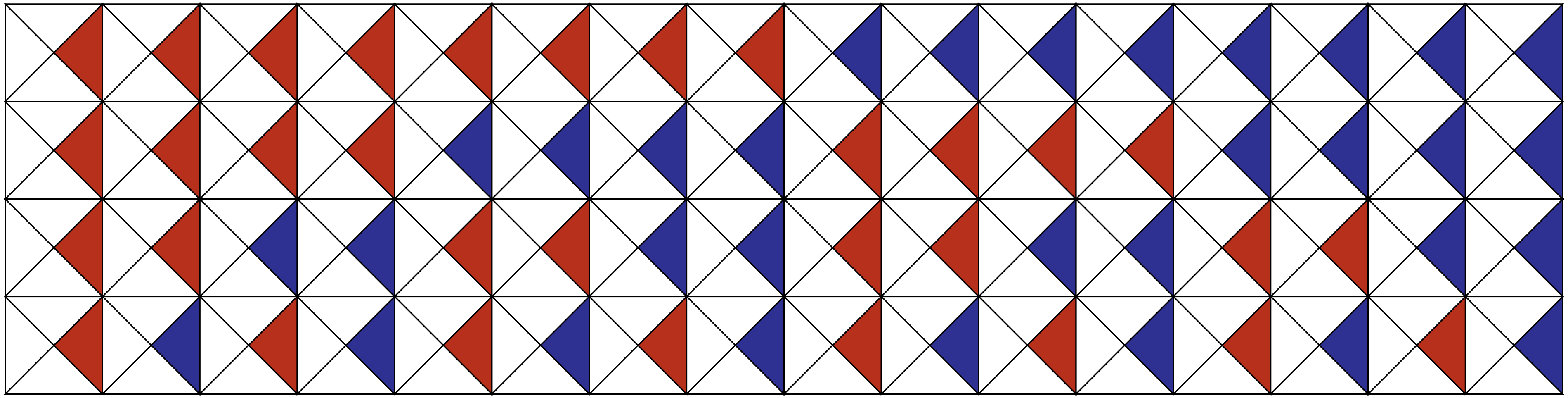


rechthoeken:
8×3 & 16×4
begin onderaan

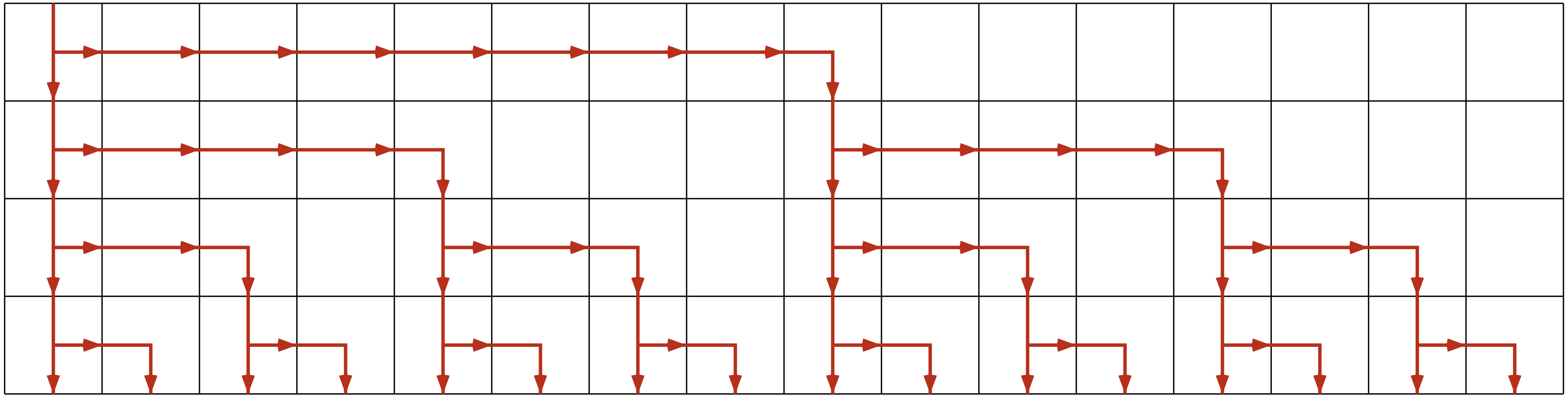






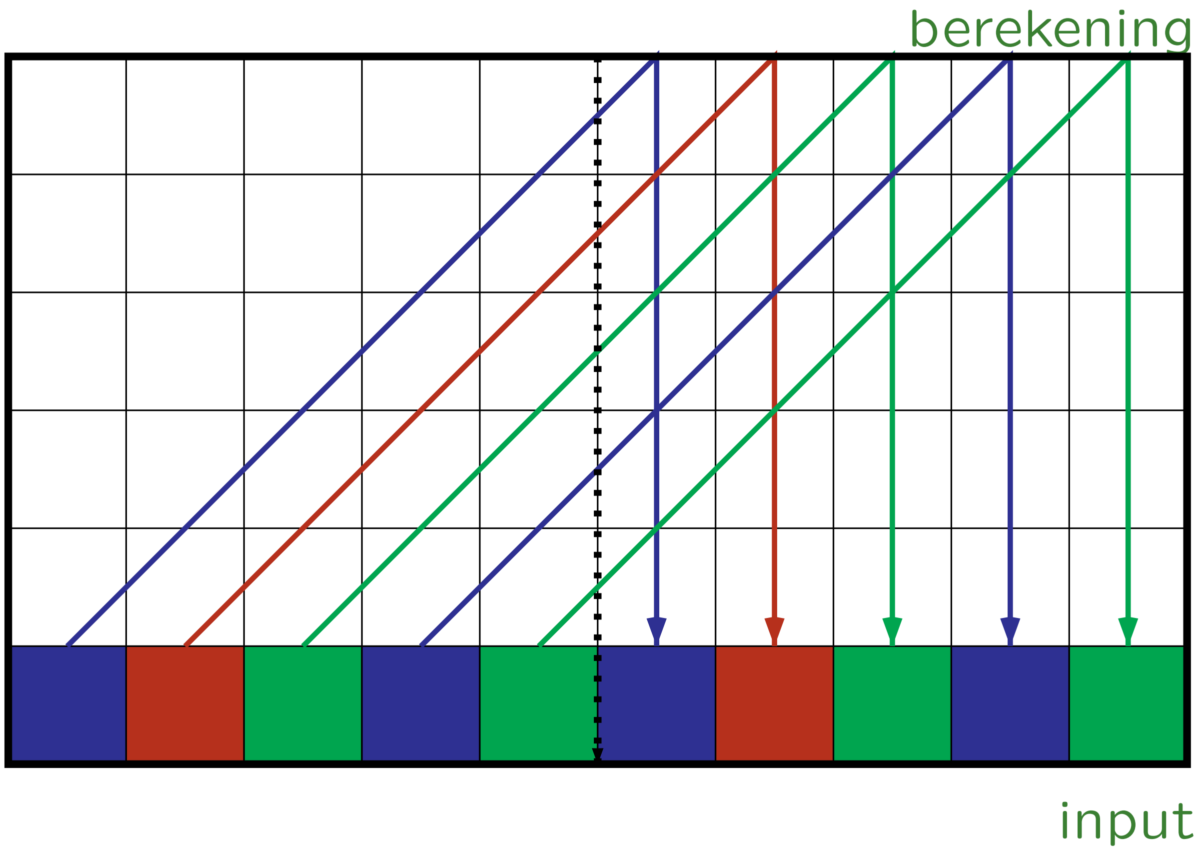


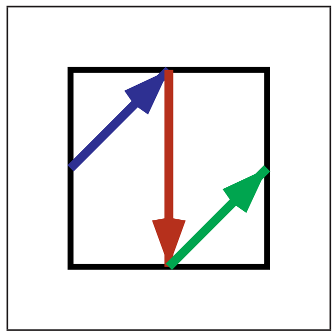
output



input (aantal hokjes)

machten van twee

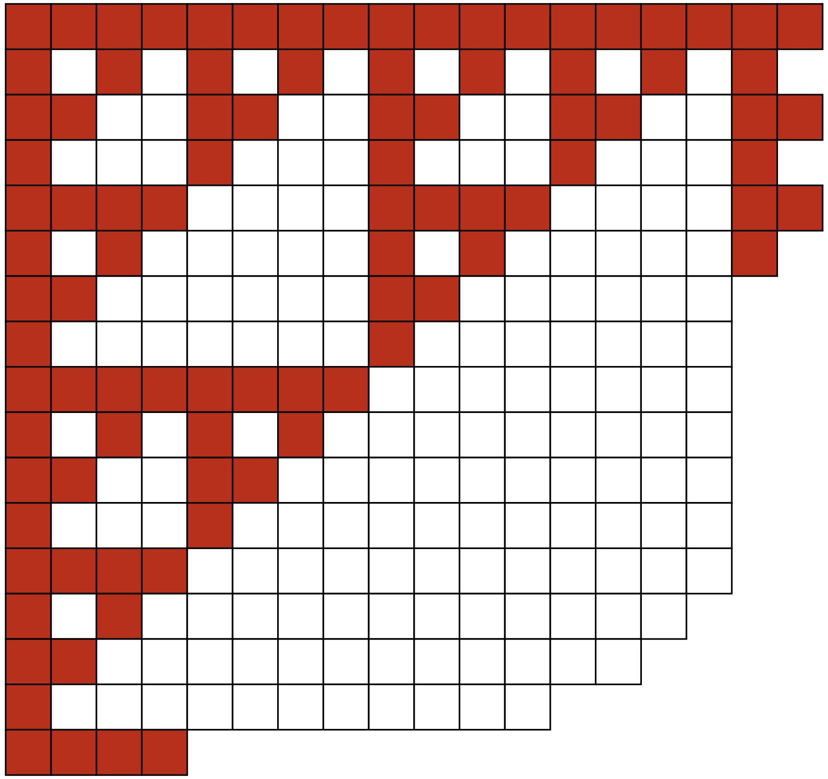


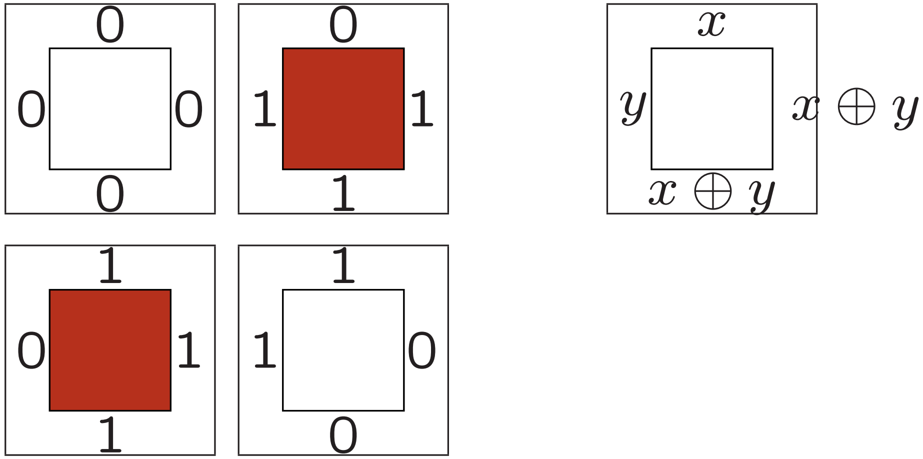


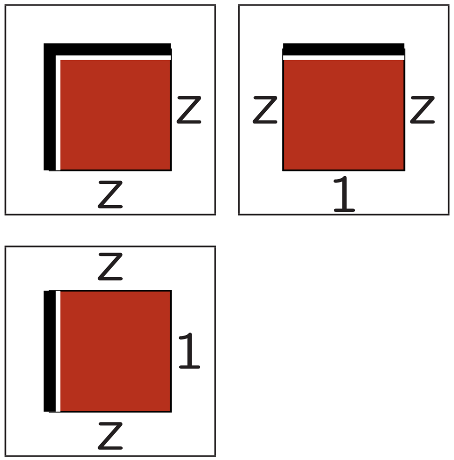
(maximaal) drie
kleuren per tegel

met tegels kun je rekenen

■ Self-Assembly







x	y	$x \oplus y$
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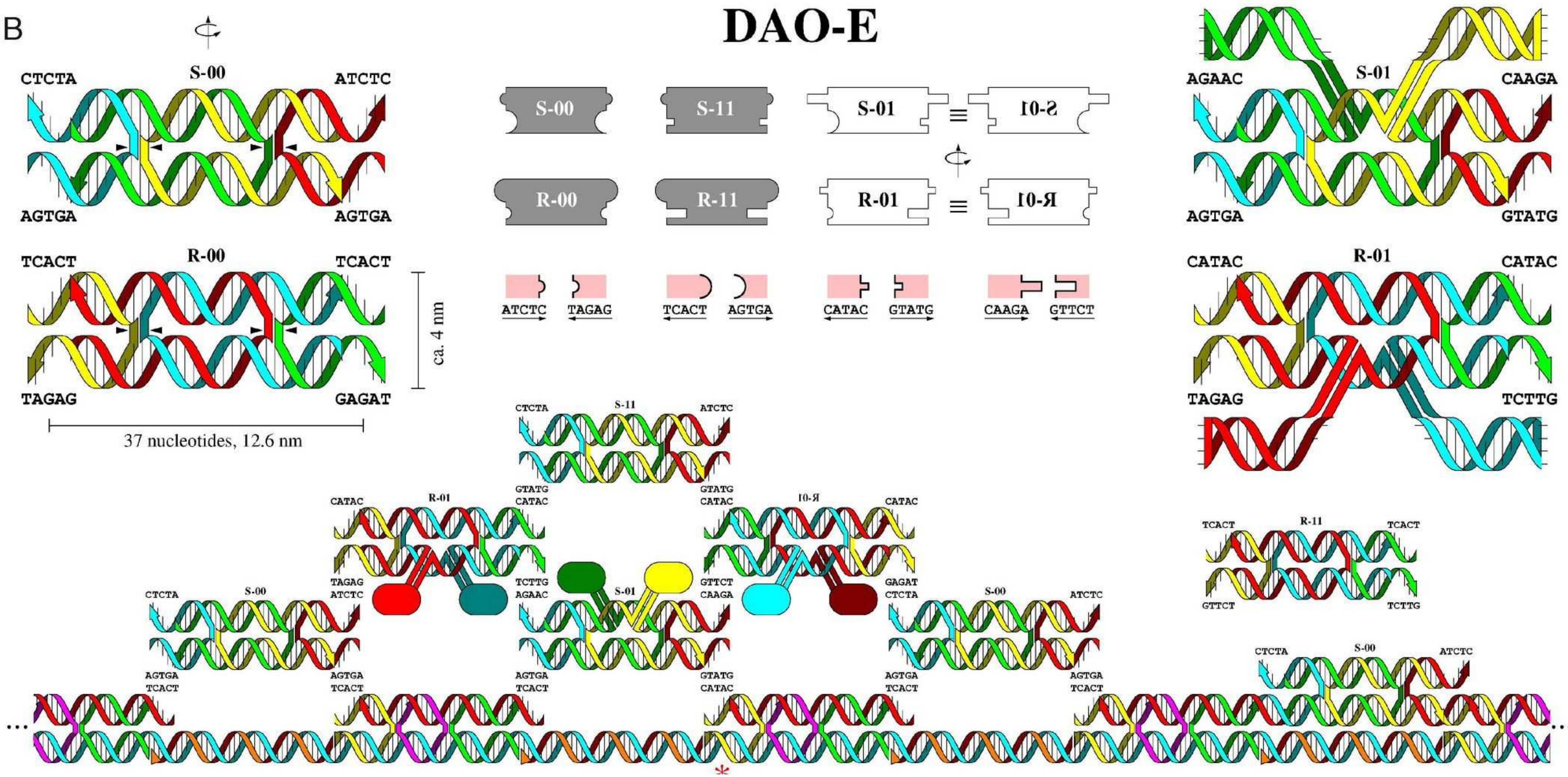
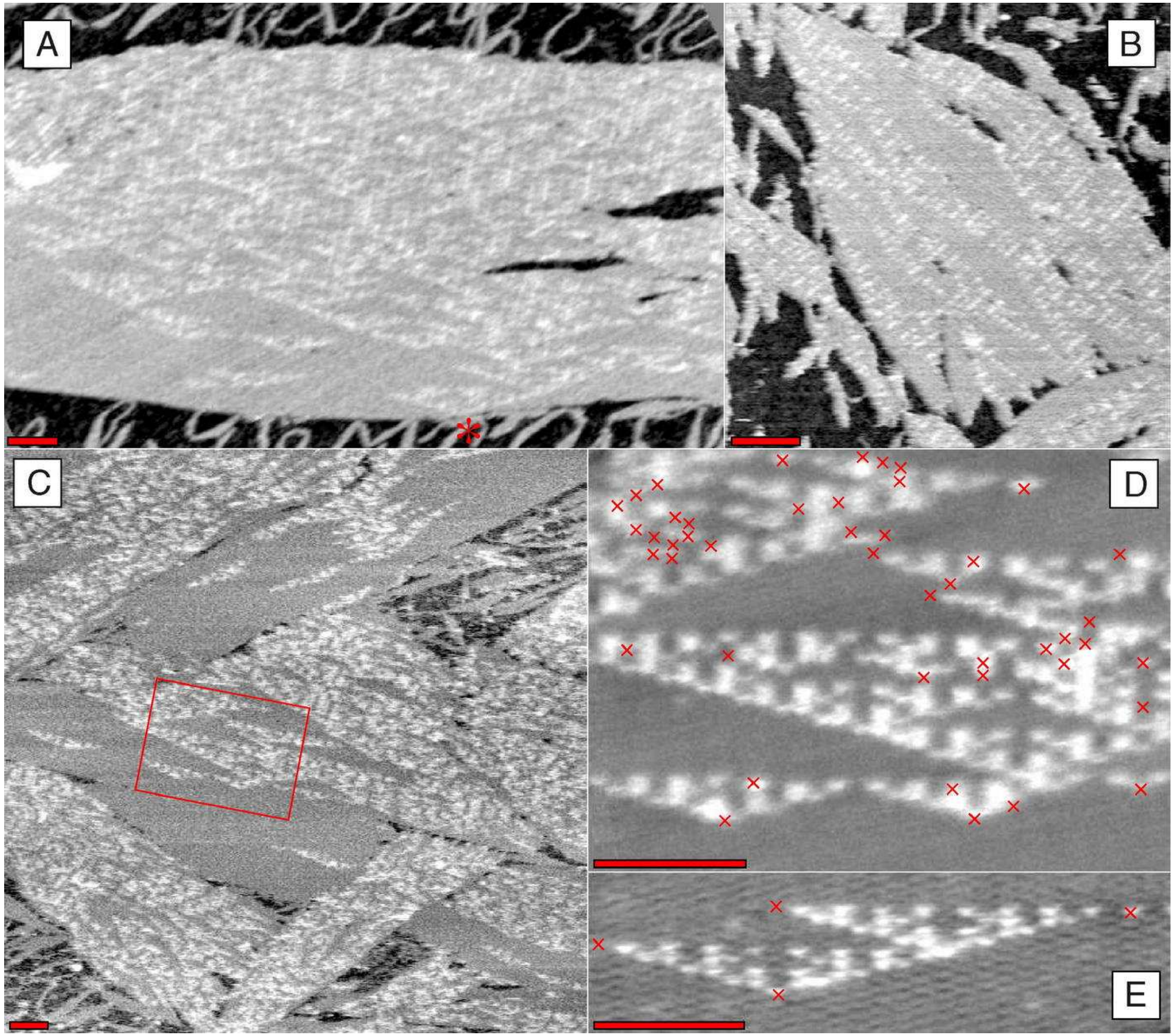


Algorithmic Self-Assembly of DNA Sierpinski Triangles (2004)

Rothemund, Papadakis, Winfree; PLoS Biology


A
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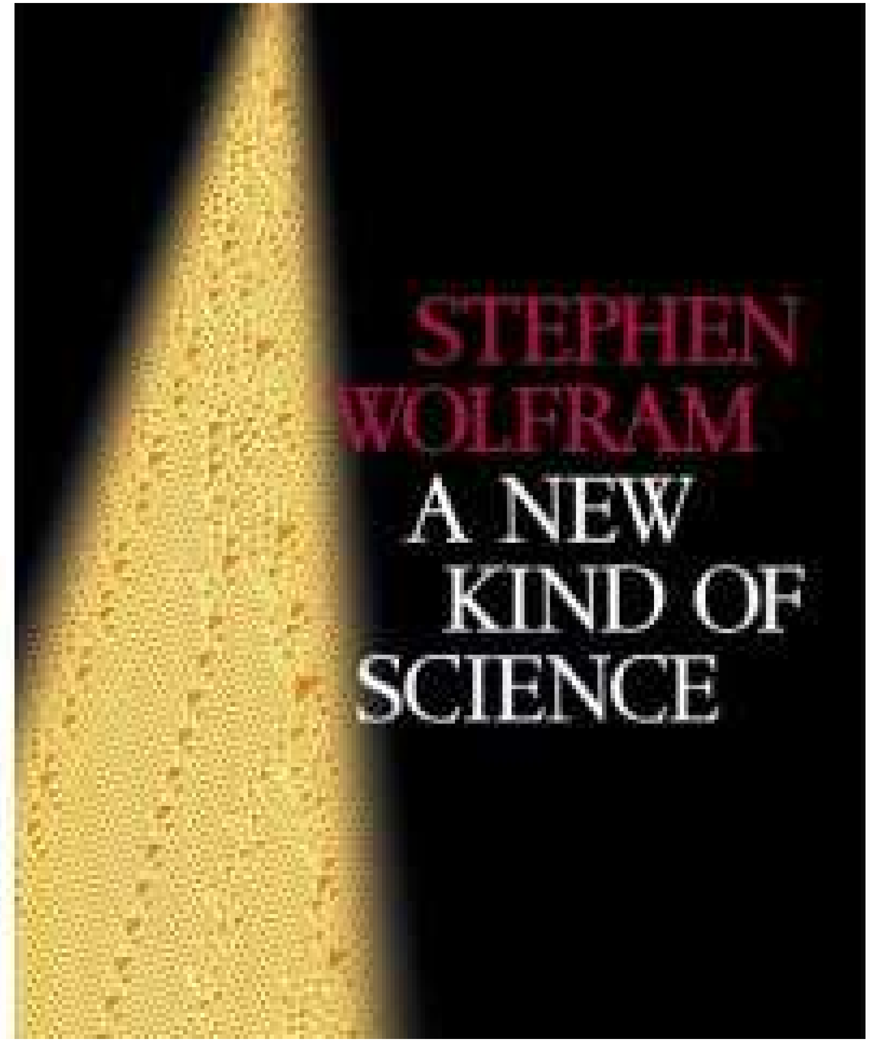
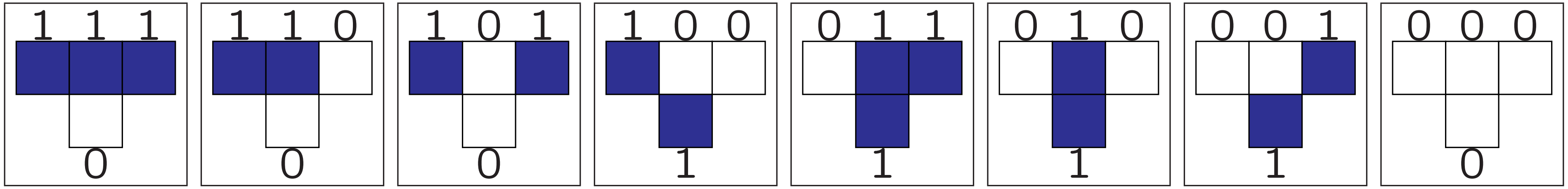
■ Cellulaire Automaten

zie ook: *collection of reviews*



Stephen Wolfram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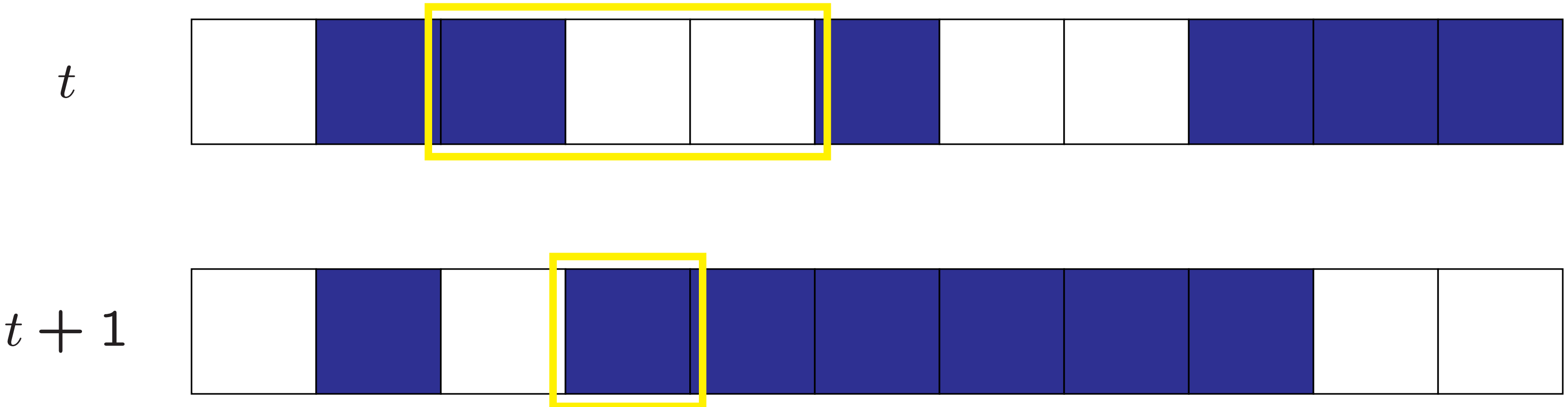
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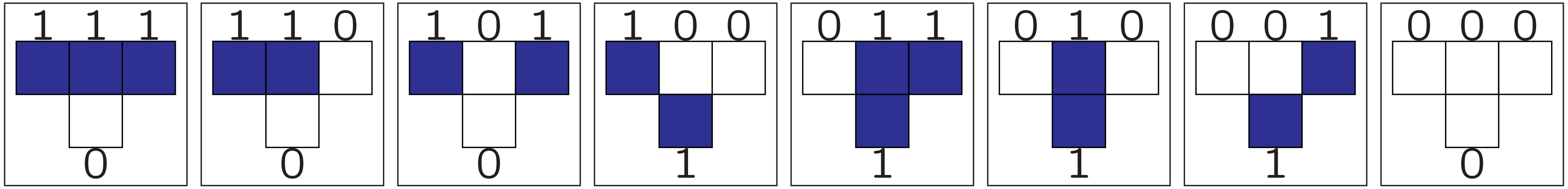
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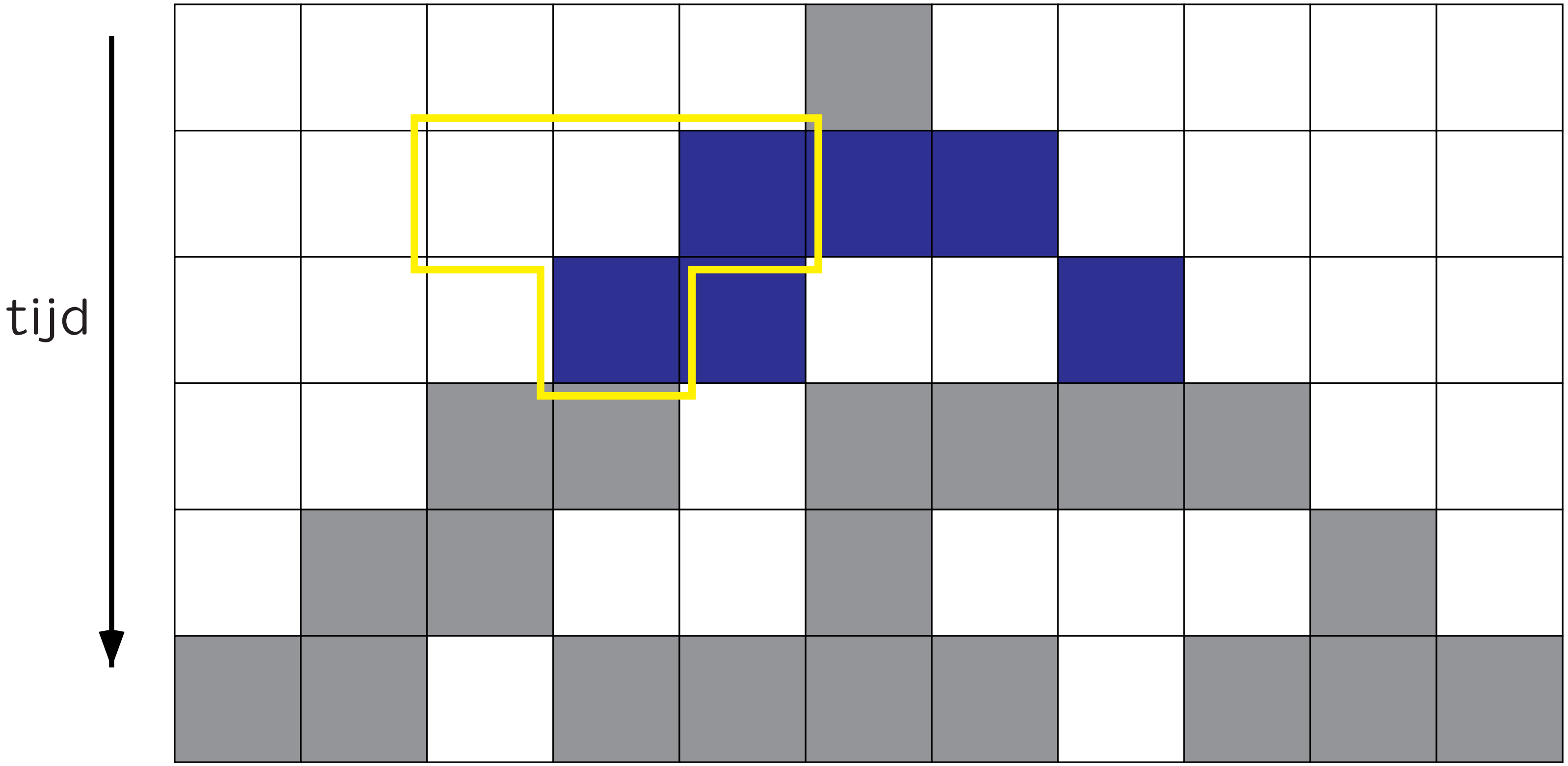
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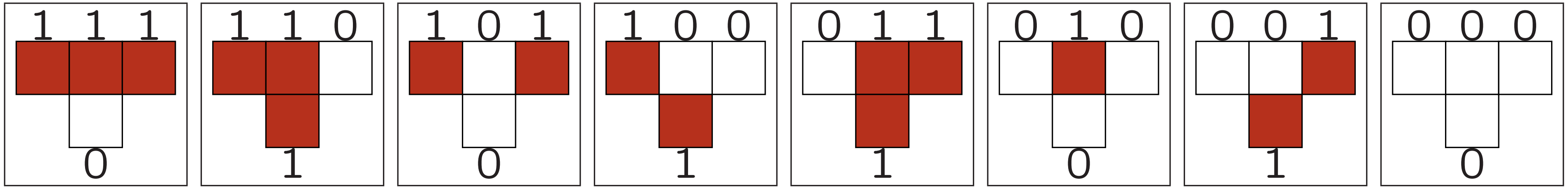
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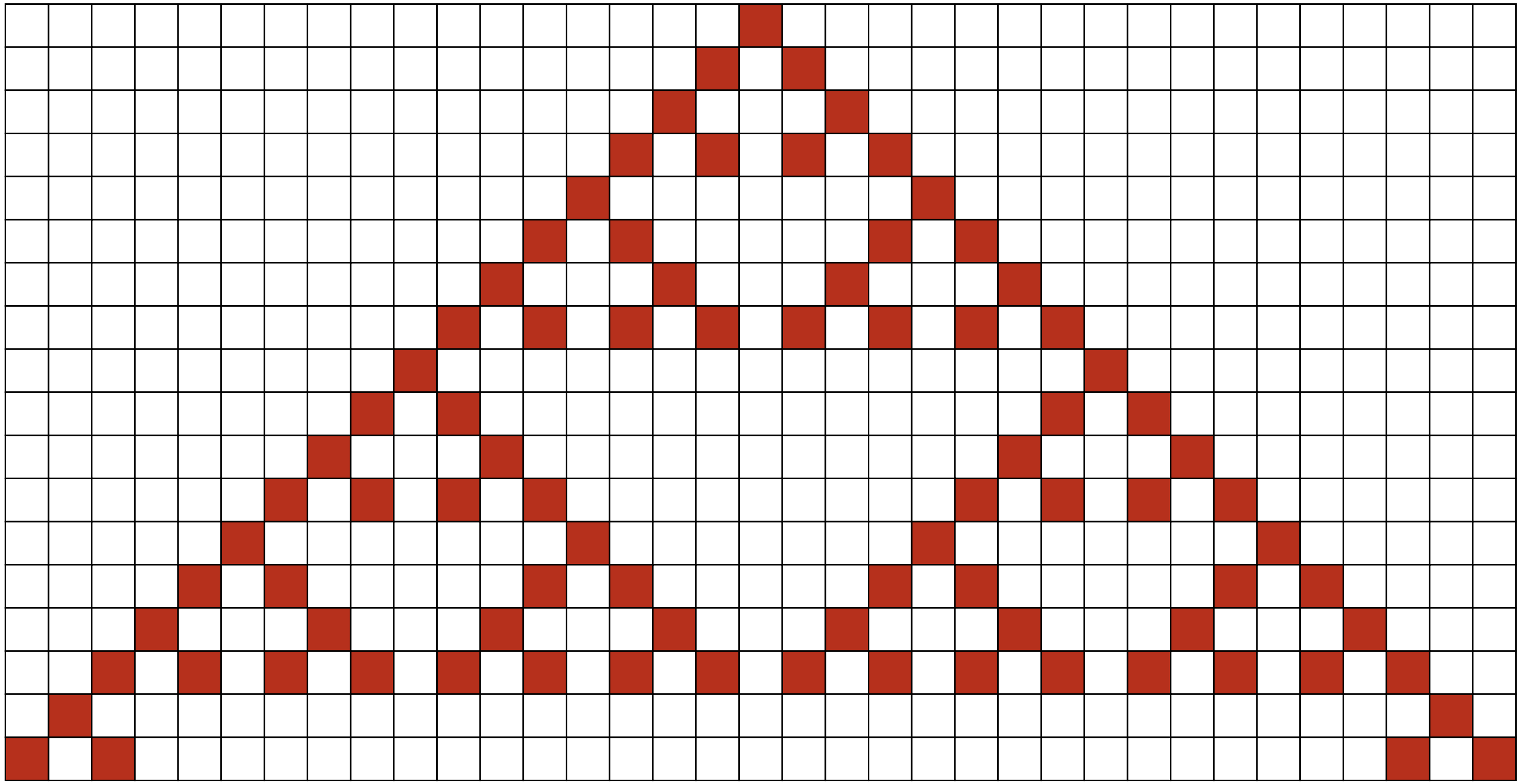

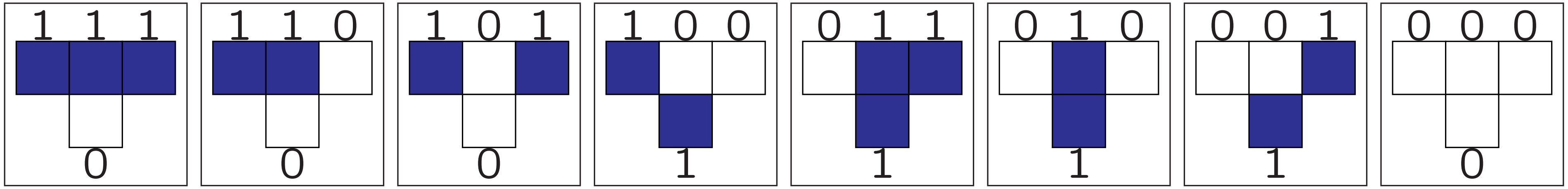
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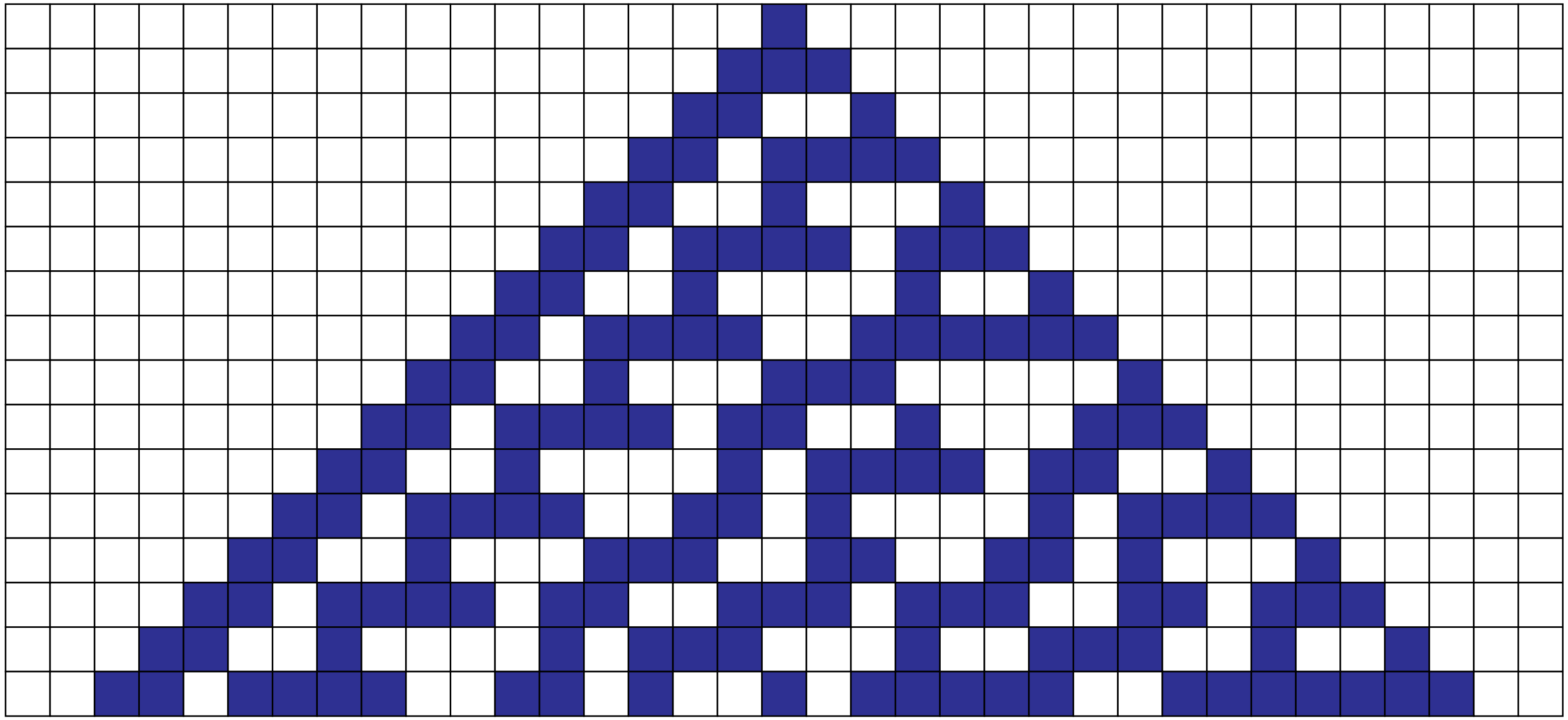

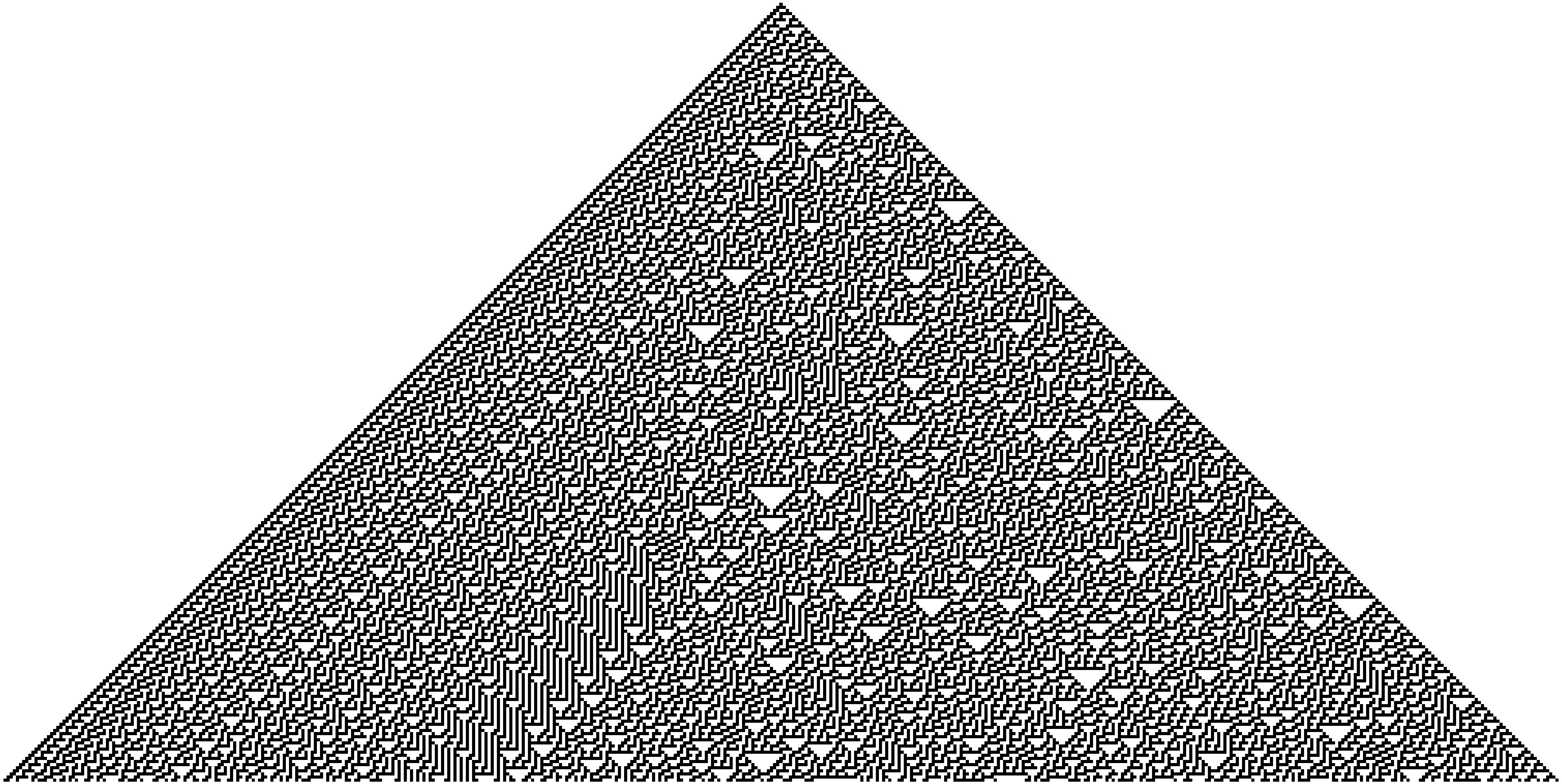

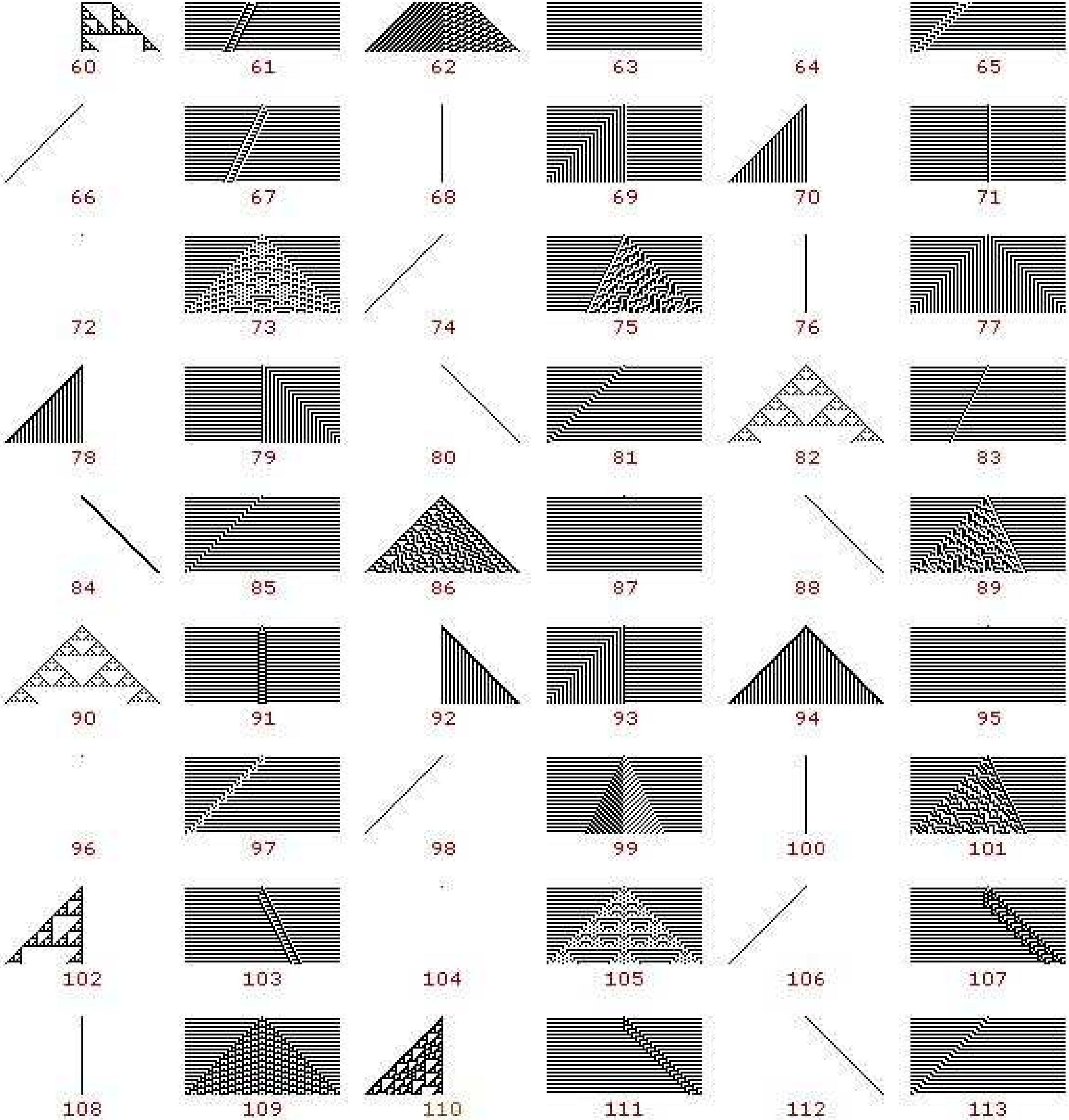
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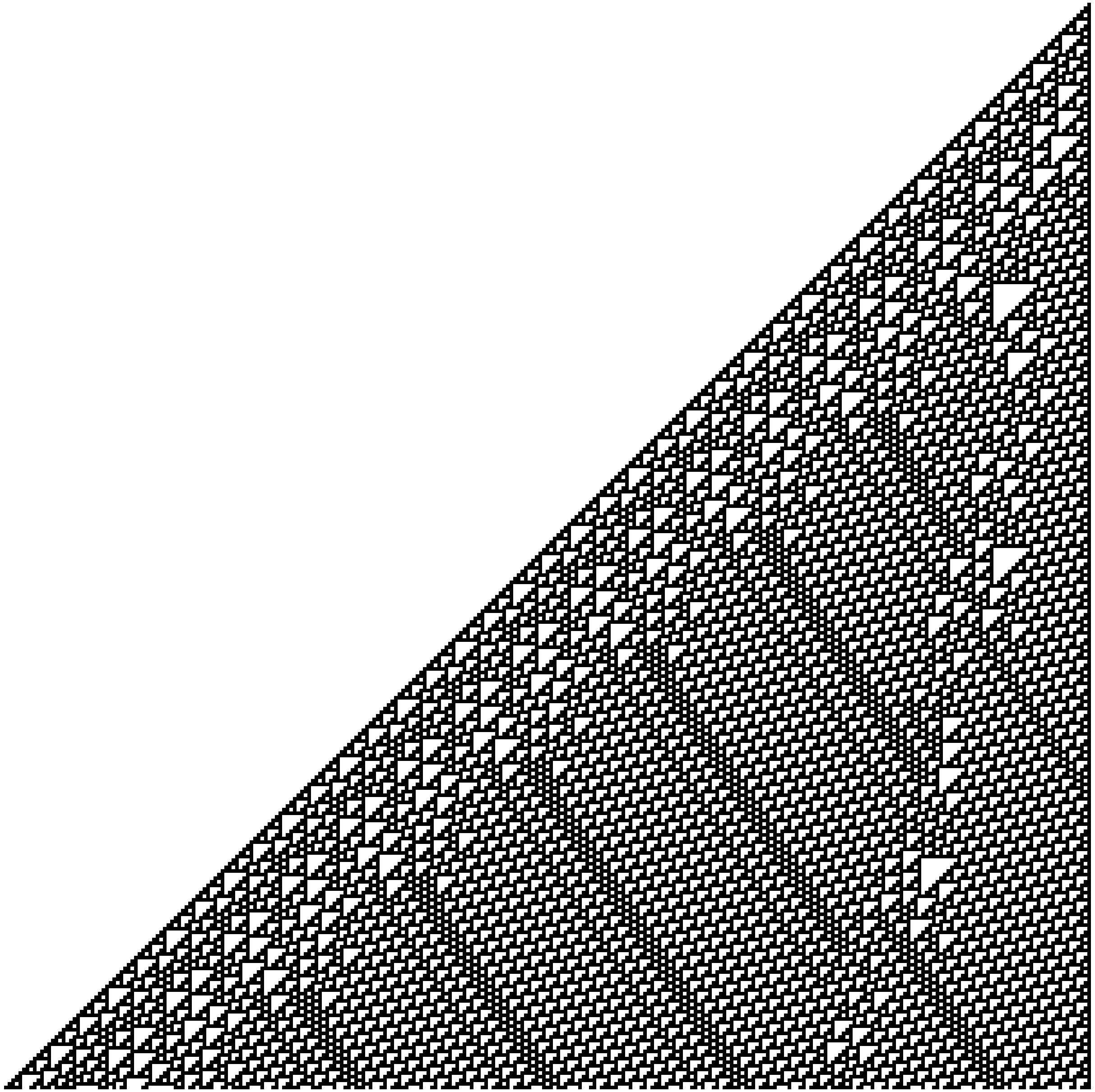
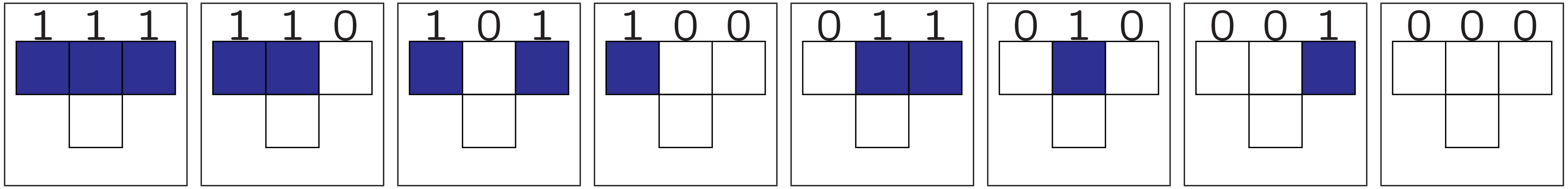


en.wikipedia.org

Conway's game of life
2-dim cellulaire automaat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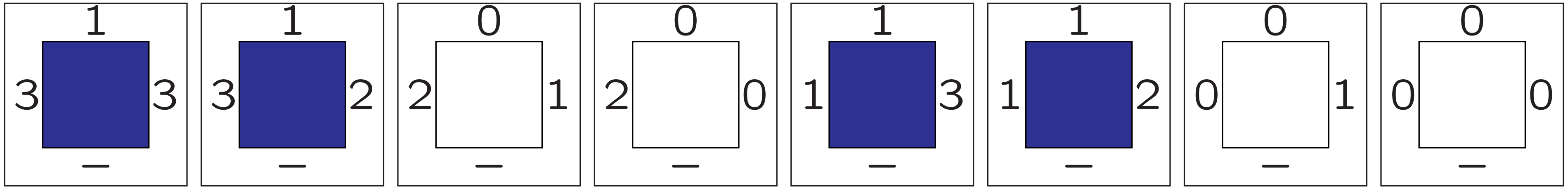
1 1 0
1 0 1
1 0 0
0 1 1
0 1 0
0 0 1
0 0 0


1
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1
3 2
0
2 1
0
2 0
1
1 3
1
1 2
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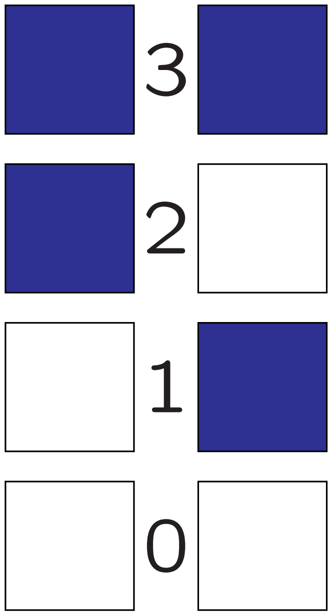
0 1
0
0 0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2
1
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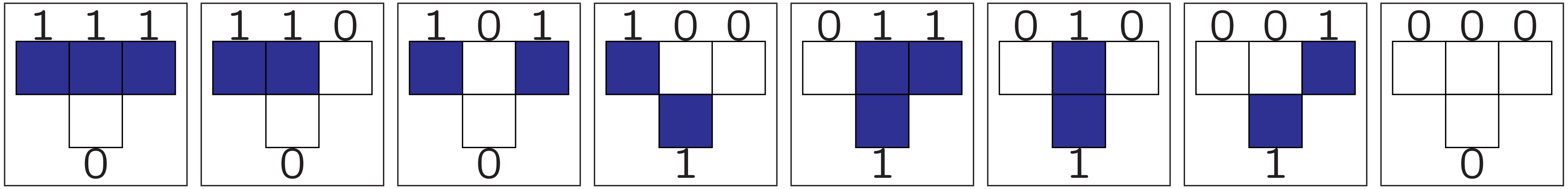
1 1 1
0
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1
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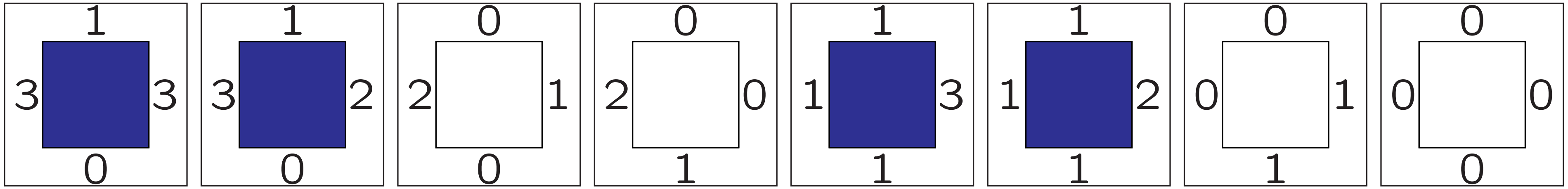
0 0 0
0


1
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3 2
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1 3
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0 1
1
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0 0
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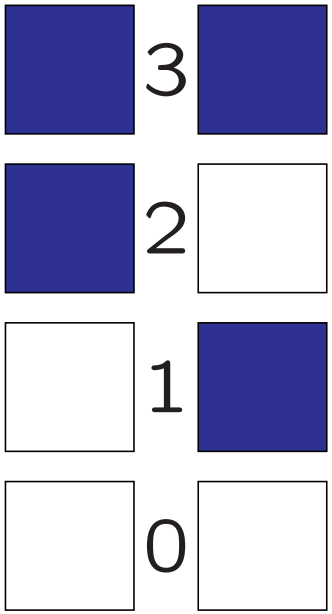
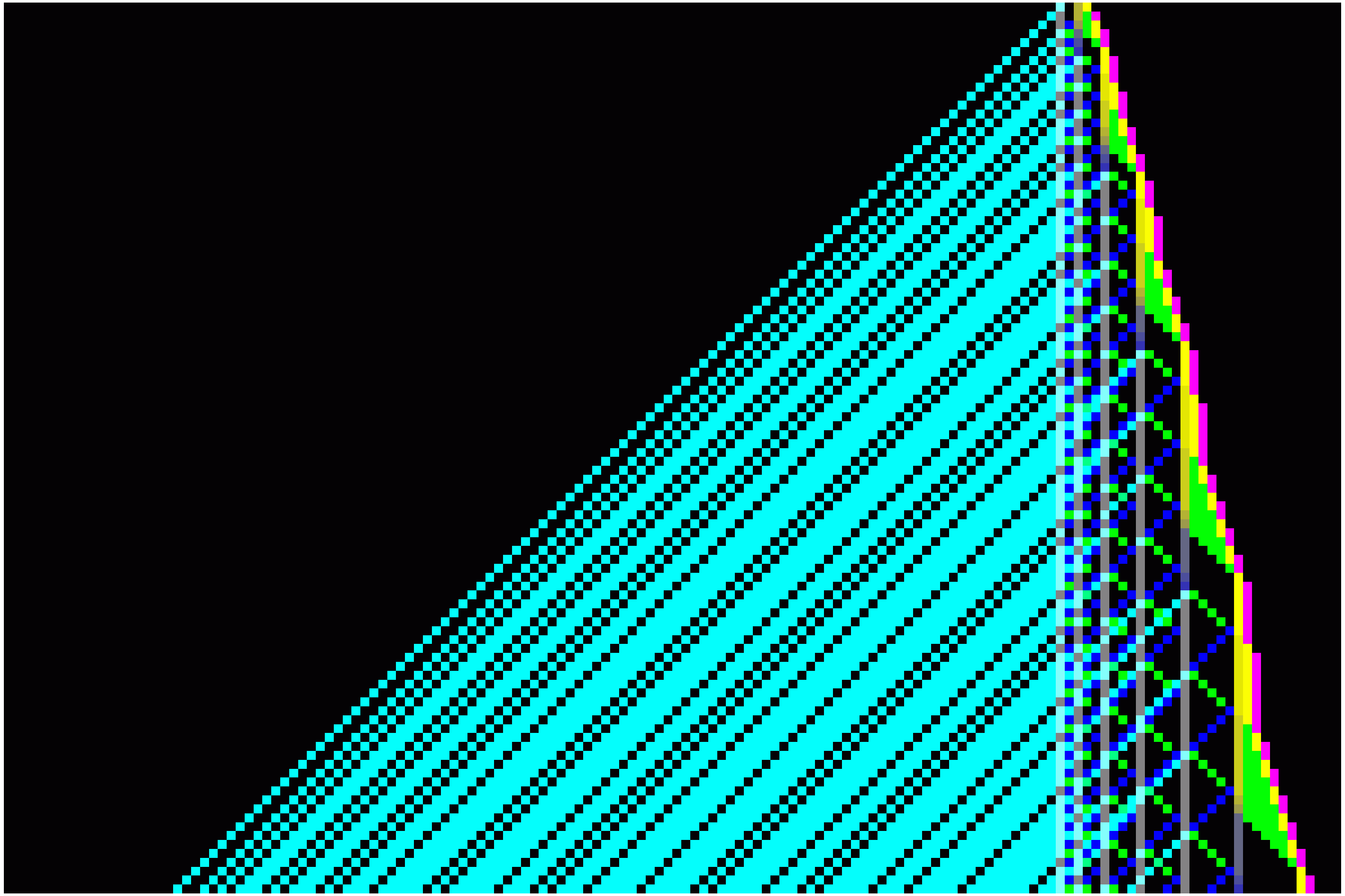

3
2
1
0



complexe patronen bij simpele regels

Turing en zijn Machine

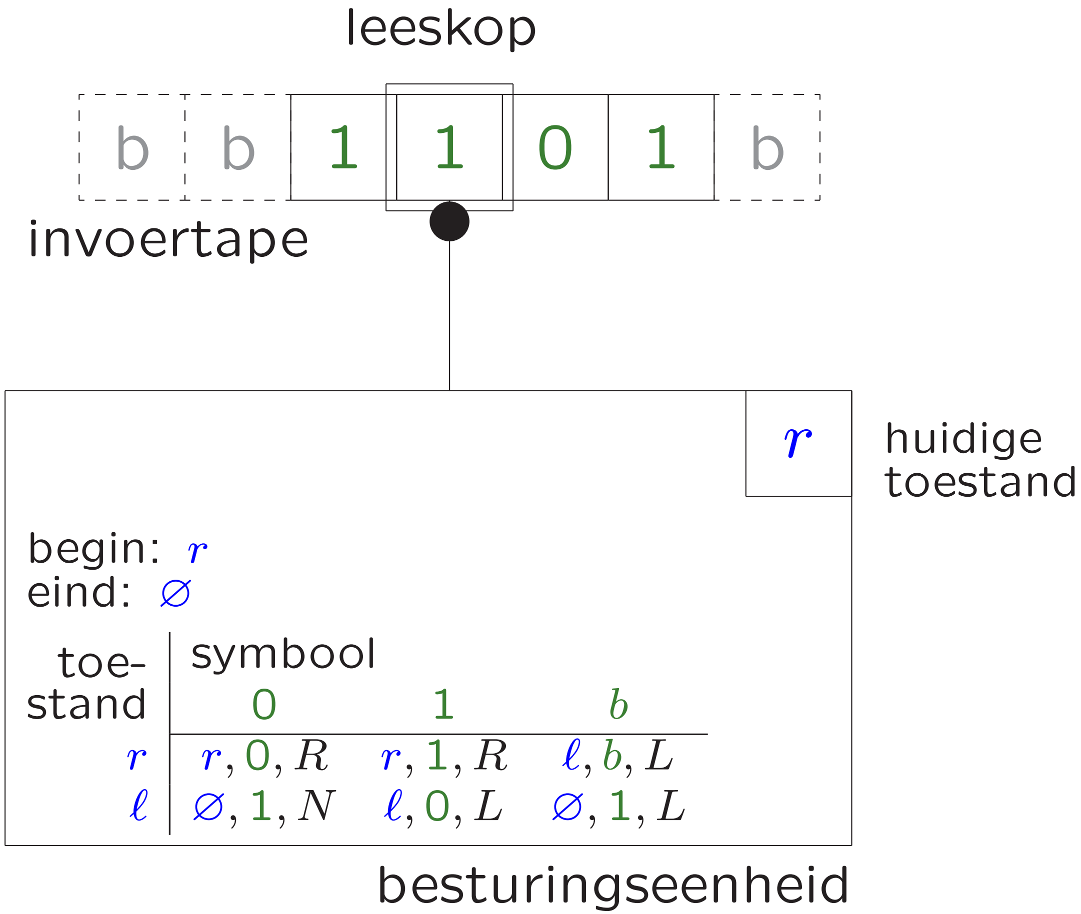
Alan Mathison Turing,
FRS OBE, 1912 – 1954

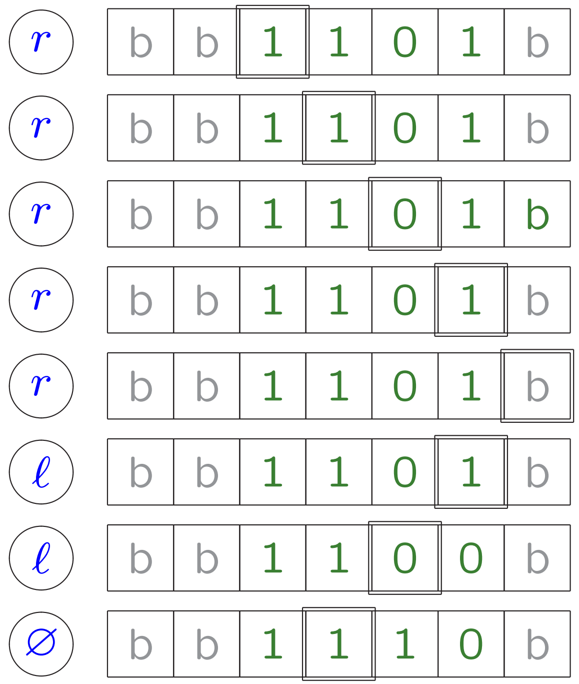
computability
wát kunnen we berekenen?
Turingmachine

Enigma
breaking the code

artificial intelligence
Turing test

morfogenese
Biologische patroonvorming

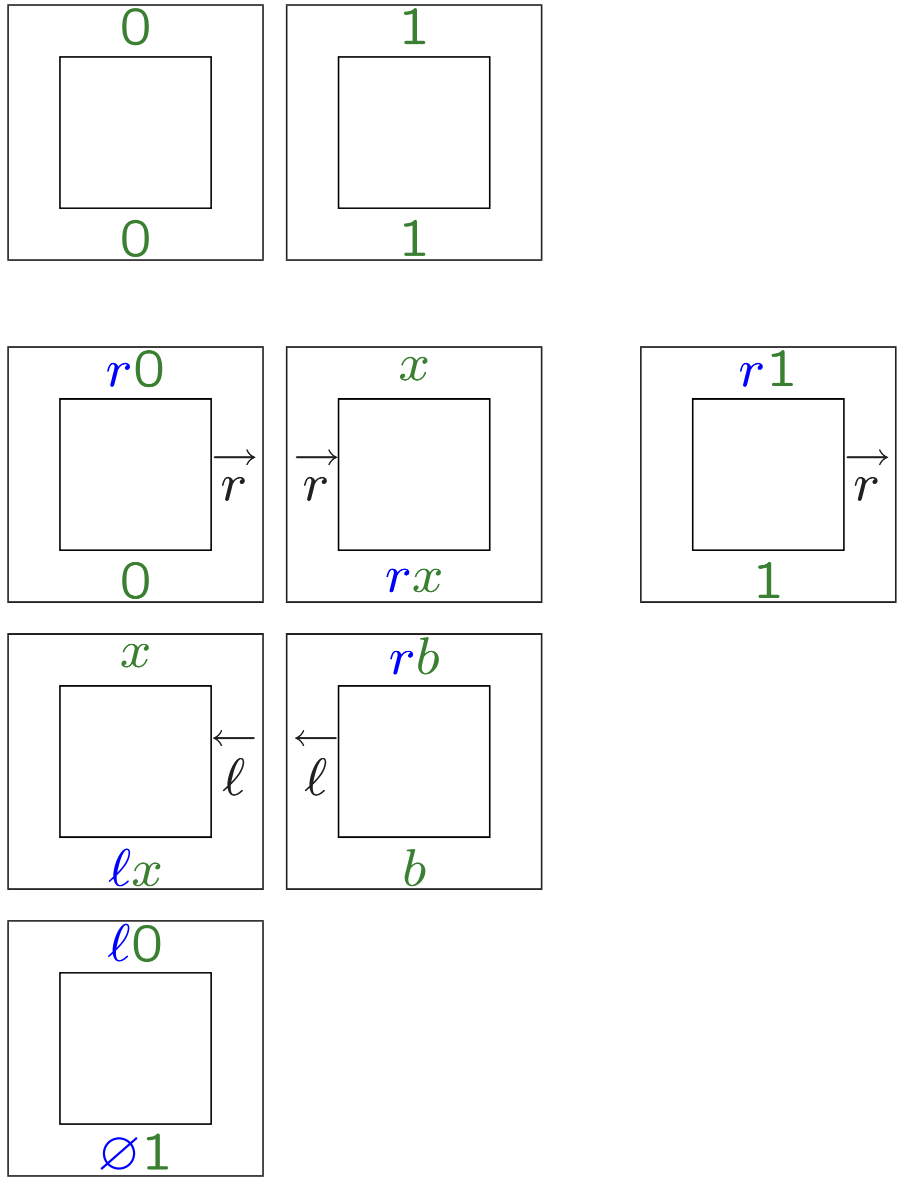


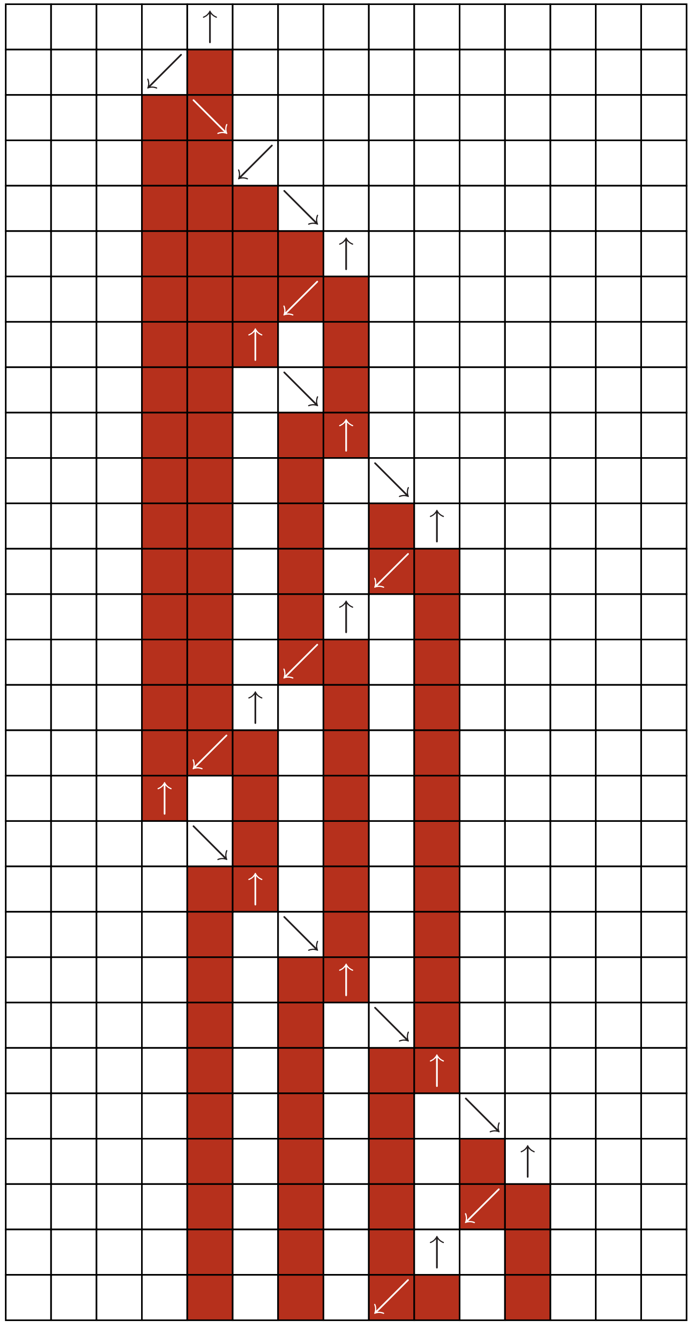


Turing, A. M. *On Computable Numbers, with an Application to the Entscheidungsproblem.* Proc. London Math. Soc. Ser. 2 42, 230-265, 1937.

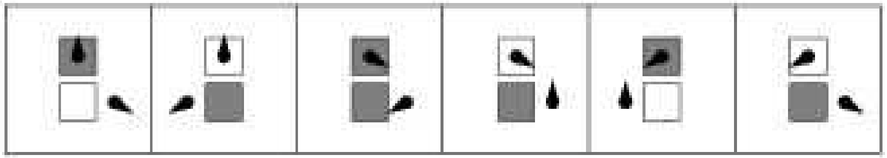
r0	1	0	1	1	B
0	r1	0	1	1	B
0	1	r0	1	1	B
0	1	0	r1	1	B
0	1	0	1	r1	B
0	1	0	1	1	rB
0	1	0	1	ℓ1	B
0	1	0	ℓ1	0	B
0	1	ℓ0	0	0	B
0	1	1	0	0	B

	0	1	b
r	$r, 0, R$	$r, 1, R$	ℓ, b, L
ℓ	$\varnothing, 1, N$	$\ell, 0, L$	$\varnothing, 1, L$

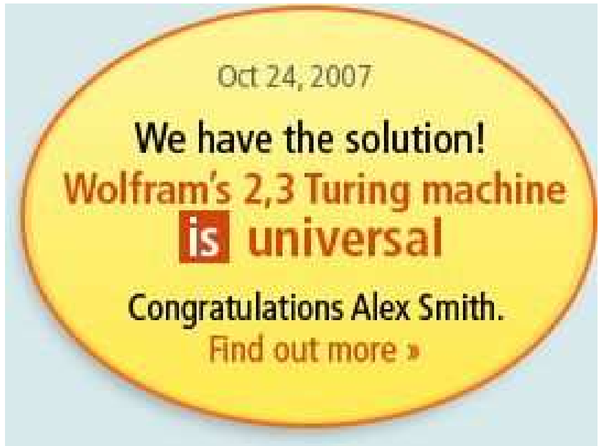




3 toestanden,
2 symbolen (kleuren)

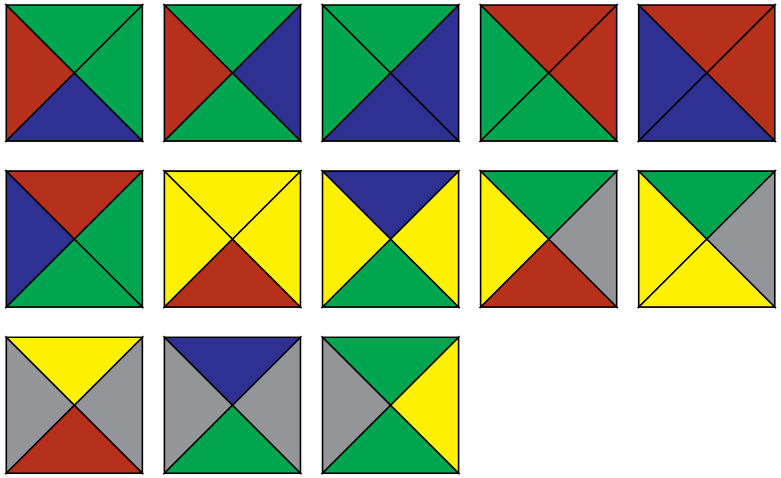


	0	1
↑	↙, 1, L	↘, 0, R
↙	↘, 1, R	↑, 0, L
↘	↑, 1, R	↙, 1, R



tegels zijn programmeerbaar

Wang tiles, 1961



patroon zonder regelmaat (lastig)

Karel Culik II, 1996

invoer: verzameling tegels

gevraagd: bestaat er een passende betegeling van het vlak (van een rechthoek) ?

er is géén algoritme dat dit probleem oplost

Berger 1966

(echt niet!)

tegels zijn onvoorspelbaar